Investigation on time-harmonic disturbances for inner-Kelvin-angle wave packets

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The frequently observed phenomenon of wave groups radiating along straight lines with an inclination angle α smaller than the Kelvin angle α_k against the ship's course, systematically produced and analyzed by experiments with the cutter Point Bower [1], has not yet found a satisfactory explanation using steady wave models (see e.g. Mei [2]). However, it is well known (see e.g. Eggers [3]) that time-harmonic disturbances advancing with the ship, essentially governed by the Brard parameter τ, the product of ship speed U and the ship borne frequency ω divided by the gravity constant g, produce a wave system which include wave cusp angles smaller than α_k. Cao [4] simulated this situation for singularity systems of time-harmonic intensity started from rest first by a linear approach, using the general time-dependent Green's function and further by a nonlinear time-stepping numerical approach using desingularized boundary integral methods. In both cases his computed wave cusp angles were in qualitative accord with analytical predictions by the method of stationary phase. Cao found that wave packets, with crests oblique to the ray direction, move away from the disturbance along a ray in a reference system fixed to the disturbance. Even this is confirmed by our stationary phase analysis, which predicts that along an additional wave cusp *outside* the Kelvin wedge waves travel away from the ship even if viewed from a system at rest. Cao's numerical results show an amplitude decay rate along the cusp like the inverse third root of the distance as predicted by the stationary phase approach and furthermore predicted by Akylas [5] in nonlinear studies. We are extending Cao's investigations to a wider range of τ and, with the linear code, are examining the decay rate along the cusp (where we have evaluated data from the experiments) and study how it can be affected by ship form characteristics.

Simultaneously, we search for the possibility of higher-order resonance interactions between such unsteady wave components and/or the steady ones in the spirit of Phillips [6] or Longuet-Higgins [7]. They showed that three wave components (in our case with two of them being identical as a degenerate case) transfer energy to a fourth wave, if both the wave number vectors and the frequencies anihilate each other, finally causing a mutual exchange of energy and hence a slow variation of amplitude between all four (actually three) components.

Phillips has condensed this particular resonance situation through a visualisation in a plane of wave number vectors (see Fig. 2) where the wave component $\overline{k_1}$, to be counted twice, determines the principal axis and the length scale in such a way that both $\overline{AO}$ and $\overline{OB}$ represent $\overline{k_1}$. Then resonance occurs for any vector $\overline{k_2}$ with a tip on a certain figure-

*Permanent Adress: Univ. Michigan, Dept. of Mechanical Engineering and Applied Mechanics of-eight curve; the resulting tertiary wave vector then is $\overline{k_1} - \overline{k_2}$. Note that the polar coordinate angle Θ represents the difference in direction of the two vectors and the polar distance actually their ratio; one might thus conceive this as a Gaussian plane for the ratio of wave vectors expressed as complex numbers.

Newman [8] has shown that for steady ship waves such resonance can occur only along the cusp line. In fact, he found resonances only for the trivial case that $\overline{k_1}$ and $\overline{k_2}$ are identical as in point O in Fig. 2. This happens along the cusp line of a Kelvin pattern due to the transition from transverse to divergent waves. However, for non-zero τ there are additional chances. This becomes evident from a plot of the dependence of the coordinate angle α on the wave front angle θ (both measured against the positive x-axis aft of the disturbance) associated through stationary phase analysis. This is shown in Fig. 1 and is equivalent to a diagram given by Eggers [3]. For larger values of τ the curves are significantly different from those for $\tau = 0$ investigated by Newman. In particular for $\tau > 1/4$, we find closed curves, though still with zeros of α for θ equal to a multiple of $\pi/2$. To cover the range of negative θ, which cannot be disregarded for positive α, the curves of Fig. 1 should be continued through rotation around the origin by 180 degrees.

The upper section of the closed curve in Fig. 1, up to the vertical tangent, represents "ring waves", just as the corresponding monotonic curve for $\tau < 1/4$ (named so with regard to the limiting case of vanishing ship speed), the lower section, including the negative part for $\tau < 1/4$ represents "speed waves", which degenerate to Kelvin waves with vanishing frequency. Note that the minimum of α for $0 < \theta < \pi/2$ represents a cusp angle of absolute value smaller than α_k and thus may correspond to the inclination of an inner-angle feature observed in experiments and to its simulation through our numerical approach.

