Time Domain Analysis of Wave Exciting Forces Acting on a Floating Body

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ABSTRACT

Formulations are done for the wave exciting forces on a floating body in plane progressive waves by linear time domain analysis. This is to present the analytical and numerical methods involved in the time domain analysis and to present typical calculation results and relevant discussions.

In this paper, on the assumption of linearity of entire system, exciting forces due to plane progressive waves of arbitrary profile are expressed by convolution integrals of wave elevation and impulsive wave response functions. The acceleration potential of the impulsive incident wave is derived, which make it possible to formulate the exciting force problems in time domain by integral equation techniques.

Numerical calculation results are given for a sphere and a spheroid($L/B = 8.0$) by this method. Their Fourier transformed results are compared with those of the conventional frequency domain calculations, which show good agreements.

SUMMARY

The exciting forces acting on a ship in waves have been conventionally treated in frequency domain. In this research, corresponding formulations are made in time domain and the results of each domains are compared to show the characteristics and validity of time domain solutions.

It is shown that the exciting forces due to plane progressive waves of arbitrary profile can be expressed by a convolution integral of wave elevation at any one point and some kernel force functions. These functions only depend on body shapes and correspond to impulsive wave response functions. These can be obtained as the wave exciting forces due to special kind of transient incident waves such as used by Davis and Zarnick[1].

This transient wave is similar to white-noise input in that it contains unit amplitude waves of all frequencies, but their phases are controlled and as a consequence of dispersive property of water waves, there occurs impulsive wave height at one point and at one instant. It is shown in Fig.1. The acceleration potential of this transient wave is derived in this paper and using this, time domain analysis is carried out for the wave exciting force problems.

To use boundary integral equation technique, Green's function with impulsive source strength[2] is adopted and by applying Green's theorem, integral equation for the acceleration potential on body surface is derived. This integral equation has the form of second kind of Fredholm integral equation with respect to spatial variables, and also has the form of second kind of Volterra integral equation with respect to time variable.

We assume that a body is floating in plane progressive waves of small amplitudes and arbitrary profiles. It is also assumed that the fluid is inviscid and incompressible and that the flow is irrotational. The total acceleration potential can be written as the sum of incident wave potential, $\Phi_{It}(P,t)$, and diffraction potential, $\Phi_{Dt}(P,t)$.

$$\Phi_t(P,t) = \Phi_{It}(P,t) + \Phi_{Dt}(P,t) \quad , \quad P = (x,y,z) \tag{1}$$

Now Φ_{It} and Φ_{Dt} are decomposed by using wave elevation $A_O(t)$ as follows[3];

$$\begin{aligned}
\Phi_{It}(P,t) &= \int_{-\infty}^{\infty} A_O(\tau) H_t(P,t-\tau) d\tau \\
\Phi_{Dt}(P,t) &= \int_{-\infty}^{\infty} A_O(\tau) \phi_{Dt}(P,t-\tau) d\tau \\
H_t(P,t) &= -\sqrt{\frac{g^3}{4\pi r_o}} \exp(-\gamma\cos\theta) \times \\
& \exp\left(i\left(\frac{\theta}{2} - \gamma\sin\theta\right)\right) erfc\left(i\, sign(t)\sqrt{\gamma}\exp\left(i\frac{\theta}{2}\right)\right)
\end{aligned} \tag{2}$$

where, $erfc$ is a complementary error function and H_t is the acceleration potential for the impulsive incident waves.

$$\begin{aligned}
\ell_o &= x\cos\beta + y\sin\beta \\
r_o &= \sqrt{\ell_o^2 + z^2} \\
\theta &= \tan^{-1}\left(\frac{\ell_o}{|z|}\right) \quad \left(-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}\right) \\
\gamma &= \frac{gt^2}{4r_o}
\end{aligned}$$

And integral equation for ϕ_{Dt} on body surface is derived as follows:

$$\begin{aligned}
2\pi\phi_{Dt}(P,t) &+ \int\int_{So} dS \phi_{Dt}(Q,t) \frac{\partial}{\partial n_Q}\left(\frac{1}{r} - \frac{1}{r'}\right) \\
&+ \int_{-\infty}^{t} d\tau \int\int_{So} dS\ \phi_{D\tau}(Q,\tau) \frac{\partial}{\partial n_Q} \tilde{G}(P,Q,t-\tau) \\
&= - \int\int_{So} dS \frac{\partial}{\partial n_Q} H_t(Q,t)\left(\frac{1}{r} - \frac{1}{r'}\right) \\
&- \int_{-\infty}^{t} d\tau \int\int_{So} dS \frac{\partial}{\partial n_Q} H_\tau(Q,\tau) \tilde{G}(P,Q,t-\tau)
\end{aligned} \tag{3}$$

where,

$$\tilde{G}(P,Q,t) = 2\int_0^{\infty} dk\ \sqrt{gk}\ \sin(\sqrt{gk}t) e^{k(z+\zeta)}\ J_0(kR) \tag{4}$$

