An Absorbing Beach for Numerical Simulations of Nonlinear Waves in a Wave Tank

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1. Introduction

For wave radiation (or diffraction) problems where the fluid domain extends to infinity in the horizontal direction, a radiation condition at infinity is required to make the problem well-posed. This condition states that the radiation waves or diffraction waves are outgoing waves only, and is known as Sommerfeld radiation condition. For computational reasons, it is necessary to reduce the computational domain to a minimum and the fluid domain will be truncated at some distance by artificial boundaries (open boundaries). To make the problem well-posed, some kind of boundary condition or treatment is required at the truncation boundaries. For free surface wave problems, this means that free surface waves approaching an open boundary should be fully transmitted or absorbed so that no wave reflection occur.

Several methods have been proposed to absorb free surface waves and are briefly reviewed in the following.

- *Periodic Boundary Condition* The solution is assumed to be periodic in space so that the values of the unknowns on one vertical boundary can be set equal to those on the other vertical boundary of the computational domain. This technique is very easy to implement and open boundaries can be chosen at a very short distance equal to the largest modeled wavelength. However, its application is very limited due to the requirement of periodicity.

- *Matching to a Simplified Outer Solution* In this technique, a simplified outer solution which satisfies the far-field radiation condition and may be obtained in analytical form is used to match the inner numerical solution at the open boundary. The problems with this technique include more difficult implementation, high computational overhead (*e.g.*, linear time-domain solution with convolution integrals), and poor simulation of the behavior of the outer part of the fluid at the open boundary (*e.g.* matching the nonlinear inner solution to a linear outer solution if the nonlinearity is still quite strong).

- *Radiation Conditions* Several types of radiation conditions have been proposed for open boundaries. The condition most widely used is the Sommerfeld condition first used by Orlanski on the open boundaries in a finite difference scheme for hyperbolic flow. For free surface problems with the Laplace equation as the governing equation, a first-order or a second-order boundary condition can be obtained using a linearized free surface boundary condition, where the first-order condition is the same as the Sommerfeld condition. Usually, these radiation conditions are only valid at a single frequency. Furthermore, successful use of a radiation condition requires an accurate estimate of the local phase velocity which can be difficult to obtain for fully nonlinear waves.

- *Absorbing Beach (Artificial Damping)* This technique uses an artificial damping (dissipation) in the form of a sponge layer or damping zone so that outgoing waves are damped near the open boundaries with as little wave reflection as possible. Two methods of implementation are possible: the first is to add an artificial damping term to the field equation. The second is to add a "damping" term to the free surface boundary condition near the open boundaries. For instance, for free surface wave problems in potential flows, a "damping" term is added to the free surface dynamic boundary condition (Betts and Mohamad 1982), or to both the dynamic and kinematic boundary conditions (Baker *et al* 1981; Cointe *et al* 1990). The technique is easy to implement and has good reflection properties for a wide range of frequencies. Although a fairly large damping zone is needed and the amount of damping needs to be tuned, the technique has become more popular for simulations of nonlinear waves because of its ability to handle a wide frequency range.

2. Energy Absorption by Beaches

The principle of the absorbing beach is to dissipate the wave energy before the waves reach the open boundary. For the case where a damping term is added to the field equation, it is clear that the wave energy lose occurs in the damping zone through internal artificial friction within the fluid domain. For example, Chan (1975) solves the momentum equation with a linear artificial damping term $\nu\vec{v}$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} + \nabla p + \vec{k} + \nu\vec{v} = 0 \tag{1}$$

where ν is zero outside the damping zone. The wave energy is absorbed at a rate (assuming $\nu > 0$)

$$\frac{dE_i}{dt} = \int\int\int_{\Omega_{damp}} \nu\vec{v}\cdot\vec{v}d\Omega. \tag{2}$$

For free surface waves using a potential flow model and a "damping" term added to the free surface boundary conditions, the mechanism of wave energy lose is different. Since no internal friction can occur within the ideal fluid, the energy can only be transmitted or absorbed through the free surface (assuming a rigid wall for the truncation boundary). Consider the free surface dynamic boundary condition (the Bernoulli's equation) with an additional term P

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}\nabla\phi\cdot\nabla\phi + p + z + P = 0 \tag{3}$$

where P vanishes outside the absorbing zone. Since a pressure acting on the free surface can take any form while the flow remains irrotational, it seems more appropriate to consider the additional term P as an additional pressure acting on the free surface. Then the energy transmission can be explained by the negative work done by the pressure P to the fluid via the free surface. The rate of energy absorption is,

$$\frac{dE_f}{dt} = \int\int_{S_{beach}} P\phi_n \, ds\,. \tag{4}$$

where ϕ_n denotes the normal derivative of the potential ϕ on the free surface.

