Unsteady Axisymmetric Flow Over a Submerged Sink

Ming Xue & Dick K.P. Yue

Department of Ocean Engineering
Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.

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1 Introduction

We consider the incompressible irrotational axisymmetric flow caused by a point sink beneath a free surface. This is a problem of some scientific interest and considerable engineering importance and has been the subject of a number of investigations in recent years.

Largely because of analytic and computational simplification, most of the studies are for the two-dimensional case and assume steady flow (Peregrine 1972; Vanden-Broeck, *et al* 1978; Tuck & Vanden-Broeck 1984; Sahin & Magnuson 1984; Hocking 1985; Collings 1986; Vanden-Broeck & Keller 1987; Hocking 1988; King & Bloor 1988; Mekias & Vanden-Broeck 1989; Mekias & Vanden-Broeck 1991; Mekias & Vanden-Broeck 1993). A notable exception is Tyvand (1992) who focussed on the *initial* evolution of the free surface using a small time expansion. Arguing that nonlinear free-surface effects are exactly cancelled by gravitational effects for a particular Froude number, he found the critical value of $\mathcal{F} \equiv Q/2\pi\sqrt{gh^3} = 1/3$ for the formation of a center dip. (Here Q is the flux rate of the sink, h its submergence with respect to the far-field/initial free surface, and g the gravitational acceleration.) This value is appreciably lower than the upper limit of 1.42 of Hocking & Forbes (1991) based on a steady-state analysis, which, as pointed by Tyvand, shows that the unsteady problem offers new insight into its steady counterpart.

Investigations of the three-dimensional problem are fewer and more recent with the exception of experiments (*e.g.*, Lubin & Springer 1967) which are, of course, somewhat easier to achieve. The salient feature of the observations is the formation of a dip on the surface above a critical Froude number. Assuming steady state and a stagnation point at the surface above the sink, Forbes & Hocking (1990) used a boundary-integral-equation (BIE) computation as well as a small-Froude-number expansion analysis to show that such solutions exist for small Froude number, in this case $\mathcal{F} \equiv Q/4\pi\sqrt{gh^5} < 0.509$. This, however, does not rule out unsteady (steady) solutions below (above) this value. Zhou & Graebel (1990) performed numerical simulations of drainage from a cylindrical basin using a nonlinear axisymmetric BIE method. Their results of the unsteady problem showed two different phenomena depending on the drain rate. For relatively large Q, a dip forms at centre of the free surface which is rapidly drawn into the drain. For smaller Q, an upward jet is formed. In that work, however, the Froude number was defined with respect to the tank radius and the precise dependence of their results on $\mathcal{F}$ is unclear. Recently, Miloh & Tyvand (1992) extended the small-time perturbation analysis of Tyvand (1992) to axisymmetric flow and identified the corresponding critical Froude number for the formation of a dip to be $\mathcal{F} = 15^{-1/2}$. This analysis depends only on the (high-order) time derivatives of the center surface elevation at small time, and its validity for the actual physical problem is unclear.

In this paper, we consider the *unsteady* axisymmetric free-surface flow around a submerged point sink which is started abruptly from rest to a constant volume rate.

2 Method of Solution

We choose time and length units such that the initial (quiescent) submerged depth h of the point sink and gravitational acceleration g are both unity. In an otherwise unbounded fluid, the problem is governed by the Froude number $\mathcal{F} = Q/4\pi\sqrt{gh^5}$, where Q is the (constant) volume flux rate of the sink which is assumed to be turned on abruptly at time $t = 0$.

We assume axisymmetry and employ the numerical technique of Dommermuth & Yue (1987) to solve the fully-nonlinear unsteady potential flow. The method is based on a mixed Eulerian-Lagrangian approach using a ring-source boundary-integral-equation (BIE) scheme for the solution of boundary-value problem at each time instant. The computational domain is closed by assuming constant finite depth H. To allow for long-time simulations, the nonlinear domain is matched to a general linear transient wavefield outside a matching cylinder of radius R. Thus, for a sufficiently large fixed R (which is a function only of the nonlinearity of the problem), the nonlinear simulations can, in principle, be continued indefinitely. For the BIE solution, the trace of the computational boundary is approximated by cubic splines over (Lagrangian) nodes. The potential and its normal derivative between adjacent nodes are represented by linear basis functions based on arclength. To maximize stability of the time integration, the nodes on the free surface are maintained at equal (arclength) spacing via a regridding procedure after each time step.

To tract the rapid cusp-like development of the free surface, the method of Dommermuth & Yue (1987) is improved in two major ways: (i) consistent fourth-order Runge-Kutta time integration is maintained by moving the surface in intermediate steps; and (ii) dynamic time stepping is used based on a Courant criterion in terms of the maximum instantaneous Lagrangian velocity. These refinements were not considered in the earlier work because of the complexities associated with the matching boundary.

3 Results

Based on extensive convergence tests for the full range of Froude numbers we consider, we choose a computational domain of $H = 6$ (to represent effectively deep water), $R = 6$, with $N = 240$ segments on the boundary. The expected error in the free-surface elevation is $O(0.2\%)$. We now perform a systematic computational search over $\mathcal{F}$ to provide a complete quantification of the problem. (Critical delineations are subsequently checked by varying the computational parameters H, R and N).

