Radial Convergence of the Second Order Free Surface Integral

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As the main source of springing excitation on Tension Leg Platforms, second order wave loads at high frequency have received increasing attention in both research activities and engineering practice. For the past ten years, a number of theoretical studies have been devoted to the subject and great progress has been made in the numerical evaluation of the second order loads. The results published were however incomplete or controversial. Only recently an excellent agreement of numerical results for a real TLP from different models [2] [5] has been obtained. The main difficulty lies in the evaluation of the contribution from the second order velocity potential. As the sum-frequency we are interested in may be as high as 2.7 rad/sec a large number of panels (nearly 14000) are required to represent the TLP's hull. Not only the cost of memory space and computer CPU time become prohibitive, but also the numerical convergence is difficult to obtain. In this paper, the slow convergence of the free surface integral for the high frequency loads in vertical direction is analysed by modeling a TLP's column as a truncated vertical cylinder where a semi-analytical solution is derived.

There are two ways to evaluate the second order wave loads on large volume structures. The first involves integration of the hydrodynamic pressure over the hull after having solved the second order problem which is quite complicated as a non-homogeneous condition over the free surface has to be satisfied (calculation of the second order potential may nowadays be achieved just for bodies of simple geometry). The second is based on the Haskind integral relation in which the second order loads contributed by the second order potential are completed by the free surface integral (Molin [4]):

$$f^{(2)}_{2Dj} = i(\omega_1 + \omega_2)\frac{\rho}{g}\iint_{z=0} \alpha_D \cdot \psi_j \, dS \tag{1}$$

whose integrand is a product of an additional radiation potential ψ_j arising from the body motion in the jth direction, and the quadratic term α_D on the right hand side of the free surface condition for the second order diffraction problem :

$$\begin{aligned}\alpha_D = {} & i(\omega_1+\omega_2)\Big[\nabla(\phi_{I1}+\phi_{D1})\cdot\nabla\phi_{D2} + \nabla\phi_{D1}\cdot\nabla\phi_{I2}\Big] \\ & -\frac{i\omega_1}{2g}\Big[(\phi_{I1}+\phi_{D1})\cdot(-\omega_2^2\frac{\partial}{\partial z}+g\frac{\partial^2}{\partial z^2})\phi_{D2} + gk_2^2(1-\mathrm{th}^2 k_2 H)\phi_{I2}\cdot\phi_{D1}\Big] \\ & -\frac{i\omega_2}{2g}\Big[(\phi_{I2}+\phi_{D2})\cdot(-\omega_1^2\frac{\partial}{\partial z}+g\frac{\partial^2}{\partial z^2})\phi_{D1} + gk_1^2(1-\mathrm{th}^2 k_1 H)\phi_{I1}\cdot\phi_{D2}\Big]\end{aligned} \tag{2}$$

where ϕ_I and ϕ_D are the first order incident and diffraction potentials, and indices $_1$ and $_2$ correspond to the wave frequencies ω_1 and ω_2.

With this integral expression, it is thus possible to get the diffraction loads without explicitly solving the second order diffraction problem. However, the numerical computation of this free surface integral is not simple as α_D and ψ_j oscillate with a weak attenuation in the radial direction. Of special concern is the fact that the infinite integral converges more slowly in the evaluation of the vertical forces for deep water, and so the truncated radius must be large enough.

To analyse the convergence rate of the free surface integral, for the sake of simplicity, one TLP column is assimilated to a truncated vertical cylinder [1]. The wave field around the cylinder can be solved by Garrett's method [3] in which the relevant eigen function expansions are used to represent the velocity potentials in the inner domain limited by the cylinder base and a circular matching surface extended from the cylinder base to the sea bed, as well as in the outer domain where the first order diffraction potential is written by :

$$\phi_D = -\frac{ag}{\omega}\sum_{l=0}^{\infty}\phi_D^l \cos l\theta \qquad (3)$$

with :

