On the Application of the Weak Scatterer Hypothesis to the Prediction of Ship Motions in Heavy Seas

by

Jacek S. Pawlowski
Institute for Marine Dynamics
National Research Council Canada
St. John's, Newfoundland, Canada

A solution to the problem of ship-wave interaction is sought by considering the perturbation introduced by the presence of a ship in the flow of steep oncoming waves. For the ship advancing with a mean forward speed U, the reference configuration of the ship is defined as the time independent configuration of static equilibrium maintained in a correspondingly advancing system of reference. This configuration together with the undisturbed free surface determines the reference fluid domain D_0 . Denoting by $\bar{x}$ radius vectors in D_0 , a transformation of D_0 onto the actual instantaneous fluid domain D is considered:

$$\bar{y} = \bar{x} + \bar{\eta}(\bar{x}, t) \tag{1}$$

where $\bar{y}$ are radius vectors in D . The transformation allows the governing equations of the fluid flow in D to be written in terms of quantities defined in D_0 , as follows:

$$\exp(\bar{\eta}\cdot\nabla)\ \nabla^2 \Phi = 0 \tag{2a}$$

$$\left[\exp(\bar{\eta}\cdot\nabla)\nabla\Phi - \frac{\partial}{\partial t}\bar{\eta}\right]\cdot(\bar{J} + \nabla\otimes\bar{\eta})^{-1}\cdot\bar{N} = 0 \tag{2b}$$

$$\exp(\bar{\eta}\cdot\nabla)\frac{\partial}{\partial t}\Phi + \frac{1}{2}\left[\exp(\bar{\eta}\cdot\nabla)\nabla\Phi\right]^2 + g\eta_3 = \frac{1}{2}U^2 \tag{2c}$$

with $\Phi = \Phi(\bar{x}, t), \bar{x} \in D_0$, denoting the velocity potential,

$$\exp(\bar{\eta}\cdot\nabla) = 1 + \bar{\eta}\cdot\nabla + \frac{1}{2}(\bar{\eta}\cdot\nabla)^2 + \ldots \tag{3}$$

and the assumption that transformation displacement field $\bar{\eta}(\bar{x}, t)$ is sufficiently smooth. Equation (2b) represents the impermeability condition, [1], applicable on the reference wetted surface of the hull, S_W , and on the reference free surface, S_F, whereas (2c) is the kinetic condition on S_F. $\bar{N}$ denotes the external normal vector.

Assuming the ship hull to be sufficiently slender, the weak scatterer hypothesis is imposed in the perturbation scheme by expressing total velocity potential Φ , as:

$$\Phi = -U x_1 + \Phi^{(1,0)} + \Phi^{(0,1)} + \Phi^{(2,0)} + \ldots \tag{4a}$$

where coordinate axis x_1, points in the direction of the mean forward velocity, with:

$$\bar{\eta} = \bar{\eta}^{(1,0)} + \bar{\eta}^{(0,1)} + \bar{\eta}^{(2,0)} + \ldots \tag{4b}$$

In (4a):

$$\Phi_W = \Phi^{(1,0)} + \Phi^{(2,0)} + \ldots \tag{5}$$

represents the velocity potential of the oncoming wave, and the following order of magnitude relations are assumed:

$$\Phi^{(2,0)} = O((\Phi^{(1,0)})^2) \quad , \; \Phi^{(0,1)} = o(\Phi^{(1,0)}), \;\; \Phi^{(2,0)} = O(\Phi^{(0,1)}) \tag{6}$$

Analogous order relations are applied also to the terms in (4b).

Relations (4) and (6) together with governing equations (2) imply that velocity potentials $\Phi^{(1,0)}$, $\Phi^{(0,1)}$ and $\Phi^{(2,0)}$ satisfy Laplace's equation in D_0. In addition, potential $\Phi^{(0,1)}$ can be represented as a sum of potentials $\Phi_U^{(0,1)}$ and $\Phi_S^{(0,1)}$, of which the former is the solution to the Neumann-Kelvin problem for the ship advancing in calm water. Scattering potential $\Phi_S^{(0,1)}$ satisfies the following impermeability condition on S_W and S_F ($\bar{N} = \bar{e}_3$):

$$\bar{N} \cdot [\nabla \Phi_S^{(0,1)} - (\frac{\partial}{\partial t} - U \frac{\partial}{\partial x_1}) \eta_S^{(0,1)}] = u_n \tag{7a}$$

with:

$$\begin{aligned} u_n = \bar{N} \cdot [(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x_1})(\bar{\eta}^{(1,0)} + \bar{\eta}^{(2,0)}) - (\frac{\partial}{\partial t} - U \frac{\partial}{\partial x_1}) \bar{\eta}^{(1,0)} \cdot \nabla \bar{\eta}^{(1,0)} + \\ - (1 + \bar{\eta}^{(1,0)} \cdot \nabla) \nabla \Phi^{(1,0)} - \nabla \Phi^{(2,0)} + \nabla \Phi^{(1,0)} \cdot \nabla \bar{\eta}^{(1,0)}] \end{aligned} \tag{7b}$$