Wave exciting forces $F_k(t)$ $(k = 1, \cdots, 6)$ are expressed as follows:

$$F_k(t) = \int_{-\infty}^{\infty} A_O(\tau) E_k(t-\tau) d\tau \qquad (5)$$

$$E_k(t) = -\rho \int\int_{S_O} n_k (H_t(P,t) + \phi_{Dt}(P,t)) dS \qquad (6)$$

The integral eq. (3) can be solved numerically to give $\phi_{Dt}(P,t)$ and hence impulsive wave response function $E_k(t)$.

Fig.2 & 3 show the impulsive wave response function, $E_2(t)$, of floating hemi-sphere for sway, and its Fourier transformed results compared with those of frequency domain calculation. Also shown in Fig.4 & 5 are those of spheroid($L/B = 8$) for heave. We can see good agreements in both results.

References

[1] Davis,M.G. and Zarnick,E.E.," *Testing Ship Models in Transient Waves*", 5th ONR, 1964.

[2] Wehausen,J.V. and Laitone,E.V.," *Surface Waves*", Encyclopedia of Physics, Vol.9, 1960

[3] Gong,I.Y.," *Time Domain Analysis of Hydrodynamic Forces acting on Three Dimensional Bodies*", Ph.D. Thesis, Seoul Natl. Univ., 1987