When $P = \nu\phi$, the "damping" used by Baker *et al* (1981), Betts and Mohamad (1982), and Cointe *et al* (1990) is recovered. In this case, the energy absorption rate becomes,

$$\frac{dE_f}{dt} = \int\int_{S_{beach}} \nu\phi\phi_n \, ds. \tag{5}$$

The term $\frac{dE_f}{dt}$ in Eq. (5) may become negative which means that the beach may not always absorb energy from the fluid but may put energy into the fluid causing undesirable wave reflection. Improved beaches may be obtained by choosing P such that $\frac{dE_f}{dt}$ is always positive and thus the beach absorbs wave energy at all times. Such a functional form is

$$P = \nu \; sign(\phi_n)\,|Q|, \tag{6}$$

where Q can be any function including nonlinear functions of ϕ and/or ϕ_n. Equation (4) then becomes

$$\frac{dE_f}{dt} = \int\int_{S_{beach}} \nu\,|Q|\,|\phi_n|\, ds\,. \tag{7}$$

3. Preliminary Numerical Results

Various types of beaches have been examined in a 2-D wave tank. Figures 2 to 6 show the results for $P = \nu\phi$ and $P = \nu\phi_n$ types of beaches. The fluid in the tank is bounded by two rigid walls, a flat bottom and the free surface (shown in Fig. 1). The problem is made dimensionless based on the calm water depth, the gravitational acceleration and the water density. The length of the tank L consists of the domain of interest (ten unit lengths) and the beach (one unit length). The waves are generated by a pneumatic wavemaker P_m at one end of the tank,

$$P_m = \begin{cases} P_o[1 + cos(\frac{\pi x}{x_o})] \, sin(\omega t) & (t \leq t_o) \\ 0 & (t > t_o). \end{cases}$$

An absorbing beach is placed at the other end of the tank. The variable ν is chosen such that $\nu = \nu_o \, (\frac{x-x_l}{L-x_l})^2$ within the beach area and $\nu = 0$ elsewhere. The mixed Eulerian-Lagrangian time-stepping procedure combined with a desingularized boundary integral technique is used to simulate the fully nonlinear waves.

We let the pressure P_m act on the free surface for one cycle ($t_o = \frac{2\pi}{\omega}$) to generate a wave train which travels towards the beach. The total energy E within the fluid will increase from zero to a certain value and remain unchanged after the forcing P_m is turned off ($t > t_o$) and before the wave train reaches the beach. After the waves reach the beach, the total energy is expected to decrease as is shown in Fig. 2 for $P = \nu\phi$ and $\nu_o = 1$. Also shown in the figure are the kinetic energy E_k and the potential energy E_p of the waves. Fig. 3 shows the total energy within the fluid and the energy absorbed, E_f, by the pressure P. The sum of the energy in the fluid and the energy absorbed by the beach which should be equal to the energy provided by the wavemaker P_m is also plotted in the figure. Fig. 4 and Fig. 5 show the results similar to those in Fig. 2 and Fig. 3 for the $P = \nu\phi_n$ type of beach with $\nu_o = 1$. As seen in Fig. 4 or Fig. 5, for $P = \nu\phi_n$ the total energy in the fluid decreases monotonically after P_m is off ($t > t_o$). This implies that the beach always absorbs energy. In the case of $P = \nu\phi$, the total energy decays but not monotonically. This becomes more obvious when $\nu_o = 10$ as shown in Fig. 6, where the total energy can exceed the energy provided by P_m. In this case at certain times the beach has provided energy into the fluid which is undesirable.

The performance of a beach is influenced by the shape of ν, the value of ν_o, as well as length of the beach. In order to accurately assess the performance of a beach, a means of determining the wave reflection is needed. For linear waves, one can use the wave reflection coefficient defined as $C_r = \frac{A_r}{A_i}$, where A_r is the amplitude of the reflected wave and A_i the amplitude of the incident wave. For fully nonlinear waves it is difficult to distinguish the reflected waves from the incident waves and consequently the reflection coefficient as defined for linear waves can not be used directly. Various techniques to measure wave reflection from the beach have been tried but none have been very satisfactory.