Potential $\Phi_S^{(0,1)}$ fulfils also the linear kinetic condition on S_F:

$$(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x_1}) \Phi_S^{(0,1)} + g \eta_{S3}^{(0,1)} = 0 \tag{7c}$$

On the basis of the weak scatterer hypothesis $u_n = O(\Phi^{(0,1)})$ on S_W, and $u_n = o(\Phi^{(2,0)})$ on S_F as a consequence of (5). Therefore the scattering potential is determined by the quasi-linear radiation problem:

$$\bar{N} \cdot \nabla \Phi_S^{(0,1)} = v_n \qquad \text{on } S_W \tag{8a}$$

$$[(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x_1})^2 + g \frac{\partial}{\partial x_3}] \Phi_S^{(0,1)} = 0 \qquad \text{on } S_F \tag{8b}$$

with:

$$v_n = u_n + \bar{N} \cdot [(\frac{\partial}{\partial t} - U \frac{\partial}{\partial x_1}) \bar{\eta}_S^{(0,1)}] \tag{8c}$$

and the appropriate radiation condition.

In time domain simulations ϑ_n is computed directly on the instantaneous wetted surface:

$$\vartheta_n(\bar{y},t) = (\bar{\vartheta} - \bar{u}_W)\cdot\bar{n} + \bar{U}\cdot(\bar{n} - \bar{N}) \tag{9}$$

with $\bar{\vartheta}$ representing the velocity of the hull, $\bar{u}_W$ denoting the velocity induced by the oncoming wave, and $\bar{n}$ representing the instantaneous normal vector. In order to be applied in (8a) $\vartheta_n(\bar{y},t)$ must be mapped on S_W, the reference wetted surface. This can be accomplished by representing $\vartheta_n(\bar{y},t)$ in terms of a set of linearly independent square integrable functions $\Psi_{Ni}(\bar{x})$ defined on the hull surface and satisfying the Lipschitz condition:

$$|\Psi_{Ni}(\bar{x}) - \Psi_{Ni}(\bar{x}+\Delta\bar{x})| \leq M|\Delta\bar{x}| \tag{10a}$$

The mapping displacement $\bar{\eta}(\bar{x},t)$ on S_W can be represented as:

$$\bar{\eta}(\bar{x},t) = \bar{\eta}_0(\bar{x},t) + \bar{\eta}_1(\bar{x},t) \tag{10b}$$

where $\bar{\eta}_0(\bar{x},t)$ denotes the part of the displacement due to the motion of the hull in space. The functions Ψ_{Ni}, having been defined on the hull surface, are invariant with respect to $\bar{\eta}_0(\bar{x},t)$:

$$\Psi_{Ni}[\bar{x}+\bar{\eta}_0(\bar{x},t)] = \Psi_{Ni}(\bar{x}) \tag{10c}$$

A representation of $\vartheta_n(\bar{y},t)$ in terms of functions Ψ_{Ni} is written as:

$$\vartheta_n(\bar{y},t) = \sum_{i=1}^{N} \beta_i(t)\Psi_{Ni}[\bar{x}+\bar{\eta}(\bar{x},t)] \tag{11a}$$

where $\beta_i(t)$ are time dependent amplitudes. With the use of relations (10), representation (11a) is transformed to:

$$\vartheta_n(\bar{y},t) = \sum_{i=1}^{N} \beta_i(t)\Psi_{Ni}(\bar{x}) + o((\Phi^{(1,0)})^2) \tag{11b}$$

since $\Psi_{Ni}(\bar{x}) = O(1)$, $\beta_i(t) = O(\Phi^{(0,1)})$, and $|\bar{\eta}_1(\bar{x},t)| = O(\Phi^{(1,0)})$. Therefore the sum on the right hand side of (11b) defines ϑ_n on the right hand side of (8a).

It should be observed that the scattering problem defined by means of relations (8), (9) and (11b) does not depend explicitly on transformation field $\bar{\eta}(\bar{x},t)$. In addition the scattering problem can be solved in reference fluid domain D_0 by a method not related to representation (11a) of $\vartheta_n(\bar{y},t)$.