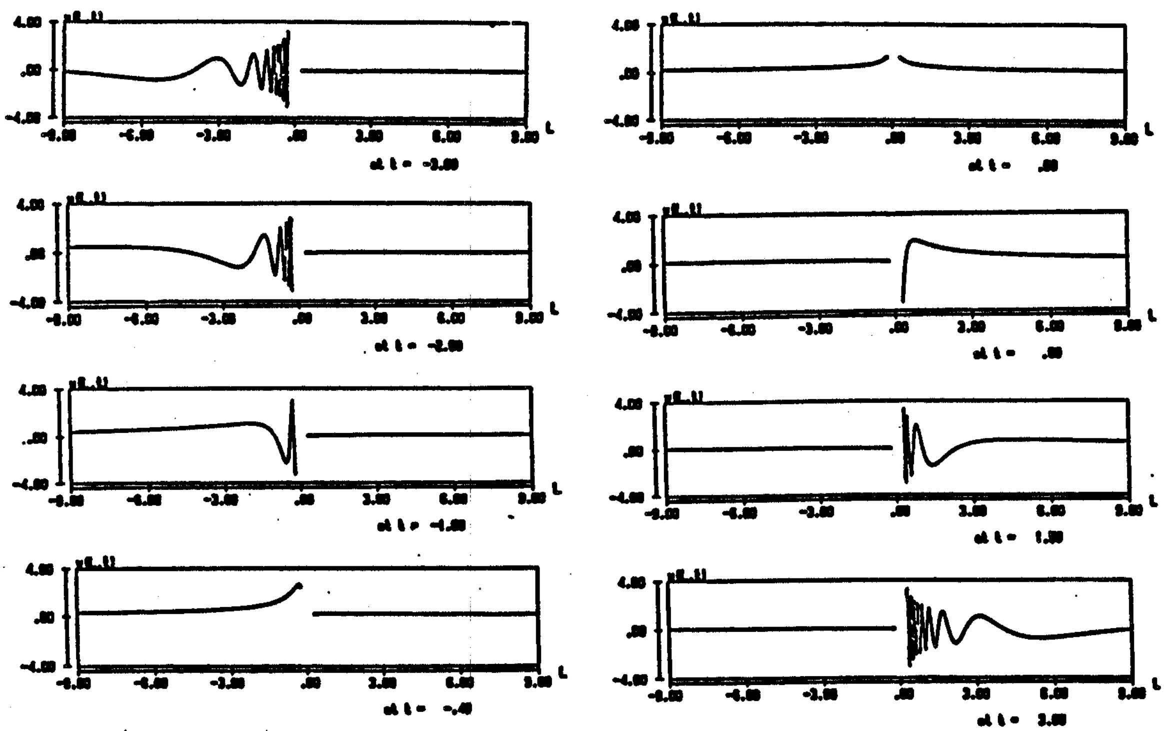

Fig. 1 Wave Profiles at various Time Steps

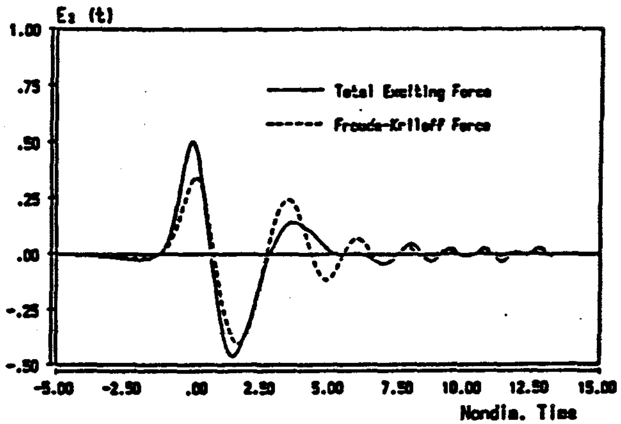

Fig. 2 Nondim. Exciting Force & Froude-Kriloff Force Kernel Func. of Floating Hemisphere for Sway

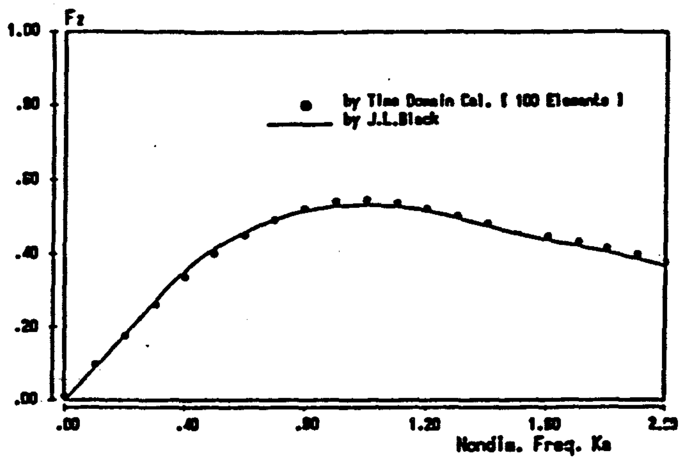

Fig. 3 Nondim. Exciting Force Coeff. of Floating Sphere for Sway

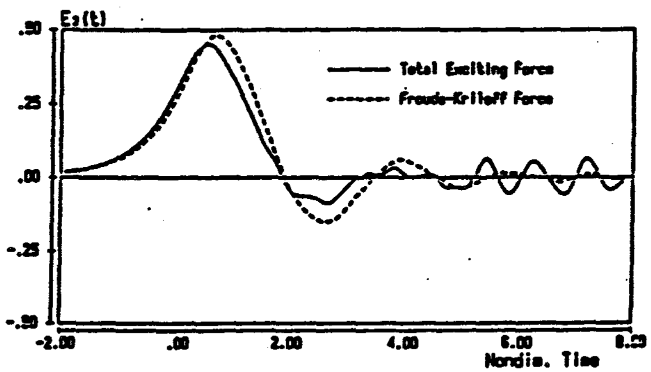

Fig. 4 Nondim. Wave Exciting Force & Froude-Kriloff Force Kernel Func. of Floating Spheroid(L/B=8) for Heave at Heading Angle = 45.000 deg.

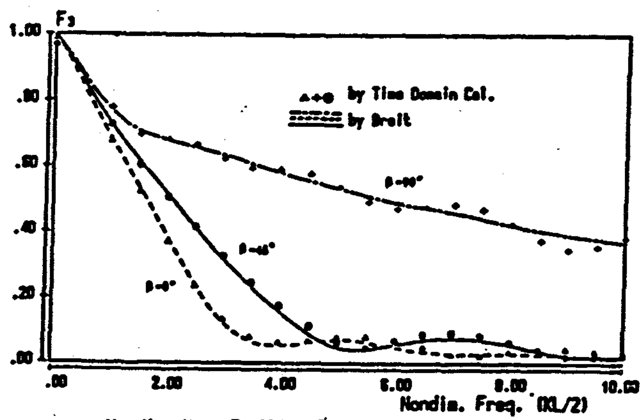

Fig. 5 Nondim. Wave Exciting Force Coeff. of Floating Spheroid (L/B=8) for Heave