For an investigation of resonance interaction, we have to consider wave propagation in a system at rest. Then the speed waves in the range $-\pi/2 < \theta < 0$, shown in the range of positive α, which travel away from the disturbance in the ship-borne system should appear with θ changed by 180 degrees, i.e. as waves following the disturbance in the system at rest. We are anxious to detect possible resonances if additional curves for τ positive are inserted in Newman's resonance diagram redrawn as Fig. 2. We found that for "self resonance", i.e. for interactions of instationary speed- or of ring-waves, we only get curves very close to that drawn for $\tau = 0$ by Newman, so that again only the trivial resonance occurs for maximum α with identical θ's. For $0 < \tau < 1/4$ the speed wave curves in the range $-\pi/2 < \theta < 0$ and $\pi/2 < \theta < \pi$ become different in shape, the first with maximum α exceeding α_k, whereas the second shows a cups angle less than α_k. These systems have non-trivial resonances with ring waves in the range of α where speed waves occur. For the case $\tau > 1/4$ and α positive, we actually found the resonance criterion satisfied within a wide range of τ for interaction of speed waves with both values of θ (from different branches) slightly exceeding $\pi/2$. These interaction angles were found within the inner cusp angle.

We are aware that the investigations of Phillips were actually based on interactions among long crested wave trains rather than local wave packets. Moreover, Akylas has questioned the uniform validity of the perturbation expansion underlying Newman's approach. To find whether such objections are relevant or if the resonance model we investigate really describes a measurable physical phenomenon, we suggest to perform a simple experiment: Install two oscillatory wave generaters operating at frequencies σ_1 and σ_2 at two points P^1 and P^2 in a wide tank, and let $\sigma_1 = \kappa\sigma_2$, with $1/2 < \kappa < 3/2$. Then for deep-water waves without capillarity, the wave numbers k_1 and k_2 of the ring waves produced are related through $k_1 = \kappa^2 k_2$, and there is at least one point P on the figure-of-eight curve with distance

κ^2 from the point A. Let Θ stand for the angle of $\overline{AP}$ against $\overline{AO}$. Then the resonance phenomenon should be observed at any point Q in the undisturbed free surface located on the periphery of the two circles connecting Q_1 and Q_2 for which $\overline{QQ_1}$ and $\overline{QQ_2}$ intersect with angle Θ.

Little experimental data on the far-field decay of ship waves seem to be available. Evaluating the results from the Point Bower, we observed a decay rate of the amplitudes more like the fourth inverse root of distance. We found a similar tendency for the bow wave cusp line of the 23m very long slender model investigated by Adachi [9] where a record over more than 8 periods along the cusp of the bow wave is available. We are continuing to check if, at least for the unsteady case, wave resonance may contribute to lowering the decay rate.

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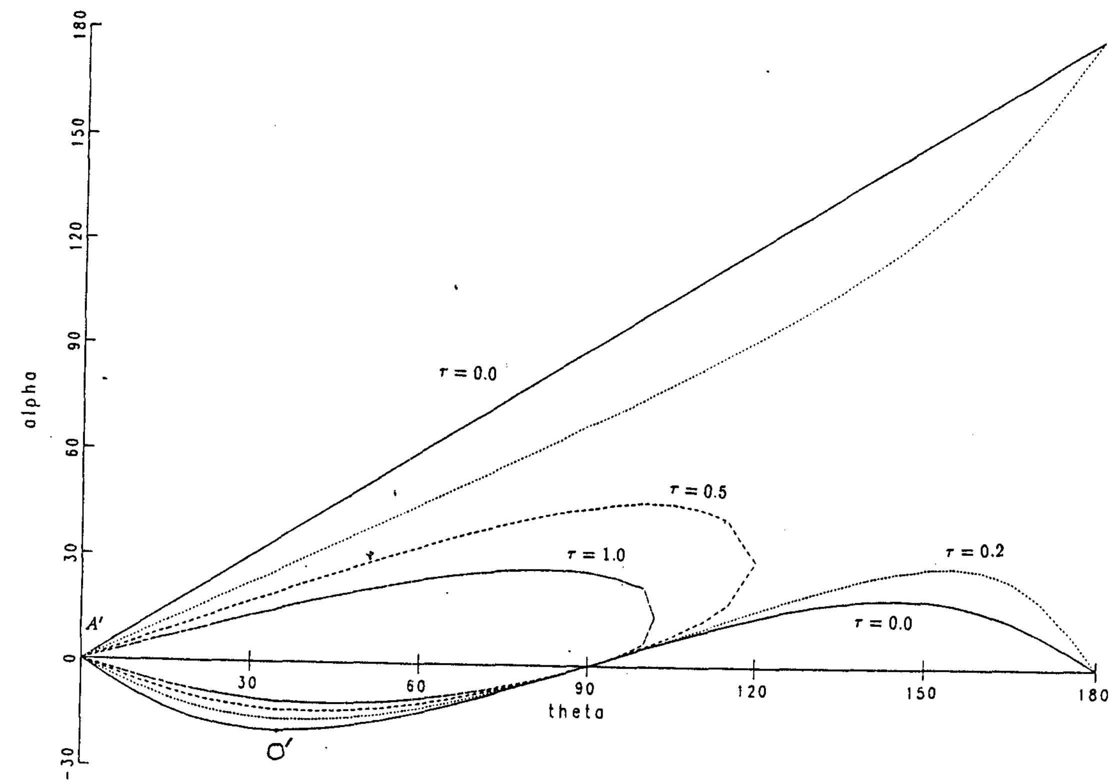

Fig. 1: The stationary wave results of α vs. θ (wave vector direction) for varying values of τ. The curves going to $\alpha = 180$ are the ring waves for $\tau < \frac{1}{4}$ (Eggers [3]). The curves for $\tau > \frac{1}{4}$ show two extrema—one with absolute value less than α_k (here shown as negative α) and one greater than α_k, representing the cusps of the two wave systems.

