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A NEO-CLASSICAL APPROACH TO WAVE-DRIFT DAMPING

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Vessels moored in deep water can experience resonant low-frequency motions in the horizontal plane, due to excitation from second-order difference-frequency forces. An understanding of the relevant damping mechanisms is essential in order to estimate the resonant response.

Conventional first-order wave damping is asymptotically small at low frequencies. For example, the horizontal exciting force in long wavelengths is proportional to the pressure gradient of the incident waves, or $O(\sigma^2)$ for waves of unit amplitude and low frequency σ. Thus, from the Haskind relations, the horizontal damping coefficients are $O(\sigma^6)$. On the other hand, the second-order wave force acting on the body tends to a finite limit equal to the mean drift force, as the difference-frequency tends to zero. Thus, in the absence of other damping effects, resonant second-order motions will occur with velocity proportional to $O(\sigma^{-6})$ and amplitude proportional to $O(\sigma^{-7})$.

The relevance of 'wave-drift damping' has been established from several experimental and theoretical efforts. The usual theoretical approach considers the added resistance in waves associated with a quasi-steady translation of the body, with small velocity U. This force is proportional to the square of the incident-wave amplitude, tending to the zero-speed mean drift force as $U \to 0$, with the leading-order correction proportional to U. The derivative with respect to U, evaluated at $U = 0$, represents a force proportional to the velocity which can be interpreted as a damping coefficient. In some references this coefficient is derived analytically, using pertinent asymptotic analysis for $U << 1$; in others the damping coefficient is evaluated by numerical differentiation, from computations of the added resistance with small nonzero velocity.

In the present work wave-drift damping is analyzed in a more classical manner, without a quasi-steady forward velocity. The essential concept is that, whereas the linear wave damping is asymptotically small with respect to the frequency of oscillations *in calm water*, this is not the case for the higher-order damping coefficient associated with low-frequency oscillations *in the presence of incident waves.*

Consider the diffraction problem for incident waves with frequency ω and amplitude A, and also the radiation problem resulting from oscillatory surge motions with displacement $\xi(t) = \xi_{01} \sin \sigma t$. Ultimately it will be assumed that $\sigma << \omega$, but this approximation may be postponed. For this combination of inputs, the first several terms in the perturbation expansion of the velocity potential are as follows:

$$\begin{aligned}\phi(\mathbf{x},t) = \mathrm{Re}\Big(&\phi_{10}\, e^{i\omega t} + \phi_{20}^{(0)} + \phi_{20}^{(2)}\, e^{2i\omega t} + \ldots + \phi_{01}\, e^{i\sigma t} + \phi_{02}^{(0)} + \phi_{02}^{(2)}\, e^{2i\sigma t} + \ldots \\ &+ \phi_{11}^{(+)}\, e^{i(\omega+\sigma)t} + \phi_{11}^{(-)}\, e^{i(\omega-\sigma)t} + \phi_{12}^{(0)}\, e^{i\omega t} + \phi_{12}^{(2+)}\, e^{i(\omega+2\sigma)t} + \phi_{12}^{(2-)}\, e^{i(\omega-2\sigma)t} + \ldots \\ &+ \phi_{21}^{(0)}\, e^{i\sigma t} + \phi_{21}^{(2+)}\, e^{i(2\omega+\sigma)t} + \phi_{21}^{(2-)}\, e^{i(2\omega-\sigma)t} + \ldots\Big)\end{aligned} \tag{1}$$

The potentials ϕ_{mn} are complex, and depend on the space coordinates $\mathbf{x}$. The two subscripts refer respectively to the order of magnitude in A and ξ. Superscripts are used when necessary to denote harmonic time dependence in the respective frequencies. The conventional first-order diffraction and radiation potentials are ϕ_{10} and ϕ_{01}. The higher-order terms in (1) satisfy inhomogeneous boundary conditions, with various nonlinear effects from interactions of the lower-order potentials and body motions. Except for the incident-wave component of the diffraction potential, each ϕ_{mn} satisfies a radiation condition in the far field.

