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SUBMERGED BODIES THAT DO NOT GENERATE WAVES

by

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Special note by EOT:

After this paper was accepted for presentation at this Workshop, Professor Tulin drew my attention to the fact that essentially all of the results in it (and more!) were obtained by him in an (otherwise unpublished) Hydronautics Inc. report (Tulin 1982). Since the work in that report is not well known, we have decided to make the present paper a joint one in recognition of the prior derivations by Tulin, although there are only a small number of changes (including correction of an erroneous conclusion) from my original draft version. For a fuller discussion of this problem, see Tulin (1982).

1. Summary

In 1989, one of us (Tuck 1989) demonstrated theoretically a winged slender body that generates no waves on the free surface of a uniform stream within which the body is submerged. This demonstration involved rather complicated manipulations involving distributions of Havelock sources and dipoles. It is our purpose here to give elementary examples, not requiring use of Havelock sources, to confirm and further illustrate this phenomenon. Similar examples and illustrations had earlier been given by Tulin (1982). We also observe that (at least when they are thin or slender) the waveless submerged bodies must have zero weight; that is, that they create negative lift, and are subject to a downward hydrodynamic force exactly equal and opposite to their buoyancy.

2. Two Dimensions

Consider the following solution of the two-dimensional Laplace equation with respect to (x, z):

$$\Phi(x,z) = \frac{x}{x^2+(z+h)^2} - \frac{x}{x^2+(z-h)^2} + \kappa \left[arctan\,\frac{z+h}{x} - arctan\,\frac{z-h}{x} \right] \qquad (2.1)$$

Physically this represents a horizontal dipole at $(x, z) = (0, -h)$ together with its reversed image at $(0, +h)$, plus an anti-clockwise line vortex at $(0, -h)$ together with its clockwise image at $(0, +h)$. This elementary function satisfies the Kelvin boundary condition

$$\kappa \Phi_z + \Phi_{xx} = 0 \tag{2.2}$$

on $z = 0$, as can be verified directly. It is a "wave-free potential", similar to that which has been used by Ursell (e.g. 1949) for time-dependent problems.

The Kelvin condition (2.2) holds for small disturbances to a uniform stream U. Thus, if we write $\phi = Ux + Ua^2\Phi$ for a total velocity potential, where a is a parameter of the dimensions of a length, then the velocity vector which is the gradient of this ϕ tends to a uniform stream at infinity, and is close to that stream almost everywhere if a/h is small.

If for example, we observe the flow in a domain close to the submerged singularity, with $x, z + h = O(a) << O(h)$, the terms of Φ in $z + h$ dominate, and we have

$$\phi \approx Ux + Ua^2 \left[\frac{x}{x^2 + (z+h)^2} + \kappa \, arctan \frac{z+h}{x} \right] \tag{2.3}$$

which is the well-known exact solution for flow past a circular cylinder centred at $(0, -h)$ with radius a, and (anticlockwise) circulation $2\pi U a^2 \kappa$ around that cylinder.

That is, a circular cylinder of small radius a, submerged to depth $h >> a$ beneath a free surface, generates no trailing waves on a stream U if there is anticlockwise circulation

$$\Gamma = 2\pi U a^2 \kappa = 2\pi a^2 g / U \tag{2.4}$$

about the cylinder. According to the Kutta-Joukowski theorem, there is then a lift force per unit span of

$$L = -\rho U \Gamma = -2\pi a^2 \rho g \tag{2.5}$$

on the cylinder. This is of a magnitude equal to the twice the weight of displaced water, and acts downward.

It is not difficult to construct other two-dimensional bodies that do not create waves, by distributing suitable dipole-vortex combinations. In particular, distributions can be constructed to model thin cambered hydrofoil-like bodies. These are subject to a downward force that is equal in magnitude to the buoyancy.

3. Three Dimensions

The three-dimensional equivalent to (2.1) is easy enough to write down, namely

$$\Phi(x, y, z) = D(x, y, z + h) - D(x, y, z - h) - \kappa \left[V(x, y, z + h) - V(x, y, z - h) \right] \tag{3.1}$$

where $D(x, y, z)$ and $V(x, y, z)$ are three-dimensional dipoles and vortices respectively, located at the origin. The former is just

$$D(x, y, z) = x r^{-3} \tag{3.2}$$

with $r^2 = x^2 + y^2 + z^2$.

Three-dimensional "vortices" are not such easy objects to specify, and it is sometimes convenient to describe V instead as a semi-infinite line of vertical dipoles of uniform strength, stretching from the origin to infinity downstream along the positive x-axis. This is equivalent to a "horseshoe vortex". However one describes it, the appropriate solution of the three-dimensional Laplace equation is

$$V(x, y, z) = -\frac{z}{r(r-x)} \tag{3.3}$$

and it can again be established directly that, with (3.2) for D and (3.3) for V, the expression (3.1) satisfies the Kelvin free-surface condition (2.2).