The weak scatterer hypothesis described above gives justification for the application of the linear free surface condition in the solution to the non-linear ship-wave interaction problem. The resulting scattering boundary value problem (8) is quasi-linear, because of its linear form and the non-linear content of ϑ_n. The derivation of the quasi-linear scattering boundary value problem given here provides an alternative to the presentation in [2], and follows a more general line of reasoning, [3]. Applications of the weak scatterer hypothesis presented in [4] and [2] confirm its usefulness. In addition, a possibility of the application of the weak scatterer hypothesis in the formulation of an effective quasi-linear radiation condition on an outer boundary is described in [5].

The theoretical model of ship-wave interaction based on the weak scatterer hypothesis assumes large (i.e. of the leading order of magnitude), $O(\Phi^{(1,0)})$, wave excitations and ship motions, but small (i.e. of the next order of magnitude), $O(\Phi^{(0,1)})$, scattering effects. The derived scattering solution contains errors of the magnitudes $o((\Phi^{(1,0)})^2)$ in the free surface and hull impermeability conditions, and therefore is $O((\Phi^{(1,0)})^2)$ consistent. In comparison, so called body non-linear models applied to surface ship scattering problems can be characterized as assuming large ship motions, $O(1)$, and small, $O(\Phi^{(1,0)})$, oncoming wave and scattering effects, with the errors of the scattering solution of the order of magnitude $O((\Phi^{(1,0)})^2)$ in the free surface and hull impermeability conditions, and therefore are $O(\Phi^{(1,0)})$ consistent.

References:

[1] Pawlowski, J.S. *"An Explicit Form of the Impermeability Condition and its Application in Hydrodynamics and Hydro-elasticity"*, Sixth Int. Workshop on Water Waves and Floating Bodies, Woods Hole, 1991, pp 207-210.

[2] Pawlowski, J.S. and Bass, D.W. *"A Theoretical and Numerical Model of Ship Motions in Heavy Seas"*, SNAME Annual Meeting, New York, N.Y., 1991.

[3] Pawlowski, J.S. *"A Non-linear Theory of Ship Motion in Waves"*, in preparation.

[4] Pawlowski, J.S., Bass, D.W. and Grochowalski, S. *"A Time Domain Simulation of Ship Motions in Waves"*, Proc. Seventeenth Symp. on Naval Hydrodynamics, 1988, pp 597-610.

[5] Pawlowski, J.S. *"Hydro-numeric Modelling of Ship Motion in Heavy Seas"*, Hydronav '91 Conf., Gdansk-Sarnowek, 1991.

DISCUSSION

YUE: You did not mention the (typical) wavelength to body dimension (B and L are comparable in this case) for the simulation/laboratory results you showed. This should be a relevant parameter for the validity of the weak scattering hypothesis.

PAWLOWSKI: In the experiment the results of which I showed the nominal wave length was 2.21 m at 30° course angle (at Fn=0.20) relative to the direction of wave propagation (the projected length 1.91 m), whereas the length of the model was 1.32 m on the waterline. The average wave height was 0.18 m. Other parameters of the model were L/B = 3.05, B/T=2.50 and C_B=0.68. The form and size of the ship relative to the length and direction of propagation of the ambient wave obviously affect the applicability of the weak scatterer hypothesis (see reply to Dr. Martin). For a discussion of the presented theoretical formulation when the weak scatterer hypothesis does not apply see my reply to Professor Grue.

GRUE: How does your theory compare with long wave slender body theory?

PAWLOWSKI: The applicability of the weak scatterer hypothesis does not depend inherently on the long wave and slender body assumptions, although the fulfillment of those assumptions may result in the applicability of the hypothesis (see my replies to Dr. Martin and to Professor Yue). It is however important to realize that if the weak scatterer hypothesis is not satisfied, the theory still gives a non-linear time domain formulation of the scattering problem, albeit not consistent in its non-linear part. In other words, the applicability of the weak scatterer hypothesis is necessary to construct a consistent non-linear scattering theory in which the linear free surface condition is used.

MARTIN: When is a scatterer weak?

PAWLOWSKI: Formally a scatterer is weak if the (disturbance) waves generated by the scatterer are sufficiently small in comparison with the ambient wave field. There is a number of physical circumstances under which this formal requirement is satisfied, [2]. For instance a sufficiently deeply submerged body of any shape becomes a weak scatterer, as does a slender ship advancing in bow waves. More generally a floating vessel which moves compliantly with the ambient waves is a weak scatterer. See also my reply to Professor Grue.