4. Acknowledgements

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5. References

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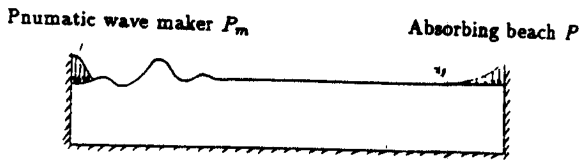

Fig. 1 Wave tank with absorbing beach.

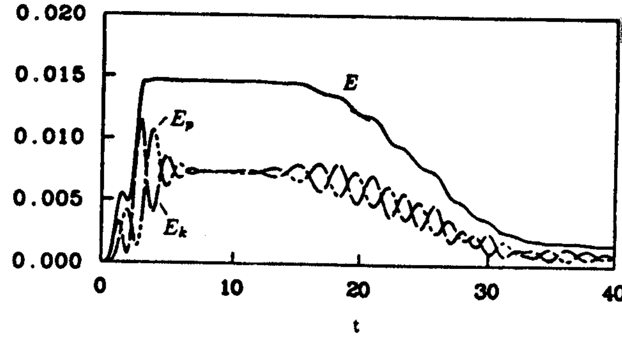

Fig. 4 Energy in the fluid domain. ($P = \nu\phi_n$, $\nu_o = 1.0$)

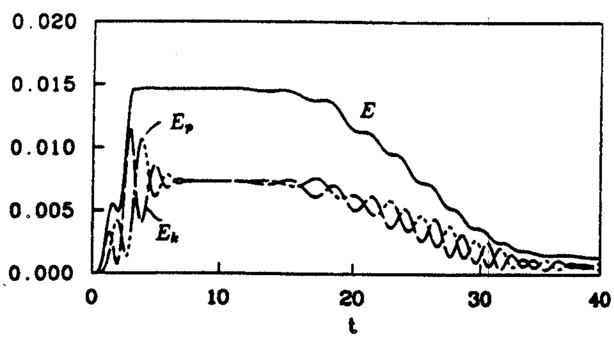

Fig. 2 Energy in the fluid domain. ($P = \nu\phi$, $\nu_o = 1.0$)

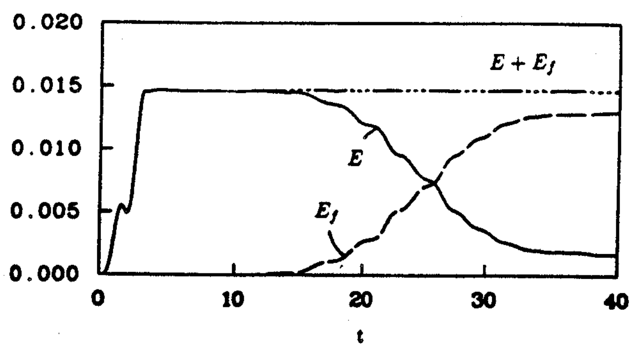

Fig. 5 Energies in the fluid domain and absorbed by P ($P = \nu\phi_n$, $\nu_o = 1.0$)

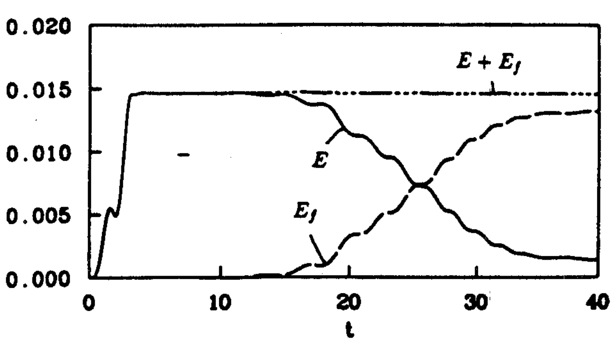

Fig. 3 Energies in the fluid domain and absorbed by P ($P = \nu\phi$, $\nu_o = 1.0$)

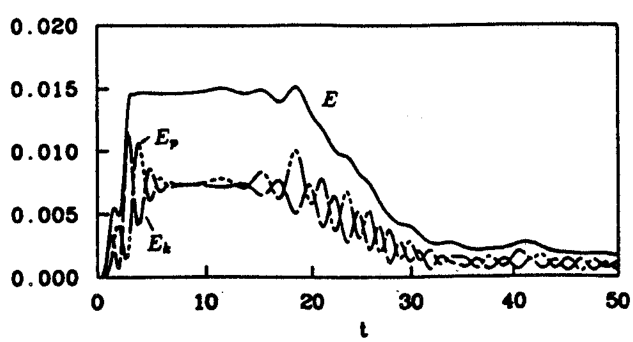

Fig. 6 Energy in the fluid domain. ($P = \nu\phi$, $\nu_o = 10.0$)