Time domain calculations of the second-order drift force on a floating object in current and waves

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Introduction

In order to be able to take non-linear effects into account for the wave-body interaction, it is necessary to solve the problem in the time domain. To get started with time-domain calculations we studied the linear problem and calculated the second-order drift forces on a sailing object. We give results of our initial research of a two-dimensional problem and some preliminary results of a three-dimensional case.

Table of nomenclature

All bold symbols represent vectors or matrices.

A_{ij}	mass coefficient in direction i for a movement in direction j	S	computational domain
B_{ij}	damping coefficient in direction i for a movement in direction j	t	time
g	gravitational acceleration	U	undisturbed horizontal velocity of the fluid
G	Green's function	x, y, z	coordinate system fixed to the ship
H	hull surface	ζ_a	wave height due to the incoming wave
$\mathbf{n}$	normal vector, pointing out of the fluid domain	ρ	density of the fluid
		$\phi, \boldsymbol{\phi}$	unsteady potential
		$\overline{\phi}$	steady potential
		ω_0	frequency

Mathematical model

We consider an object, floating in a uniform current with velocity U and regular incoming waves. The coordinate system is chosen in such a way that the undisturbed free surface coincides with the line $z = 0$ and the point of gravity coincides with the origin. We assume the fluid to be incompressible and we neglect the effect of viscosity. If we assume the flow to be irrotational, we can introduce a velocity potential, which has to satisfy the Laplace equation. The free-surface condition is linearized around $z = 0$ and around the steady potential, and is correct up to $\mathcal{O}(U^2)$. As an approximation for the steady potential we used the double-body potential which arises from the case when the free surface is approximated by a rigid wall. The linearized version of the body boundary we used, can be found in Nakos [1].
Unlike the fluid domain our computational domain cannot be infinite. Therefore we have to impose a radiation condition on the artificial boundaries. We chose partial-differential equations to absorb outgoing waves, but they will be mentioned later.
The first-order forces acting on the ship are calculated by integrating the pressure over the hull surface, using Bernoulli's equation. Then the added mass and damping coefficients and the coupling coefficients are determined by fitting the first-order forces to the inducing movement of the ship. Knowing these coefficients the movements can be calculated in the case of incoming waves by solving the equations of motion. Finally, the second-order force, or drift force, follows from formulae equivalent to formulae given by Zhao [5] and Pinkster [2].

Numerical Algorithm

To solve the problem described above, we used Green's second theorem on the boundary of the computational fluid domain:

$$\frac{1}{2}\phi(\boldsymbol{x},t) = \int_{\partial S} \left(\phi(\boldsymbol{\xi},t)\frac{\partial G}{\partial n_\xi}(\boldsymbol{x},\boldsymbol{\xi}) - G(\boldsymbol{x},\boldsymbol{\xi})\frac{\partial \phi}{\partial n_\xi}(\boldsymbol{\xi},t)\right) d\Gamma \tag{1}$$

with ∂S the boundary of the domain.
For all the relevant quantities can be obtained by the values of the potential on the boundary, only values of $\boldsymbol{x}$ on the boundary will be considered.
We discretize the integral equation (1) by dividing the boundary into intervals and assuming the quantities to be constant on such an interval. The collocation points are the midpoints of these intervals. This gives a set of linear equations which can be rewritten as:

$$\boldsymbol{A}\boldsymbol{\phi}(t) = \boldsymbol{B}\boldsymbol{\phi}_n(t) \tag{2}$$

with $\boldsymbol{A}$ and $\boldsymbol{B}$ matrices built up by the Green's function and its derivative, and $\boldsymbol{\phi}_n$ a vector containing the normal derivatives of ϕ.
This matrix equation is solved by substituting the boundary conditions themselves in (2) and integrating the resulting matrix equation in time. An other possibility would be to solve the matrix equation and then, seperately, integrate the boundary condition in time. However, it appeared that this method leads to instable time integration and is only applicable in the case that there is no current.
Our method leads to the following overall matrix equation:

$$\boldsymbol{A}_1\boldsymbol{\phi}_{i+1} = \boldsymbol{A}_2\boldsymbol{\phi}_i + \boldsymbol{A}_3\boldsymbol{\phi}_{i-1} + \boldsymbol{f}_{i+1} \tag{3}$$

with $\boldsymbol{f}$ a time-dependent vector, resulting from the body boundary condition and parts of the spatial derivatives. With the condition that the fluid is initially undisturbed, which means $\boldsymbol{\phi} = \boldsymbol{0}$ and $\frac{\partial \boldsymbol{\phi}}{\partial t} = \boldsymbol{0}$, this system can be solved.
For more details on out numerical algorithm, we refer to Prins [3].

