# OSCILLATION OF A SLIGHTLY-SUBMERGED CYLINDER IN A VISCOUS FLUID

**P. Ananthakrishnan & Ronald W. Yeung**

*Department of Naval Architecture & Offshore Engineering*

*University of California at Berkeley, CA 94720, USA.*

We consider in this paper the viscous flow associated with a slightly-submerged two-dimensional body undergoing forced oscillation. This serves as a follow-up study on the inviscid-flow analysis presented in an earlier paper at the Øystese workshop [1]. While the earlier work focussed primarily on the capability of our grid-generation method, the use of reference and computational spaces, and a finite-difference methodology of the inviscid-fluid formulation, we are now in a position to present the parallel solution of the same problem for the case of a viscous fluid, thus enabling one to address the importance of viscous effects in typical motion problem of floating bodies.

The nonlinear viscous-flow problem is formulated using primitive variables, *i.e.* velocity $\mathbf{u}(\mathbf{x},t)$ and pressure $p(\mathbf{x},t)$. The governing field equations are the incompressible Navier-Stokes equations. No-slip condition is satisfied on the body contour. The free-surface kinematic condition is stated in a Lagrangian form. Similar to the case of inviscid fluid, this form can be used to advance the position of the free surface at each time step. Free-surface flows in a viscous fluid require the consideration of both tangential and normal stress components at the free surface (see *e.g.* [2]). For the primitive-variables formulation, we were able to obtain a Dirichlet-type boundary condition for the pressure and Neumann-type conditions for the velocity components in the computational (mapped) space. An approximate condition is however used at the (numerical) open boundary.

The solution of the Navier-Stokes equations in conjunction with the above-mentioned boundary conditions still requires some innovative treatments. A fractional-step procedure similar to the one reported in [3] is used. In brief, this consists of introducing an auxiliary velocity $\mathbf{u}(\mathbf{x},t^*)$, which is an artifice advanced by using the Navier-Stokes equations without the pressure-gradient term:

$$\mathbf{u}(t^*) = \mathbf{u}(t_{k-1}) + \delta t\ \mathcal{R}(t_{k-1}), \tag{1}$$

where $\mathcal{R}(t_{k-1})$ represents the diffusion and convection terms at time $t_{k-1}$ and $\delta t$ the time-step size. It is not difficult to establish that the auxiliary field can be decomposed as

$$\mathbf{u}(t^*) = \mathbf{u}(t_k) + \delta t\ \nabla P(t_k), \tag{2}$$

where $\nabla \cdot \mathbf{u}(t_k) = 0$ and $P = p/\rho + gy$, $g$ being the acceleration of gravity and $\rho$ the fluid density. If the divergence of Eqn. (2) is taken, it follows that the pressure field satisfies the following Poisson equation:

$$\nabla^2 P(t_k) = \frac{1}{\delta t}\nabla \cdot \mathbf{u}(t^*) \tag{3}$$

which needs to be solved subject to the appropriate Dirichlet conditions on the free surface and the open boundary and a Neumann condition on the body. Once known, the pressure field can be substituted back in Eqn. (2) to determine the complete velocity field $\mathbf{u}(t_k)$. This solution procedure is one form of the so-called *projection method* (see [4]), a more detailed description of which for the present application is reported in [3] and [5]. In our actual implementation, curvilinear coordinates generated using the variational method of [1] are incorporated in all of the differential operators. In the numerical discretization, upwind differencing is used for discretizing the convection terms and a central differencing for the diffusion terms. The Poisson equation is solved using LU decomposition of the banded matrix.