Our computational investigation reveal three distinct flow regimes depending on the Froude number $\mathcal{F}$:

1. $\mathcal{F} > 0.1930$. This 'super-critical' regime is marked by the rapid cusp-like collapse of the free surface towards the sink. The decrease of the surface elevation is everywhere monotonic in time. We are able to compute well after the cusp is developed although the simulations eventually break down as the surface approaches the sink. Figures 1a and 1b show typical results for the case of $\mathcal{F} = 0.24$. Plotted are the time evolution of the free-surface elevation directly above the sink, $\eta(r = 0, t)$, as well as an instantaneous surface profile $\eta(r, t^*)$ at a late stage $t = t^*$ of the development. Note that the critical value for (eventual) collapse based on the small-time expansion of Miloh & Tyvand (1992) of $\mathcal{F} = 15^{-1/2} \simeq 0.258$ is well within this regime.

2. $0.1930 > \mathcal{F} > 0.1924$. This 'trans-critical' regime is characterised by a sharp reversal of the free surface immediately above the sink, eventually developing into a sharp *upward* cusp. Figures 2a and 2b show typical results for $\mathcal{F}$ =0.1927. The free surface initially drops, reaches a minimum depth at the center, and then abruptly rebounds to form a sharp upward spike. This upward jet resembles that observed by Milgram (1969) for the case of the sudden deceleration of a cylindrical layer of fluid under free fall. Our simulations are able to follow the full development of the upward jet but eventually fail as the spike develops.

3. $\mathcal{F} < 0.1924$. In this 'sub-critical' regime, the flow is marked by (spatially) wave-like behavior of the free surface. The temporal evolutions are much slowly (longer time scale) than the higher Froude number cases with the surface elevation resembling that of a damped oscillator. The detailed spatial and temporal features, however, are quite varied and depends on the specific value of $\mathcal{F}$. An example result for $\mathcal{F} = 0.1$ is shown in figures 3. Because of the matching boundary, we are able to continue simulations to well beyond $O(10)$ characteristic time (limited only by computational effort). At this time, we find evidence of approach to asymptotic steady state for small r. This provides partial confirmation of Forbes & Hocking (1990)'s finding of steady state solutions for sufficiently small $\mathcal{F}$, although our critical value is substantially lower then their value of 0.509 for the existence of steady-state solutions.

It remains to resolve whether the initial starting rate of the sink would affect the present findings (Zhou & Graebel 1992). Work is ongoing on this and other issues related to the sub-critical solution and will be reported at the workshop.

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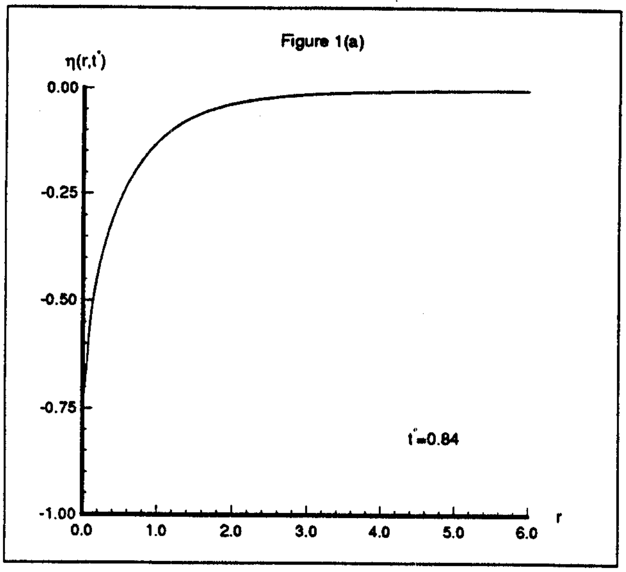
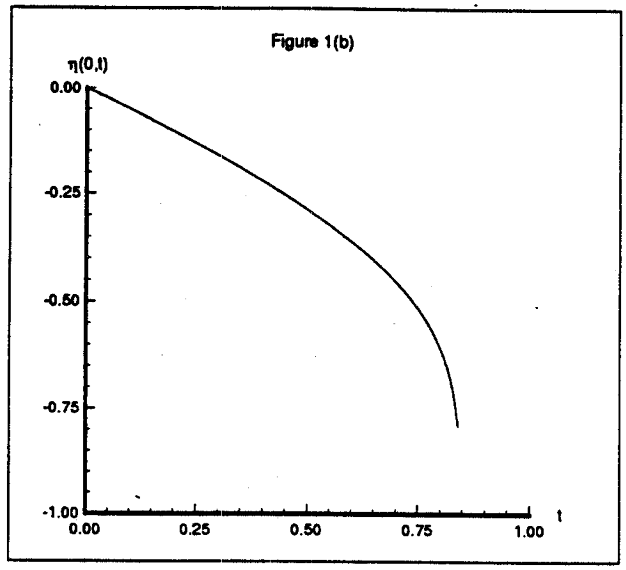