Results

The numerical algorithm presented in the previous section has been used to calculate the second-order horizontal drift force and all other relevant quantities, such as added mass and damping coefficients and movements of the hull. We studied a two dimensional case, a cylinder of infinite length, and are currently studying a sphere.

Two dimensional

We studied a cylinder of infinite length with radius R. The draught of the cylinder equals this radius.
The radiation condition we used, was a partial differential equation based on the plane-wave representation. For we have two outgoing waves on each side, i.e. we only consider $\tau = \frac{U\omega}{g} < 0.25$, we get a second-order partial differential equation:

$$\left(\frac{\partial}{\partial t} + c_1\frac{\partial}{\partial n}\right)\left(\frac{\partial}{\partial t} + c_2\frac{\partial}{\partial n}\right)\phi = 0 \tag{4}$$

After substituting the phase velocities this equation reduces to

$$\frac{\partial^2 \phi}{\partial t^2} + U(\frac{1}{\tau} \pm 2)\frac{\partial^2 \phi}{\partial n \partial t} + U^2\frac{\partial^2 \phi}{\partial n^2} = 0 \tag{5}$$

On the right boundary the plus sign should be taken, on the left the minus sign. Here the steady potential $\overline{\phi} = Ux$ is used. We could also have used the double-body potential as we do at the free surface, but because the artificial boundary is chosen relatively far away from the ship, these two are approximately equal.

The accuracy of the results was mostly influenced by the number of panels on the free surface, especially for low frequencies. This can be explained by the fact that we used a computational domain of the length of 3 wavelengths. Therefore the domain becomes very large for these frequencies and the steady potential nearly equals Ux, the potential of the uniform velocity, at almost every collocation point. By taking more panels the region where the steady potential differs significantly from Ux can be represented more accurately. The relative error can be estimated roughly at a few percent, except for the sway coefficients, where the large gradient in the free surface was not represented well enough.
As the incoming potential we used a plane harmonic wave, progressing in the positive x-direction. We calculated the results for six different Froude numbers: $Fn = -.035,\ 0,\ .035,\ .07,\ .105,\ .14$, with the Froude number given by $\frac{U}{\sqrt{gR}}$. All coefficients agreed well with the results found by Zhao [5] and Vugts [4].
The results for the drift force are given in figure 1. The slightly negative values at low frequencies vanishes if one increases the accuracy of the algorithm.

Three dimensional

At the moment we are studying a sphere as a first start of our three dimensional computations. To derive a radiation condition we used the Green's function satisfying the boundary condition at the free surface. The asymptotic behavior of this function for large r at zero forward speed gives rise to the following condition:

$$\frac{1}{c}\frac{\partial \phi}{\partial t} + \frac{\partial \phi}{\partial r} + \frac{1}{2r}\phi = 0$$

First results indicate that this boundary condition absorbs the outgoing waves very well. Up to now we have calculated the added mass and damping coefficients for heave and sway for zero forward speed, see figure 2 and 3. Our results agree very well with the the results of Pinkster [2], who used the frequency domain. We are now calculating the second-order forces and implementing non-zero forward speed.

Conclusions

In our latest research we tackled the time-dependent ship-wave problem. A numerical algorithm has been developed to solve the equations which are correct up to second order in the current velocity U. This algorithm is stable for every velocity and for every time step we tried, at least in two dimensions. Therefore it is a very powerfull algorithm and it is now being implemented for three-dimensional problems.
The calculations agree well with the results Zhao [5] and Pinkster [2] found using the frequency domain.

References

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[5] Zhao,R. and Faltinsen,O.M. 1988 Interaction between waves and current on a two-dimensional body in the free surface, *Applied Ocean Research*, **10**, 87.

