Hydroelastic modelling of slamming against wetdecks

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1 Introduction

The structural part connecting the two hulls of a catamaran is often referred to as the wetdeck. Slamming against the wetdeck influences the global motions of the vessel (Zhao and Faltinsen [1992]) and causes local and global hydroelastic effects. In this paper we will study the local hydroelastic effects by considering a horizontal elastic plate that is forced with constant velocity through the crest of a regular wave system. Slamming against rigid two-dimensional bodies has for instance been studied by Wagner [1932], Zhao and Faltinsen [1992],[1993], and Cointe [1991]. Meyerhoff [1965] studied slamming on elastic wedges penetrating an initially calm free surface by extending Wagner's theory.

2 Theory

The wetdeck is modelled as a two-dimensional beam with length L_B corresponding to the length between two transverse stiffeners. Local deformations of the plate field between the longitudinal stiffeners is a 3D effect which is not covered by this approach. The coordinate system, definition of the beam as well as the wave envelop are shown in Figure 1.

The x-axis is pointing toward the stern of the catamaran, the y-axis is pointing toward the

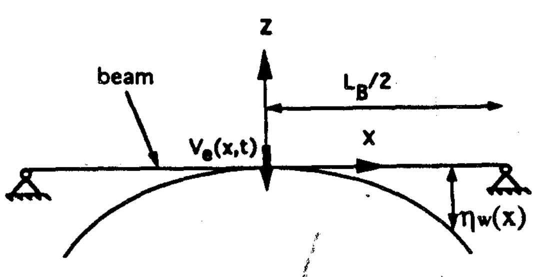

Figure 1: Local coordinate system.

starboard side and the z-axis is pointing upwards. The waves are propagating in the positive x-direction. The reference frame xz is moving with the phase velocity of the waves so that $x = 0$ corresponds to the wave crest. Vertical velocities due to the waves as well as gravity effects are neglected. The flow is assumed symmetric about $x = 0$ and the beam is simply supported at the transverse stiffeners. In addition; the vertical velocity V in the slamming region due to global ship motions is assumed constant in time.

The 2-dimensional beam equation is set up by assuming small deflections $w(x,t)$. Axial force effects, shear deformations as well as rotatory inertia effects are neglected. The properties of the beam are assumed constant along its length. $w(x,t)$ is expressed in terms of "dry" normal modes:

$$w(x,t) = \sum_{n=1}^{\infty} a_n(t)\psi_n(x) \qquad (1)$$

where $a_n(t)$ and $\psi_n(x)$ are the principal coordinate and the eigenfunction of the n'th vibration mode; respectively. t is the time variable. Multiplying the beam equation with the eigenfunction $\psi_m(x)$ of an arbitrary mode m and integrate over the beam, one obtains the modal beam equation of mode m:

$$\left[M_B \ddot{a}_m(t) + EI \cdot a_m(t)\left(\frac{m\pi}{L_B}\right)^4\right] \int_{-\frac{L_B}{2}}^{\frac{L_B}{2}} \psi_m^2(x)dx$$

$$= \int_{-c(t)}^{c(t)} p(x,w,t)\psi_m(x)dx \qquad (2)$$

where M_B is the mass per unit area of the beam, EI is the bending stiffness, p is the hydrodynamic pressure on the beam and $2c(t)$ is an approximation of the wetted area. Dot stands for the time derivative. As indicated in equation

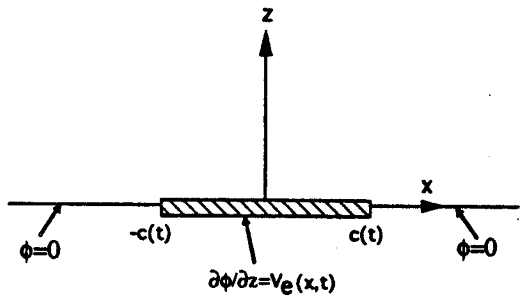

Figure 2: Simplified hydrodynamic slamming model.

(2) the pressure p is also dependent on the deflection $w(x,t)$.

The pressure p is obtained by solving a hydrodynamic boundary value problem (HBVP). In order to solve this HBVP one needs to define an inner (Wagner's analysis [1932]) and an outer region. The inner and the outer solutions can be matched in the same way as shown by Cointe [1991]. Here; only the outer solution is considered. Assuming ideal fluid, there exists a velocity potential ϕ which satisfies Laplace equation in the fluid domain. Boundary conditions are $\phi = 0$ on the free surface and $\frac{\partial \phi}{\partial z} = V_e(x,t) = V + \dot{w}(x,t)$ on the wetted surface. The boundary conditions are transformed to $z = 0$. $V_e(x,t)$ is called the effective velocity. It can be shown similarly as Cointe [1991] did that p can be approximated by the "$-\rho\frac{\partial \phi}{\partial t}$" term in Bernoulli's equation. The HBVP causes a singular pressure near the edges ($x = \pm c(t)$). Assuming small angles between the body and the free surface near the edges this singularity will match the inner solution.