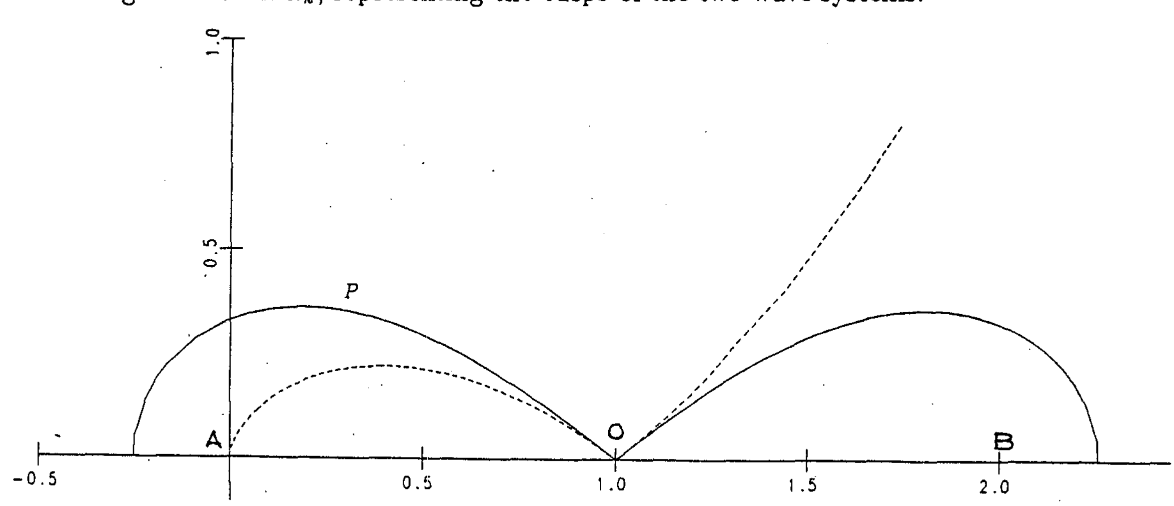

Fig. 2: Resonance of Ship waves systems (redrawn from Newman [8]). Shown is the plane of wave number vectors, the scale is arbitrary. The vector which is to be doubled should be mapped onto both $\vec{AO}$ and $\vec{OB}$. Any point P on the figure-of-eight curve (half is ommitted here due to symmetry) specifies two other members ($\vec{AP}$ and $\vec{PB}$ of the "triad". The dotted curve inserted by Newman is an image of the $\tau = 0$ curve in our Fig. 1; the point A' ($\theta = \alpha = 0$) and the point O' (where α has an extreme value) correspond to A and O, respectively.

DISCUSSION

NOBLESSE: I agree with the authors that the unsteady wave patterns generated by a ship advancing in waves are likely to be at the origin of some of the far-field features observed via remote sensing. On the other hand, preliminary numerical calculations of the slopes and curvatures of the free surface for the steady wave pattern of a ship advancing in calm water indicate a highly pronounced peak along a narrow ray at an angle roughly half the Kelvin cusp angle. Thus, it is possible that both the unsteady and the steady components of a ship wake contain features likely to be observed at large distances.

EGGERS & SCHULTZ: The inner-angle phenomena investigated experimentally are certainly of such complex character that from the data available through the restrictive instrumental arrangements we cannot determine the pertinence of steady or unsteady wave models. And it is certainly true that effects of slope and curvature are often disregarded in studies of Kelvin patterns, although they may help to explain local peculiarities such as wave breaking. However, all linear steady theories predict that such features decay like $R^{-1/2}$ inside a Kelvin wedge (for which the position of its apex is definitely moot!). If the phenomenon referred to should persist in the far field, it should be possible to prove analytically that they have a lower rate of decay. We would like to encourage the discusser to use the analytical tools he has created within the last years to clarify the situation.

MILOH: I wonder if you be kind enough to give us your learnt interpretation of the physical meaning of the narrow bright V-shape clearly seen in the photograph which you showed us?

EGGERS & SCHULTZ: The bright vee seems to come from turbulence and breaking waves in the viscous wake. The bright vee can become dark further behind the ship. Then the likely physical mechanism is surfactant (or perhaps turbulence) damping of ambient waves as described by Peltzer et al. in the last ONR symposium.