Analogous perturbation expansions apply for the hydrodynamic pressure and the resulting force acting on the body. Thus F_{10} is the first-order exciting force, $F_{20}^{(0)}$ is the second-order mean drift force, and the first-order force due to the surge motion is expressed in the usual form

$$F_{01} = -(i\sigma^2 A_{00} + \sigma B_{00})\xi_{01} \tag{2}$$

Here A_{00} and B_{00} are the conventional added-mass and damping coefficients. Similarly, the wave-drift damping coefficient B_{20} is defined as the corresponding coefficient of the force

$$F_{21}^{(0)} = -(i\sigma^2 A_{20} + \sigma B_{20})\xi_{01} \tag{3}$$

This force is of second-order in the wave amplitude, and of first-order in the surge amplitude.

The surge force can be derived from pressure integration in the form

$$\begin{aligned} F_x = \iint_{\bar{S}_b} p n_x dS = -\rho \iint_{\bar{S}_b} (\phi_t + \tfrac{1}{2}\nabla\phi\cdot\nabla\phi + \xi(t)\phi_{xt})n_x dS + O(\xi^2) \\ + \tfrac{1}{2}\frac{\rho}{g}\oint_{\bar{C}} (\phi_t^2 + \phi_t\nabla\phi\cdot\nabla\phi - \frac{1}{g}\phi_t^2\phi_{zt} + \ldots)n_x d\ell \end{aligned} \tag{4}$$

The term proportional to $\xi(t)$ and the neglected higher-order terms account for the transfer of the pressure from the oscillatory $(\bar{S}_b)$ to the mean $(\bar{S}_b)$ position of the body surface. Since the linear term is in phase with the surge displacement, and the neglected terms are of higher order in ξ relative to the force (3), these can all be ignored in computing the component of (3) in phase with the surge velocity. The waterline integral in (4), along the mean contour $\bar{C}$, accounts for the oscillatory intersection between the body and free surface.

After substituting the appropriate components of (1) in (4), and using (3) to separate the damping coefficient,

$$\begin{aligned} \sigma\xi_{01}B_{20} = \rho\,\mathrm{Re}\iint_{\bar{S}_b} \left(i\sigma\phi_{21}^{(0)} + \nabla\phi_{20}^{(0)}\cdot\nabla\phi_{01} + \tfrac{1}{2}\nabla\phi_{10}^{*}\cdot\nabla(\phi_{11}^{(+)} + \phi_{11}^{(-)})\right)n_x dS \\ - \tfrac{1}{2}\frac{\omega\rho}{g}\,\mathrm{Re}\oint_{\bar{C}} \phi_{10}^{*}\left[\omega(\phi_{11}^{(+)} + \phi_{11}^{(-)}) + \sigma(\phi_{11}^{(+)} - \phi_{11}^{(-)}) - i\nabla\phi_{10}\cdot\nabla\phi_{01}\right]n_x d\ell \end{aligned} \tag{5}$$

Hereafter the frequency of surge oscillations is assumed to be asymptotically small, thus $\sigma << \omega$ and $\sigma^2\ell << g$ where ℓ is the characteristic length scale of the body. With the surge velocity defined as $\dot{\xi}(t) = \sigma\xi_{01}\cos(\sigma t)$, the corresponding potential ϕ_{01} is real, of order σ, and non-wavelike in a suitably defined inner region. Substantial analysis is required to simplify (5), using appropriate boundary conditions on the body and free surface, together with Green's and Stokes' theorems. The contribution from the higher-order potentials $\phi_{20}^{(0)}$ and $\phi_{21}^{(0)}$ can be evaluated in this manner, in terms of an integral over the free surface of lower-order potentials.