$$\phi_D^l = b_0^l \frac{\text{ch}k_0H\,\text{ch}k_0(z+H)}{2k_0H+\text{sh}2k_0H}\frac{\mathbf{H}_l(k_0r)}{\mathbf{H}_l(k_0R)} + \sum_{m=1}^{\infty} b_m^l \frac{\cos k_m(z+H)}{2k_mH+\sin 2k_mH}\frac{\mathbf{K}_l(k_mr)}{\mathbf{K}_l(k_mR)} \qquad (4)$$

and the first order radiation potential due to heaving motion :

$$\psi_3 = c_0 \frac{\text{ch}k_0H\,\text{ch}k_0(z+H)}{2k_0H+\text{sh}2k_0H}\frac{\mathbf{H}_0(k_0r)}{\mathbf{H}_0(k_0R)} + \sum_{m=1}^{\infty} c_m \frac{\cos k_m(z+H)}{2k_mH+\sin 2k_mH}\frac{\mathbf{K}_0(k_mr)}{\mathbf{K}_0(k_mR)} \qquad (5)$$

In above expressions R is the cylinder's radius, $\mathbf{H}_l$ and $\mathbf{K}_l$ are Bessel functions and the coefficients b_0^l, b_m^l, c_0 and c_m are determined by the matching condition over the surface which separates the inner and outer domains. The propagating and evanescent wave numbers are defined by :

$$\omega^2 = gk_0\text{th}k_0H \quad \text{and} \quad -\omega^2 = gk_m\tan k_mH \quad \text{for } m\geq 1 \qquad (6)$$

Note that the lowest value k_1 of the evanescent modes is between $\pi/2H$ and π/H.

At the small wave periods that we are interested to, it can be easily shown that the coefficients b_m^l $(m\geq 1)$ tend to $O(e^{-k_0D})$ and :

$$b_0^l = -\frac{2k_0H+\text{sh}2k_0H}{\text{ch}^2k_0H}\varepsilon_l i^l \mathbf{J}_l'(k_0R) + O(e^{-k_0D}) \quad (\varepsilon_0=1,\ \varepsilon_l=2 \text{ for } l\geq 1) \qquad (7)$$

We obtain the same expression as for a vertical cylinder standing on the sea bed. The first order wave field is then attenuated in the order of $O(e^{-k_0D})$ in vertical direction. Introducing the equation (3) for ϕ_D and the series expression for ϕ_I into (2), the non-homogeneous term α_D can be written in series expansion as well :

$$\alpha_D = a_1a_2\omega_1\omega_2(\omega_1+\omega_2)\sum_{l=0}^{\infty}\alpha_D^l(r)\cos l\theta \qquad (8)$$

where the components α_D^0 and α_D^1 are the most useful in the free surface integral for heave forces, surge forces and pitch moments. They tend asymptotically to the order $O(1/r)$ as $r\rightarrow\infty$. The free surface integral for the heave forces is written then :

$$f_{2D3}^{(2)}/(\rho gRa_1a_2) = \frac{2\pi i\omega_1\omega_2(\omega_1+\omega_2)^2R^2}{g^2}\int_R^{\infty}\alpha_D^0(r)\psi_3(r)rdr/R^2 \qquad (9)$$

This integral is semi-analytically integrated and its truncated radius depends largely on the asymptotic behaviour of the additional heaving potential $\psi_3(r)$ in the radial direction.

In the limiting case of small periods (high frequencies) of oscillation for the heaving potential, it is found that the coefficient c_0 in the equation (5) is as small as $O(e^{-k_0D})$. The potential field arises from the heaving oscillation at high frequency is thus dominated by the evanescent components ("local" waves) in the radial direction : outgoing propagation waves do not appear and the attenuation rate is dependent on $\mathbf{K}_0(k_mr)$ which is known to decay in the order of $O(e^{-k_mr})$. However, k_m is inversely proportional to the waterdepth and therefore the "local" waves in deep water may not be negligible even at a large distance from the cylinder center.