Figure 1(a)



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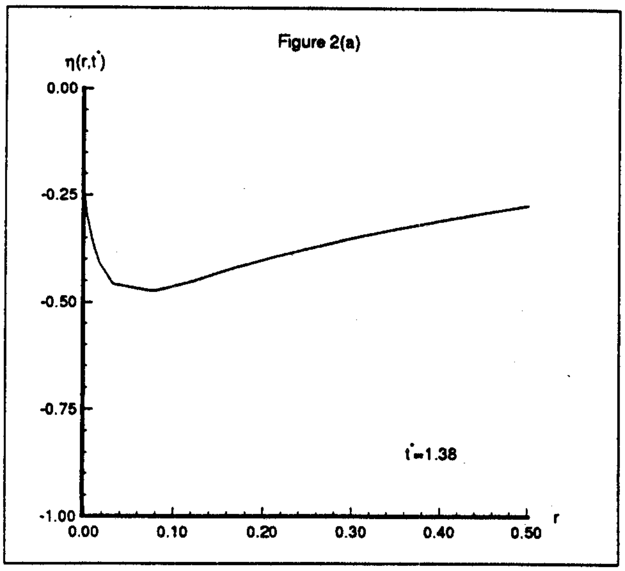
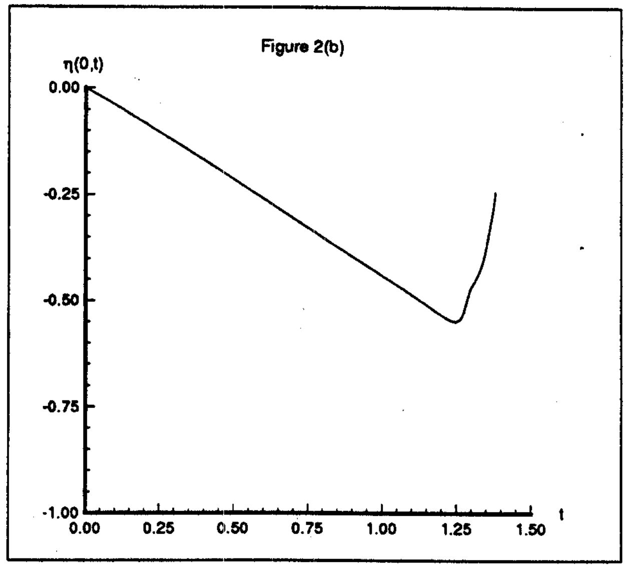


Figure 2(a)



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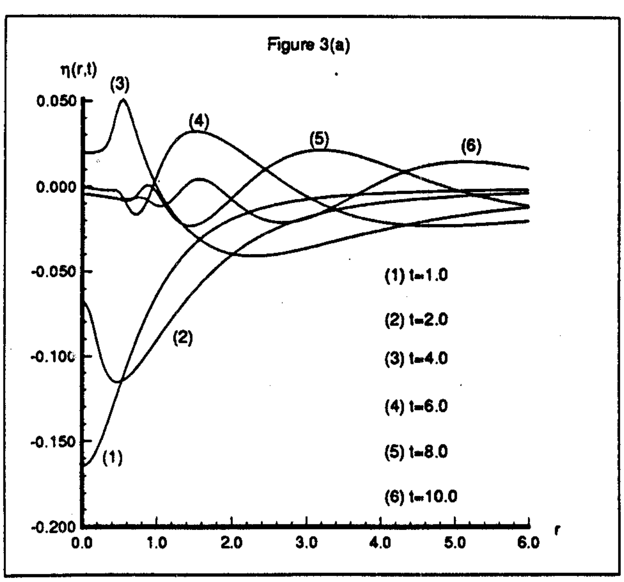

Figure 3(a)

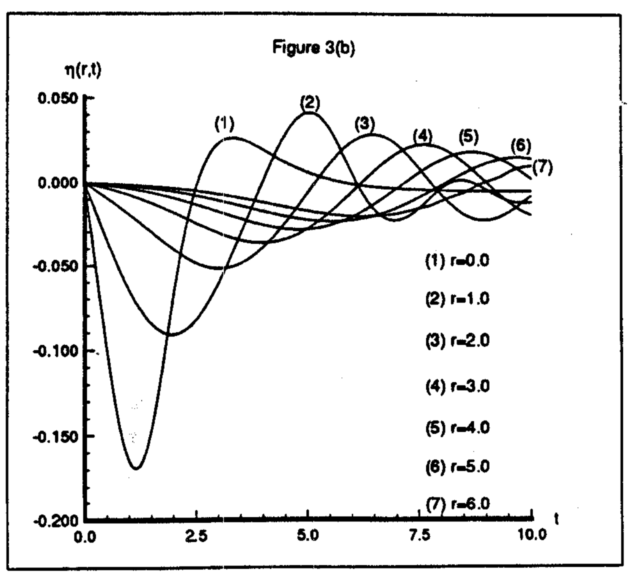

Figure 3(b)