The potentials $\phi_{11}^{(\pm)}$ are of primary importance. On the (mean) body surface these potentials satisfy the inhomogeneous boundary conditions

$$\phi_{11n}^{(\pm)} = \pm\tfrac{1}{2} i\xi_{01}\phi_{10xn} \tag{6}$$

The corresponding free-surface conditions (on $z = 0$) are

$$\begin{aligned} g\phi_{11z}^{(\pm)} - (\omega \pm \sigma)^2\phi_{11}^{(\pm)} = &\frac{i\omega}{2g}\phi_{10}\left(-\sigma^2\phi_{01z} + g\phi_{01zz}\right) \\ &\pm \frac{i\sigma}{2g}\phi_{01}\left(-\omega^2\phi_{10z} + g\phi_{10zz}\right) - i(\omega \pm \sigma)\nabla\phi_{10}\cdot\nabla\phi_{01} \end{aligned} \tag{7}$$

The solutions can be expanded in the forms

$$\phi_{11}^{(\pm)} = \pm\tfrac{1}{2}D_0 + \tfrac{1}{2}S_1 \pm \tfrac{1}{2}D_2 + \dots \tag{8}$$

where (S, D) denote the sum and difference $\phi_{11}^{(+)} \pm \phi_{11}^{(-)}$, respectively. The potentials D_j and S_j are of order σ^j, and of the same order as $\phi_{11}^{(\pm)}$ with respect to the wave and surge amplitudes. Since the right side of (7) is proportional to $\phi_{01} = O(\sigma)$, the leading-order coefficient in (8) satisfies a homogeneous free-surface boundary condition. The appropriate potential which satisfies (6) and the radiation condition is

$$D_0 = i\xi_{01}\left(\phi_{10x} + iK\cos\beta\,\phi_{10}\right) \tag{9}$$

The potential S_1 is subject to a homogeneous body boundary condition and the free-surface condition

$$gS_{1z} - \omega^2 S_1 = 2\omega\sigma^2 D_0 + i\omega\sigma\phi_{10}\phi_{01zz} - 2i\omega\sigma\nabla\phi_{10}\cdot\nabla\phi_{01}, \quad \text{on } z = 0 \tag{10}$$

After further analysis the wave-drift damping coefficient is evaluated in the form

$$\begin{aligned} \sigma\xi_{01}B_{20} = &\tfrac{1}{2}\rho\,\mathrm{Re}\iint_{S_b}\phi_{10xn}^{*}S_1\,dS - \tfrac{1}{2}\frac{\omega\rho}{g}\mathrm{Re}\; i\iint_{S_f}\phi_{01z}\phi_{10}^{*}\phi_{10zz}\,dS \\ &+ \tfrac{1}{2}\frac{\omega\rho}{g}\mathrm{Re}\oint_{C}\phi_{10}^{*}\left(-i\nabla\phi_{10}\cdot\nabla\phi_{01} + \sigma D_0\right)dy \end{aligned} \tag{11}$$

where the second integral is over the free surface.

This rather complicated expression can be simplified considerably by using Green's theorem to replace the integral over the body by integration over the free surface and a control surface S_c which surrounds the body. The contributions from the waterline and free surface cancel in the final result:

$$\sigma\xi_{01}^2 B_{20} = -\tfrac{1}{2}\rho\,\mathrm{Re}\; i\iint_{S_c}\left(D_{0n}^{*}S_1 - D_0^{*}S_{1n}\right)dS \tag{12}$$

This equation can be confirmed from a complementary analysis based on energy flux, but that approach is no simpler. Since the work done by the wave-drift damping coefficient is proportional to σ^2 it is necessary to analyse the potential D_2 in (8), and also the potential $\phi_{12}^{(0)}$ in (1).

The principal complication in evaluating (12) is the need to solve for the potential S_1, which satisfies the inhomogeneous free-surface condition (10). Special attention is required for the first term on the right side of (10), since the potential D_0 is a solution of the homogeneous free-surface condition. The solution can be expressed formally as

$$S_1 = \frac{2\omega\sigma^2}{g}\frac{\partial}{\partial K}D_0 + \bar{S}_1 \tag{13}$$

where $\bar{S}_1$ satisfies (10) without the term $2\omega\sigma^2 D_0$ on the right-hand-side. For large values of the horizontal radius R, the scattered component of D_0 is asymptotic to $f(\theta)R^{-1/2}\exp(Kz - iKR)$, where $f(\theta)$ is a slowly-varying function of the angular coordinate θ about the vertical z-axis. Thus the first term on the right side of (13) is secular with its amplitude proportional to $R^{1/2}$. The remaining terms on the right side of (10) are of order $1/R^3$, and the corresponding solution $\bar{S}_1$ is regular with the same far-field form as a first-order radiating wave.