Figure 1 presents the real part of the heaving potential over the free surface at a sum-frequency ($\omega_1 + \omega_2 = 2.65$ rad/sec) for a vertical circular cylinder of radius $R=12.5$ m and draft $D=3R$. The water depth varies from $H=1.5D$ to $H=4.0D$. As the propagating waves are nearly absent, the imaginary part of the potential is negligible. At large radial distance, the potential at greater depths is more important than that for small water depth. The evanescent mode waves are shown to extend much further when the water depth increases. Consequently, the convergence ratio for heave forces is lower. Comparison with the convergence ratios for surge forces and pitch moments is shown by figure 2 at different truncated radial distances. The minimum truncated radial distance derived from the figure is 20 times the cylinder radius for heave forces while 10 times the radius is sufficient for surge forces or pitch moments. Figure 3 presents the evaluation of the free surface integral values as a function of the truncated radial distance, for the difference-frequencies varying from $\omega_1 - \omega_2 = 0.0$ to $\omega_1 - \omega_2 = 0.2$ rad/sec. It is shown again that the convergence of the integral for heave forces is slow.

For large volume structures like Tension Leg Platforms, the contribution to the radiation potential due to structural pontoons' heaving motion at high frequency is characterized by a locally sharp variation on the free surface just above pontoons and a small propagating wave component at large radial distance. This may be explained by the two opposite effects from upper and lower sides of pontoons during heaving interaction with water. The radiation field at large radial distance principally arises from heaving motion of columns' bases. The convergence rate of the free surface integral for TLP's is then quite similar to that for one vertical cylinder. It is then expected that a much larger truncated radius is needed to obtain accurate evaluation of the vertical forces upon TLP's.

Finally, figure 4 shows the quadratic transfer functions for heave forces upon a Tension Leg Platform column, in a series of bichromatic waves defined by the difference-frequency from 0.0 to 0.2 rad/sec. The heave forces at a given high frequency decrease rapidly when the difference of wave frequencies increases, due to the cancellation effects in the free surface integral. These effects are shown to be more important for higher wave frequencies.

References

[1] **X.B. Chen** 1990 : "Analytical evaluation of the second order loads upon a Tension Leg Platform column". IFP's report No:38 739.

[2] **X.B. Chen, B. Molin** and **F. Petitjean** 1991 : "Faster evaluation of resonant exciting loads on Tension Leg Platforms", Proceeding of the 8th International Symposium on Offshore Engineering, Brasil.

[3] **C.J.R. Garrett** 1971 : "Wave forces on a circular dock". J. Fluid Mech. Vol:46, Part 1, pp129-139.

[4] **B. Molin** 1979 : "Second order diffraction loads upon three-dimensional bodies", Applied Ocean Research. Vol.1, pp197-202.

[5] **J.N. Newman** and **C.H. Lee** 1992 : "Sensitivity of Wave Loads to the Discretization of Bodies" BOSS '92, London.

Figure 1 - Radiation potential by heaving motion at the sum-frequency ($\omega_1+\omega_2$=2.65 rad/sec) of the vertical cylinder in different waterdepths : ■— $H = 1.5D$, ○— $H = 2.0D$ and ▲— $H = 4.0D$.

