Standing waves in numerical tanks

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Introduction

At last year Workshop [1], we reported experimental and numerical results related to the nonlinear propagation of bichromatic wave trains. In particular, we presented numerical simulations of the propagation of such wave trains using the Sindbad code that simulates a two-dimensional wave tank [2].

This code uses an "absorbing" beach in order to avoid the reflection of the waves at the end of the tank. This beach was previously shown to be quite efficient for the absorption of the wave components to which it is tuned. In fact, the reflection coefficient in that case is smaller than 2% (in amplitude), a result better than that of most real tanks. During these simulations for bichromatic waves, however, large amplitude standing waves appeared for some critical values of the parameters (difference frequency used and length of the tank). These standing waves interacted with the fundamental components, resulting in an erroneous evolution of the wave train with the distance from the wavemaker in the critical conditions. It appeared important to get a better knowledge of the standing waves of the numerical tank. These were therefore studied analytically in the linear case.

The numerical beach (linear problem)

In the numerical beach, the classical free surface boundary conditions are modified by adding a restoring term. For the linearized free surface boundary conditions, this yields:

$$\phi_t + g\eta - \nu_1(x)\,\phi = 0$$
$$\eta_t - \phi_z - \nu_2(x)\,\eta = 0$$

where the "damping" coefficients ν_i (i=1 or 2) are equal to zero outside the numerical beach ($x \leq x_0 = L-\beta\lambda$) and to $\alpha_i\,\omega\,(x-x_0)^2/\lambda^2$ in the numerical beach ($x \geq x_0$). λ and ω are the wave length and wave frequency to which the beach is tuned. The non-dimensional parameters α_i and β are constant (and usually taken of order 1).

We can therefore write the following free surface boundary condition for ϕ:

$$\phi_{tt} + g\phi_z + (\nu_1 + \nu_2)\,\phi_t + \nu_1\,\nu_2\,\phi = 0\,, \quad \text{for } z = h$$

Standing waves in the numerical tank

We look for standing waves in the numerical tank by writing

$$\phi = \sum_{n=0}^{N} A_n \cos k_n x \frac{\cosh k_n z}{\cosh k_n h} \exp(i\omega t) ,$$

where $k_n = n\pi/L$, h is the depth, and where it is implied that the real part has to be taken. Such a potential satisfies Laplace equation and all the boundary conditions of the problem, except the free surface boundary condition.

Writing that this solution satisfies the free surface boundary conditions implies:

$$[g k_m \tanh k_m h - \omega^2] A_m + \frac{2-\delta_{0m}}{L} \sum_{n=0}^{N} A_n \int_0^L (i\omega(\nu_1+\nu_2)+\nu_1\nu_2) \cos k_n x \cos k_m x \, dx = 0$$

This leads to the following eigen values problem:

$$[B] \{A\} + i\,\omega\,[C]\{A\} = \omega^2 \{A\} \qquad (1)$$

where $\{A\} = (A_0, A_1, ..., A_N)$ is a complex vector,

$$B_{mn} = g k_n \tanh k_n h \, \delta_{mn} + \frac{2-\delta_{0m}}{L} \int_0^L \nu_1 \nu_2 \cos k_n x \cos k_m x \, dx,$$

$$C_{mn} = \frac{2-\delta_{0m}}{L} \int_0^L (\nu_1 + \nu_2) \cos k_n x \cos k_m x \, dx.$$

and $\delta_{nm} = 1$ if $n = m$, $\delta_{nm} = 0$ if $n \neq m$.

The above integrals can be computed analytically and the eigen values problem (1) is solved in an iterative manner. This yields the shape of the eigen modes and their eigen frequencies.These frequencies are complex; their imaginary part is of particular interest since it corresponds to the damping of the mode. In order to check the result, the resulting mode shape can be used as initial condition in the Sindbad code.

Example

We consider a tank of depth 1.8 m, we take $\alpha_1 = \alpha_2 = 1$, $\beta = 2$ and we tune the beach to a period 1.5 s (ω = 4.189 rad/s, λ = 3.513 m). We show in the following table the first two eigen modes of the numerical tank for two different lengths:

	L = 66 m		L =100 m	
Frequency rad/s	Real part	Imaginary part	Real part	Imaginary part
Mode 1	0.105	$1.4\ 10^{-4}$	0.068	$3.8\ 10^{-5}$
Mode 2	0.315	$1.2\ 10^{-3}$	0.205	$3.4\ 10^{-4}$

These numerical values correspond to those of [1] for which the difference frequency of the excitation was equal to 0.2 rad/s. As that appears in the table, this value is very close to the second natural frequency of the tank of length 100m. This is, indeed, the critical length for which an erroneous evolution of the wave train was observed.

A new absorbing beach

Since we are capable of computing the eigen frequencies and the damping of the modes, it seems interesting to test other numerical damping zones and to check their efficiency for the damping of long waves.

We now propose a new beach; the principle is to separate the absorbing zone in two parts: a first part for the absorption of wave components and a second part for the damping of long waves.

$$\nu_i = \alpha_i\, \omega \sin[\pi(x-x_0)/2\lambda] \qquad (x_0 \le x \le x_0+\lambda)$$

$$\nu_i = \alpha_i\, \omega \qquad (x_0+\lambda < x \le L)$$

As mentioned before, λ and ω denote the wave length and the wave frequency to which the beach is tuned. Since the second zone is a priori devoted to the damping of natural modes, its length must be important. In return, a large mesh may be used to "lessen the increase" of computer time.