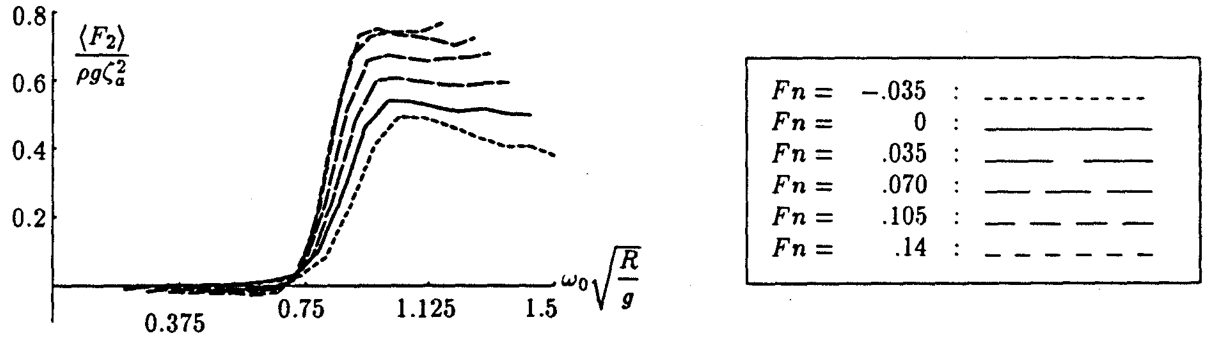

Figure 1: The drift force for six velocities

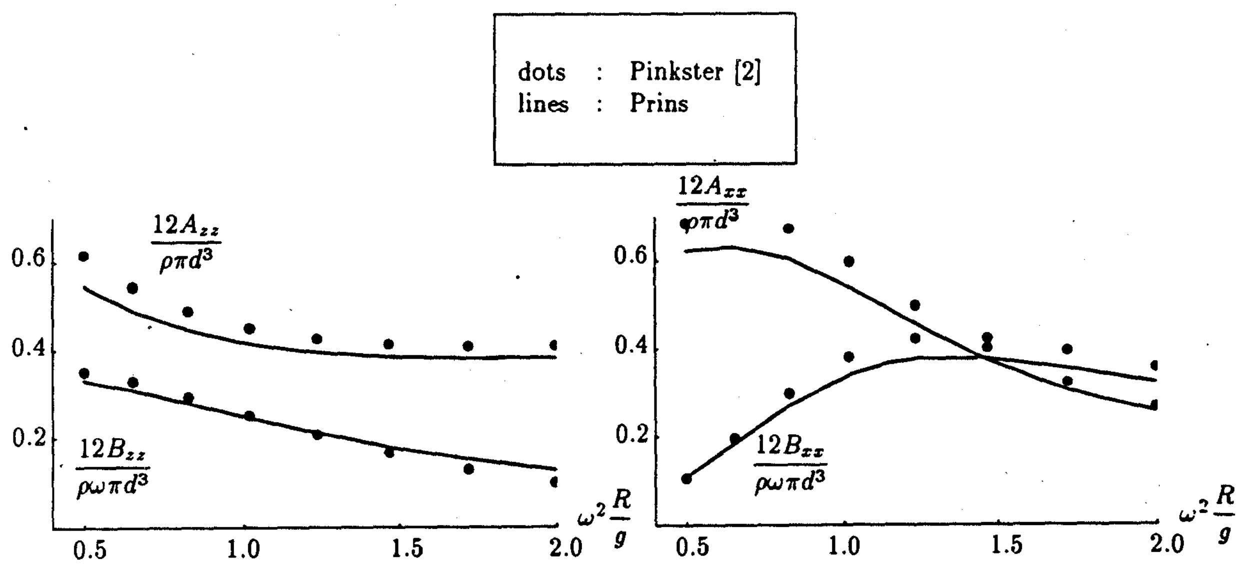

Figure 2: Added mass and damping coefficient in heave

Figure 3: Added mass and damping coefficient in surge

Discussion

Mr. Clement: Can you justify your use of the product of two Summerfeld-like operators as an absorbing condition on the presence of a system of two outgoing waves in 2D?

Author's Reply: This is based on the linearity of our problem. The wave can be seen as a sum of two independent waves. Each of these waves is absorbed by one of the two terms in the porduct-operator. This has been used before by for instance Romate (PhD-thesis, University of Twente, Netherlands).

Mr. Grilli: Can you tell us, in connection with your stable time-stepping algorithm, what is the exact nature of the operator with which you express the normal derivative of the potential on the free surface, as a function of the potential?

Author's Reply: Maybe the notation I used caused some confusion. By rewriting the free-surface condition, the normal derivative of the potential can be made explicit easily. This expression is used to eliminate the normal derivative of the potential from the matrix equation. For more details I would like to refer to my paper in the Journal of Ship Research.

Mr. Vada: I have analyzed the stability of time-marching schemes in combination with a piece-wise constant solution in space and the results are basically the same as for higher order panels, although the critical value of β (i.e. the maximum allowable time step) is slightly modified. It is important, however, to obtain stability to use an implicit formula for at least one of the FSC's. It is also vital to use a spatial filter to remove short waves, typically waves shorter than $5\Delta x$. I think the reason for the failure of your computations with the first method you tried may be found here.

Author's Reply: We studied analytically the eigenvalues of the free-surface condition. Some eigenvalues have positive real parts. This implies that no time-integration scheme can be stable, for the analytical solution is unstable itself. Therefore we combined the boundary condition with the matrix equation.
(Further discussion revealed that the discrepancy in our results may be due to numerical damping of unstable modes. The resulting modes are then the realistic ones, as found by our new algorithm as well.)