Since the secular solution in (13) is nonuniform in the far field, analytical problems arise regarding the correct interpretation of the integral over the control surface in (12). Another issue is the computational problem of evaluating the derivative in (13). Both of these problems can be overcome in the following manner. First substitute (13) in (12), with the control surface situated at an intermediate radius where KR=O(1). From the chain rule it follows that

$$\begin{aligned}\sigma\xi_{01}^2 B_{20} = &-\tfrac{1}{2}\rho \,\mathrm{Re}\, i \iint_{S_e} (D_{0n}^* \bar{S}_1 - D_0^* \bar{S}_{1n})dS \\ &- \tfrac{1}{2}\frac{\rho\omega\sigma}{g}\,\mathrm{Re}\, i \frac{\partial}{\partial K}\iint_{S_e} (D_{0n}^* D_0 - D_0^* D_{0n})dS.\end{aligned} \tag{14}$$

In this form the evaluation of the derivative with respect to the wavenumber is simplified, and the control surface can be located arbitrarily far from the body. The integrals over S_e can be evaluated in terms of the far-field scattering amplitudes $f(\theta)$, in a similar manner as the mean drift force.

This work differs in its basic approach from the papers by Nossen, Grue, & Palm (1991) and Emmerhoff & Sclavounos (1991), which are based on the concept of a small quasi-steady forward velocity. In those works the potential corresponding to (8) satisfies a homogeneous body boundary condition, and there is no explicit consideration of the component (9). Nevertheless there are many similarities in the analysis, and the final results are expected to be equivalent.

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REFERENCES

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DISCUSSION

GRUE: Is there an easy way to separate out the effect on the wave drift matrix of the yaw angle compared to the yaw angular velocity. I would expect the yaw angle (the position) to be important to the forces and moments. Is the same true with respect to the yaw angular velocity?
NEWMAN: In fact, the wave-drift damping coefficients do not depend on either the yaw angle or yaw velocity, both of which are assumed small. (This situation is the same as in conventional damping coefficients derived from linear theory.) But if by 'yaw angle' you mean the O(1) angle of incidence (β) of the waves in relation to the body, it is clear that this angle is of importance.

EATOCK TAYLOR: If this approach is 'neo-classical', one can ask to what stylistic school the conventional approach belongs. Analogies in the world of painting might be 'abstract expressionist', or simply 'abstract'. 'Realism' would perhaps not be suitable. In architecture it might be suggested that 'brutalism' is the appropriate 'ism' to apply! But I would argue for 'functionalism'. On a more serious note, I would be interested to know if there are any numerical difficulties we should be aware of, in the exciting prospect of obtaining a 3x3 damping matrix by use of the 'neo-classical' method.
NEWMAN: The most significant numerical difficulty I forsee is in the boundary condition (6) on the body, which requires double spatial derivatives of the first-order diffraction potential.

PALM:
1. I think this work is important since it derives a unified theory for the three horizontal modes. For surge and sway, however, which has been considered in the quasi-steady theories, I do not expect that your approach leads to any new results. For, if you instead are working in a body-fixed frame of reference the non-Newtonian force is proportional to ξ^2 and may be neglected. Hence we are considering a fixed body in a slowly varying current. It seems to me obvious that the first approximation as $\sigma \to 0$, is obtained by considering the body imbedded in a *steady* current.
2. You must in your approach assume that the horizontal displacement of the body, ξ, is small, which is not realistic. This assumption is not made in the quasi-steady approaches, but we end up with the same results.
Do you have any comments?
NEWMAN: I accept the first point, although it is not completely 'obvious' in my view. Regarding the small-amplitude assumption it should be noted that in both approaches the objective is a damping coefficient which is independent of ξ, or the coefficient of the force component proportional to $\dot{\xi}$. Since this coefficient is independent of ξ the magnitude of ξ is irrelevant.