However, tests of this new beach suggest that the natural modes may be relatively well damped using a short beach : as example, we consider again a tank 66m long and 1.8m deep, and we use a beach of total length 6m which is tuned to a period 1.5s. We show in the following table the first two eigen modes of the tank for two different arrangements of the α_i coefficients :

	$\alpha_1=\alpha_2=1$		$\alpha_1=0.3$, $\alpha_2=0$	
Frequency rad/s	Real part	Imaginary part	Real part	Imaginary part
Mode 1	0.105	$1.1\ 10^{-4}$	0.111	$6.1\ 10^{-2}$
Mode 2	0.316	$9.9\ 10^{-4}$	0.329	$5.5\ 10^{-2}$

The simulation using the linear version of the Sindbad code and taking as initial condition the second mode corresponding to the case $\alpha_1=0.3$,and $\alpha_2=0$ is shown on the figure. As can be seen, this mode is well damped. An additional simulation was performed in regular waves (T=1.5s) using the linear version of the code; it was therefore verified that this beach remained efficient at "short" wave frequencies.

Acknowledgements

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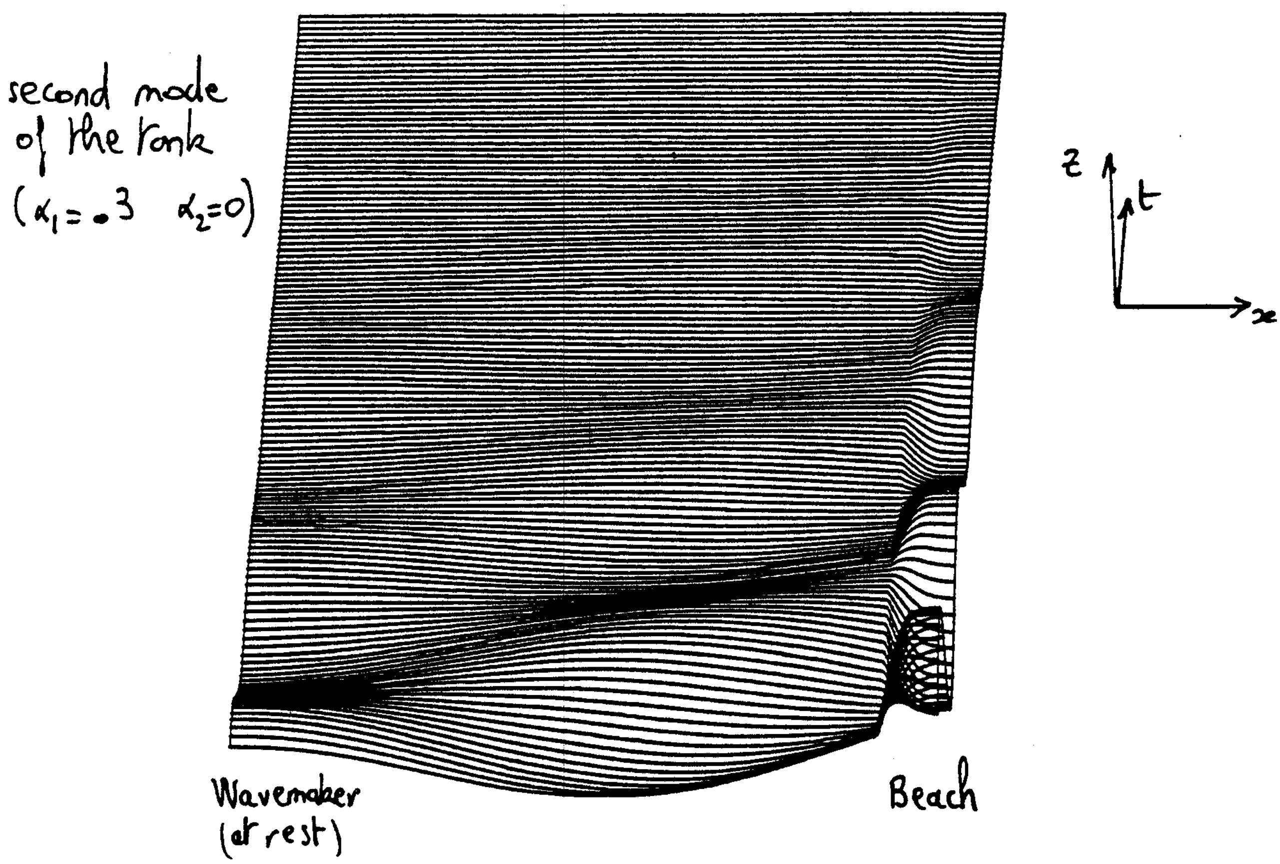
References

[1] Boudet, L., and Cointe, R., 1991, "Nonlinear propagation of bichromatic wave trains," 6th IWWWFB, Woods Hole, Mass.

[2] Cointe, R., 1989, "Quelques aspects de la simulation numérique d'un canal à houle", Thèse de Doctorat de l'Ecole Nationale des Ponts et Chaussées, Paris.



DISCUSSION

YUE: In anticipation of long (difference-frequency) waves, it appears that you must use a long beach (the one with the constant v_i) — the numerical results you showed had a length less than one wave length which seems inadequate.

BOUDET & al.: Your question refers to the new beach. Indeed, we were expecting that it would be necessary to use a second beach of length ≈ one wave length of the long waves to damp them. However, it appeared (for reasons that are still unclear to us) that, in the particular case described here, a length equal to the wave length of the "short waves" was sufficient — at least in the linear case since we did not perform nonlinear simulations with the new beach yet.