In the absence of circulation, the total potential $\phi = Ux + \frac{1}{2}Ua^3\Phi$ generates a submerged sphere of small radius a in the neighbourhood of $(x, y, z) = (0, 0, -h)$. Unfortunately, in contrast to the two-dimensional situation, no similar conclusion holds with $\kappa \neq 0$. It is possible to construct a closed submerged body using just one isolated dipole and one vortex, by allowing the location of the vortex to differ slightly from that of the dipole, but this body is not a sphere, and has little practical interest.

More significantly, one can distribute the combined dipole-vortex singularity over suitable domains to generate waveless bodies of almost any type. For example, the line distribution

$$\phi = Ux + \frac{1}{4\pi}\int S(\xi)\Phi(x-\xi, y, z)\, d\xi \tag{3.4}$$

generates from the horizontal dipoles a submerged slender body with section area $S(x)$. But Φ consists of both horizontal dipoles and horseshoe vortices, and the contribution from the latter demands that the slender body has attached to it slender wings of a span whose x-derivative is proportional to $S(x)$, as discussed in Tuck (1989). If one computes the net lift L provided by this wing, the result is

$$L = -\rho g \int S(x) dx \tag{3.5}$$

which is exactly equal and opposite to the buoyancy of the slender body. [A sign error in the special example used to illustrate this question in Tuck (1989) suggested incorrectly that the lift was positive].

Similarly, one can generate a "waveless hydrofoil" by distributing the waveless dipole-vortex combination over a portion B of the plane $z = -h$, namely

$$\phi = Ux + \iint_B \mu(\xi, \eta)\Phi(x-\xi, y-\eta, z)\, d\xi d\eta \tag{3.6}$$

for some $\mu(\xi, \eta)$ to be determined. If we were to consider only the vortex part of Φ and ignore the images in the free surface, this would lead to the usual lifting surface equation of aerodynamics (c.f. Ashley and Landahl, 1965, Tuck 1991), with $\mu(\xi, \eta)$ proportional to the (unknown) bound vorticity, and determined by the (known) camber of the mean surface of the hydrofoil.

However, in the present case, the horizontal dipole part of Φ indicates that $\mu(\xi, \eta)$ is also proportional to the thickness of the hydrofoil. Hence, for any given thickness distribution, there is a unique camber (which can be computed by a simple quadrature over B) such that the hydrofoil generates no waves. Equivalently, for any given mean surface shape (including one with zero geometrical camber but non-zero angle of attack), there is a unique thickness distribution that eliminates waves. However, determination of that thickness distribution is a much more difficult task, since it requires solution of the lifting surface integral equation. In any case, the net lift force turns out again to be exactly equal and opposite to the buoyancy due to the hydrofoil's (necessarily non-zero) thickness.

4. Conclusion

Submerged bodies seem to exist in both in two and three dimensions that generate no waves when placed in a uniform stream. However, the examples given here suggest that these bodies must be subject to a downforce. This downforce is exactly equal in magnitude to the buoyancy for the slender or thin examples given here, and exceeds it for the bluff (circular cylinder) example. Hence no free body can be waveless. A fixed body can be waveless if support is provided for its weight. The results may however have some significance for minimisation (rather than total elimination) of the waves and wave resistance of free bodies of non-zero weight.

There is scope for much further work, e.g. to use global arguments not related to singularity representation to generalise the slender-body result that the required negative lift equals the buoyancy, to generalise to include nonlinearity and hence to bring the bodies close to the surface, to compute the camber of waveless hydrofoils, etc. The zero-weight result for thin bodies is the submerged-body equivalent of the result of Krein (see Kostyukov 1968, p. 354) that the (Michell) wave resistance of a thin surface ship can never vanish if that ship has finite non-zero displacement.

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DISCUSSION

MARTIN: Here is a simple comment on your potential V. Let $L(x,y,z) = -\log(r-x)$. Then, $\frac{\partial L}{\partial x} = \frac{1}{r}$, which is obviously harmonic, whereas $\frac{\partial L}{\partial z} = V$. Potentials such as L and V occur in the exact solution to the problem of a point force in an elastic half-space, as obtained by R.D. Mindlin in 1935.

TUCK: There are a number of alternative derivations and alternative formulas for the horseshoe vortex potential. An obvious rearrangement is:

$$V = \frac{z}{y^2+z^2}(1+x/r)$$

displaying the singularity on the positive (but not negative) x-axis. My paper (Tuck 1992) on lifting surfaces contains some further formulas, and this function appears in many standard aerodynamic reference texts.