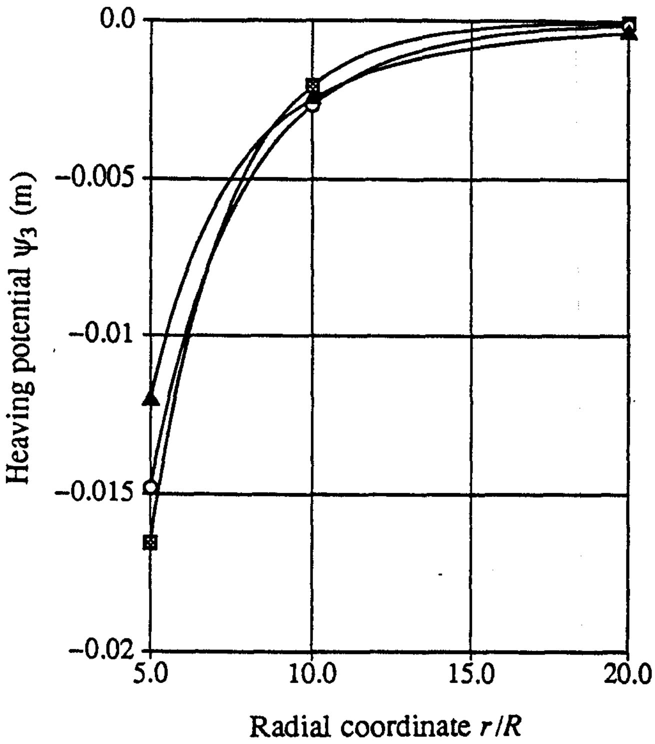


Figure 2 - Ratios of the free surface integral ($\omega_1+\omega_2$=2.65 rad/sec) truncated values at R_T to the values truncated at $30R$: ■ — surge forces, ○ — heave forces and ▲— pitch moments.

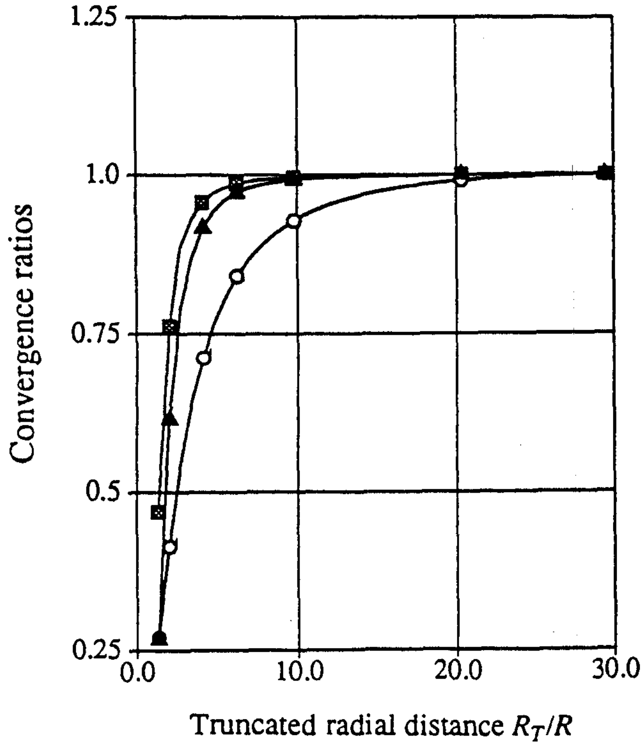


Figure 3 - Truncated values of the free surface integral of heave forces at $(\omega_1+\omega_2) = 2.65$ rad/sec for the vertical cylinder in waterdepth $H=4D$: ■— $(\omega_1-\omega_2) = 0.0$, ○—$(\omega_1-\omega_2) = 0.1$ and ▲— $(\omega_1-\omega_2) = 0.2$ rad/sec.