Three different methods have been used to solve the HBVP described in Figure 2. The two first methods are analytical solutions where the second being a simplified analytical solution. The third method is a boundary element method. Each of these approaches will now be discussed.

In the first analytical method the flow is expressed in terms of a vortex distribution on the wetted surface of the beam. The vortex density $\gamma(x,t)$ is solved by an integral equation (See page 180 in Newman [1977]) and is derived in an analytical form. The effective velocity on the wetted surface is rewritten in terms of a Fourier series:

$$V_e(\theta(x,t)) = \overline{A}_0(t) + \sum_{k=1}^{\infty} \overline{A}_{2k}(t)\cos(2k\theta) \quad (3)$$

where $\overline{A}_0(t)$ and $\overline{A}_{2k}(t)$ are Fourier coefficients and θ is the transformed variable on the wetted surface ($x = c(t)\cos\theta$). An analytical expression of the hydrodynamic pressure on the beam is developed. Then the modal total force described by the right hand side of equation (2) can be rewritten as:

$$\int_{-c(t)}^{c(t)} p(x,t)\psi_m(x)dx = F_{exc,m}(t) - \sum_{n=1}^{\infty} B_{mn}(t)\dot{a}_n(t) - \sum_{n=1}^{\infty} A_{mn}(t)\ddot{a}_n(t) \quad (4)$$

where $F_{exc,m}(t)$ is the modal excitation force which may interact with the global ship motions and $B_{mn}(t)$ is the coupled modal damping coefficient which is proportional to the time derivative ($\frac{dc(t)}{dt}$) of the wetted surface. $A_{mn}(t)$ is the coupled modal hydrodynamic added mass. The two last terms of equation (4) are defined as the modal damping force and the modal added mass force; respectively. Equation (4) indicates that there are interaction effects between all the vibration modes. The modal damping force and the modal added mass force are moved to the left hand side of (2) when integrating the set of differential equations numerically. This ensures the numerical stability of the time integration.

The second method is an analytical approach where one assumes that the vertical velocities due to the beam vibrations are constant in space on the wetted surface of the beam. The solution of the velocity potential is the classical solution for a flat plate with heave velocity in infinite fluid (See for instance page 122 in Newman [1977]). In the same way as described by equation (4) the modal total force on the beam is decomposed into different force terms.

In the boundary element method the velocity potential is expressed in terms of Green's second identity. This formulation breaks down with zero plate thickness. The geometry of the flat plate is therefore modelled as a diamond with height to length ratio 0.01. The numerical method predicts the slamming force on a rigid flat plate with relative error 1%. For the hydroelastic part the following difficulties occurred: a) Decomposition of the total hydrodynamic force on the beam b) Convergence problems. Each of these problems will now be discussed. It turned out to be difficult to decompose the total hydrodynamic force on the beam into excitation, damping and added mass terms as illustrated by equation (4). In particular we did not manage to identify the damping terms. This implies that way we could not move the hydrodynamic added mass and damping terms to the left hand side of the equation system. Two reasons are proposed to cause the convergence problems. As a consequence of a) the right hand sides were large compared to the left hand sides in the set of differential equations. This may cause numerical instabilities in the time integration. In the boundary element method the structural and the hydrodynamic parts were not solved simultaneously since an iteration for the solution was carried out at each time step. It is believed that this also may cause numerical instabilities in the time integration. Due to these problems the boundary element method was unsuccessful.

How to obtain the wetted area $2c(t)$ in the outer solution will now be discussed. Following Wagner's analysis [1932], half the wetted area is the solution x of the integral equation:

$$\eta_w(x) = \int_0^t \left[\frac{\partial\phi(x,t)}{\partial z} - (V + \dot{w}(x,t))\right] dt \quad (5)$$

where $\eta_w(x) = \zeta_a(1 - \cos\nu x)$ denotes the air-gap between the beam and the wave envelop. Here ν is the wave number and ζ_a is the wave amplitude. $\frac{\partial\phi}{\partial z}$ is the vertical velocity outside the wetted part of the beam along $z = 0$. Integral equation (5) was solved numerically in the analytical method as well as the boundary element method and expressed analytically in the simplified analytical method.

3 Results and discussion

The set of second order differential equations described by equation (2) is simulated in time by using a fourth order Runge-Kutta integrator. Linear structural damping ξ_m is implemented in the numerical formulation and added along the diagonal of the damping matrix. Aluminium is selected as the beam material in this study. Input data of the simulation are shown in Table 1. Acoustic effects are taken into account in an approximate way in the initial phase of the impact. Cavitation is not considered, but could actually occur for high impact velocities.

Figure 3 shows the total hydrodynamic force on the beam as a function of time. The total hydrodynamic force is identical with the modal total force in equation (4) when $\psi_m = 1.0$. The figure also includes the total hydrodynamic force

Description	Unit	Value
Area inertia I	$[m^4/m]$	0.000011
Mass M_B	$[kg/m^2]$	36.6
Beam length L_B	$[m]$	1.5
Eigenmodes N	[-]	30
Time step dt	[s]	0.000001
Wave period T_w	[s]	5.7
Wave amplitude ζ_a	$[m]$	2.0
Fall velocity V	$[m/s]$	-6.0
Structural damp. ξ_m	[-]	0.01

Table 1: Input data.

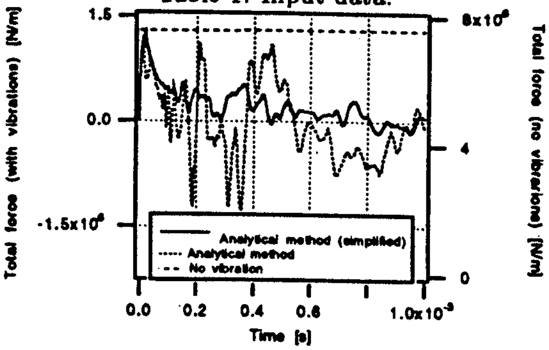

Figure 3: Total hydrodynamic force on the beam as a function of time.

on the beam in the non-vibratory case. The total hydrodynamic force in the non-vibratory case is almost constant in time. By including beam vibrations the total hydrodynamic force is significantly reduced. This reduction is mainly due to the added mass effect and the force components proportional to the vibration velocity. This means that the two latter terms of equation (4) play an important role when predicting the modal total force on the beam. A rough estimate of the pressure by dividing the total hydrodynamic force by the wetted area shows that cavitation will occur at time $t = 0.00019$ [s], $t = 0.00031$ [s] and $t = 0.00036$ [s], i.e. that the total pressure is equal to the vapor pressure. Since the effect of cavitation is neglected, part of the negative force in Figure 3 is unrealistic. The agreement of the total hydrodynamic force due to the two analytical methods is not satisfactory. The differences are believed to be due to inadequate modelling of the vertical velocity on the wetted surface in the simplified analytical method.

We will now focus the attention on the analytical non-simplified method and decompose the corresponding total hydrodynamic force F_{TOTAL} on the beam into excitation force F_{exc}, damping force $F_{damping}$ as well as added mass force $F_{addedmass}$. These force components are

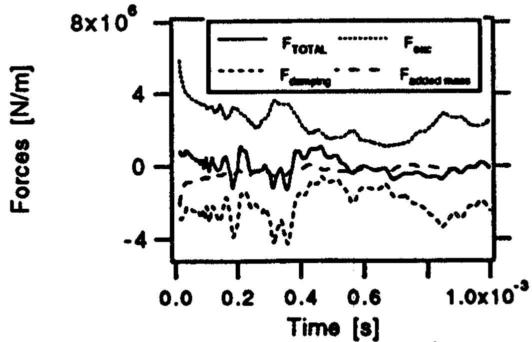

Figure 4: Force components on the beam as a function of time.

identical with the modal excitation force, modal damping force and modal added mass force; respectively, when $\psi_m = 1.0$ (See equation (4)). Figure 4 reveals each of these force components as a function of time. From the figure it is clear that the local minima of the total hydrodynamic force are due to the local minima of the damping force. These minima correspond to local maxima in the damping coefficients as well as the time derivative $2\frac{dc(t)}{dt}$ of the wetted surface. In that way $\frac{dc(t)}{dt}$ becomes an important parameter when analysing wetdeck slamming. Large values of $\frac{dc(t)}{dt}$ is caused by a small relative angle between the beam and the free surface near the edges ($x = \pm c(t)$). This means that hydroelastic effects are less important when considering slamming on wedges with moderate or large deadrise angles but become important when analysing wetdeck slamming.

The simplified analytical method underestimates both the modal damping and the modal added mass coefficients compared to the analytical non-simplified method. As expected modal added mass is very large compared to the physical mass.

A stress estimate in the wetdeck shows that yielding of the material may occur. The maximum deformation of the beam is below 0.3 % of L_B.

The total hydrodynamic force will cause realistic global accelerations of the vessel. This implies that the assumption of $\frac{dV}{dt} = 0$ is not valid and the instantaneous ship response to the local slamming force is required. Interaction between the local slamming force and the global ship accelerations will be accounted for in two steps. First we will assume that the ship is rigid. Later the ship will be modelled as a global elastic structure. Linear ship motion in the time domain will be expressed in terms of an analytical transformation of results from the frequency domain method developed by Faltinsen and Zhao [1991].

4 Verification

In order to verify the computer code and the mathematical formulation, miscellaneous methods may be used. Verification methods based on conservation of energy, mass and momentum are commonly used but are difficult to use in this case. This is due to the inadequate modelling of the flow near the edges. In order to conserve energy, mass as well as momentum during the time simulation, the jet flow near the edges has to be accounted for in a proper way (See Zhao and Faltinsen [1993]). Anyway; convergence tests by varying the time step, number of eigenmodes as well as number of Fourier components used to represent the velocity on the wetted surface of the beam, have successfully been carried out. Green's second identity has been used for ϕ and an auxiliary function ψ to check the solution. Here ψ is the flat plate solution with unit heave velocity and the same free surface condition as in the impact problem.

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