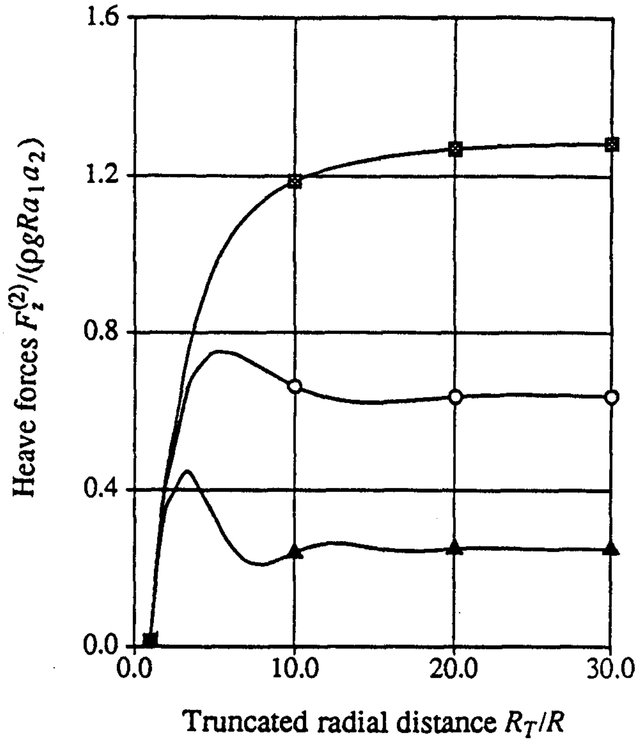
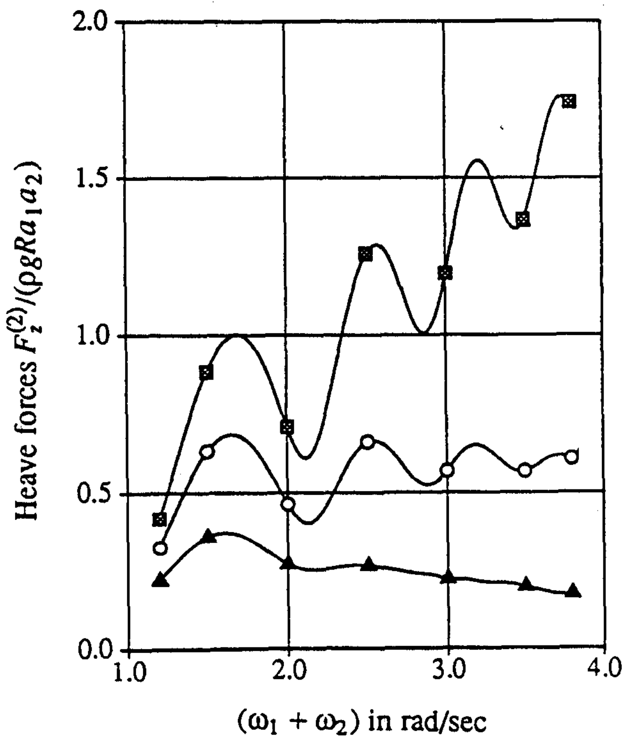


Figure 4 - Quadratic transfer function of the heave forces non-dimensioned by $(\rho g R a_1 a_2)$ on the vertical cylinder in waterdepth $H=4D$: ■— $(\omega_1-\omega_2) = 0.0$, ○—$(\omega_1-\omega_2) = 0.1$ and ▲— $(\omega_1-\omega_2) = 0.2$ rad/sec.



DISCUSSION

CLARK: There are some effective algorithms for evaluating the far-field contribution to the second-order force due to the wave-like components in the free surface integral, equation (1), (e.g. Kim and Yue). The question is do we need an asymptotic method for evaluating the wave free (evanescent) contribution in the free surface integral and what form would it take?
Ref.: M.H Kim and D.K.L. Yue, J.F.M. vol 200, pp 235-264, 1989.

CHEN: The second-order free surface integral (Eq. 1) is usually carried out by dividing the infinite surface into two regions: the interior and exterior regions. The numerical quadrature method is applied on the interior region surrounding the body. On the exterior region, asymptotic expressions of the integrand are used to evaluate the far-field contribution. For the second-order heave forces at large wave frequencies, it is shown in this paper that the wave-like contribution is negligible. To calculate the free surface integral on the interior region, the numerical quadrature algorithm may be the only way to obtain a good precision for a floating body of arbitrary geometry. It is possible to formulate asymptotic expressions for the evanescent contribution of the free surface integral in the case of vertical cylinders, although the exact numerical integration (Eq. 9) is very fast.