Representative results are shown in this abstract for illustrative purposes. The heave oscillation of a submerged square cylinder of width $2b$, with an equilibrium submergence clearance $d$ that is one-eighth of the cylinder width, is considered. The cylinder is oscillated sinusoidally with a relatively small motion amplitude $a$, being 5% of the body width. For the case presented, the nondimensional frequency $\omega\sqrt{b/g}$ is 2.09 and the Reynolds number, defined as $\sqrt{b^3 g}/\nu$, is $10^3$. Here, $\nu$ denotes the kinematic viscosity coefficient. In Fig. 1, the time history of the computed heave force is shown. It can be seen that the response is rather sinusoidal and that steady state is reached within 2 to 3 oscillations. The time history of the body displacement ($a\ \sin\omega t$) is also plotted to clarify the phase lag of the force curve. From the phasing, it is possible to determine the intensity of wave and linear viscous damping. Comparison with the inviscid-fluid results reveal that the viscous damping, primarily coming from boundary-layer shear is not significant, at least for this configuration and motion amplitude. The time evolution of the free-surface elevation in the right half of the domain is displayed in Fig. 2. Following the wave characteristics, one notices that the sharp corners of the body generates two waves, one propagating away from the body and the other towards the centerline. It is interesting to observe that waves traveling towards the centerline results in a wave sloshing above the cylinder. Again, the basic features are very similar to the inviscid-fluid results. Velocity-vector and vorticity-contour plots at two instants of time that are one half period apart are shown in Fig. 3. Generation of small vortices at the sharp edges can be observed in the plots. These vortices do not appear to affect the global force magnitude in a substantial way. Preliminary results for other modes of motion and motion amplitudes, however, portray rather different behavior, *i.e.* in terms of the effects of viscosity. These will be reported in more detail in a future report.

## References

1. R. W. Yeung and P. Ananthakrishnan, Solution of Nonlinear Water-Wave and Wave-Body Interaction Problems using a new Boundary-Fitted Coordinates Method, *Proceedings, Fourth International Workshop on Water Waves and Floating Bodies*, Øystese, Norway, 1989.

2. J. V. Wehausen and E. V. Laitone. *Surface Waves*, Handbuch der Physik IX, Springer-Verlag, 1960.

3. P. Ananthakrishnan. *Surface Waves Generated by a Translating Two-dimensional Body: Effects of Viscosity.* Ph.D. Thesis, Department of Naval Architecture and Offshore Engineering, University of California at Berkeley, November 1991.

4. A. J. Chorin. Numerical solution of the Navier-Stokes equations. *Mathematics of Computations*, 23:341–353, 1969.

5. R. W. Yeung and P. Ananthakrishnan, Oscillation of a Floating Body in a Viscous Fluid, *Journal of Engineering Mathematics*, 26:211–230, 1992.

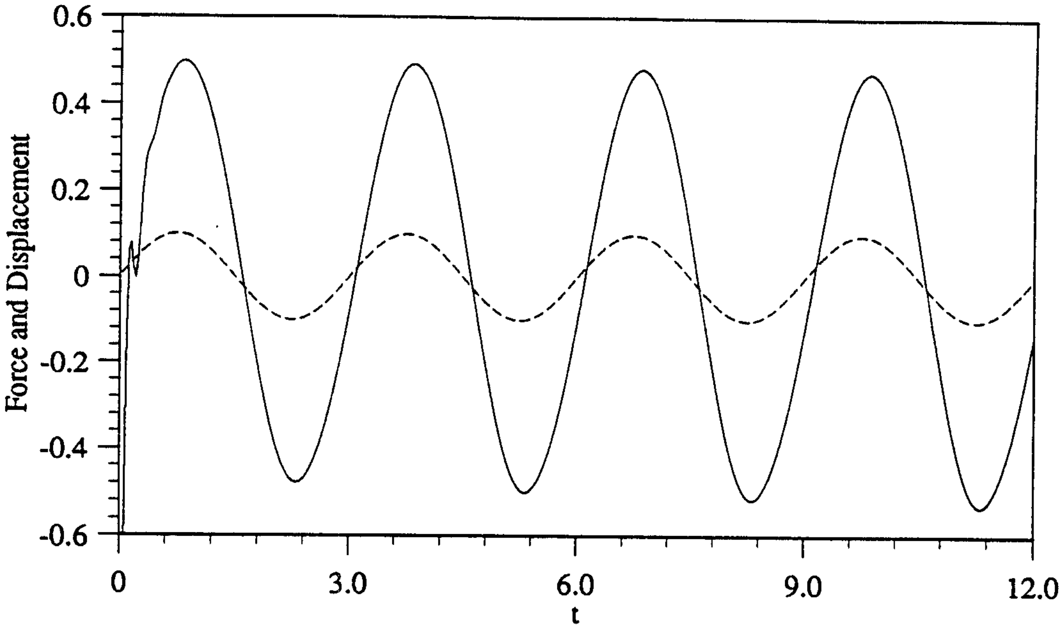


**Fig.1** Time history of heave force (solid line) and body displacement (dashed line) for $a/b = 0.1$, $d/b = 0.25$, $\omega\sqrt{b/g} = 2.09$, $Re = \sqrt{b^3 g}/\nu = 10^3$.

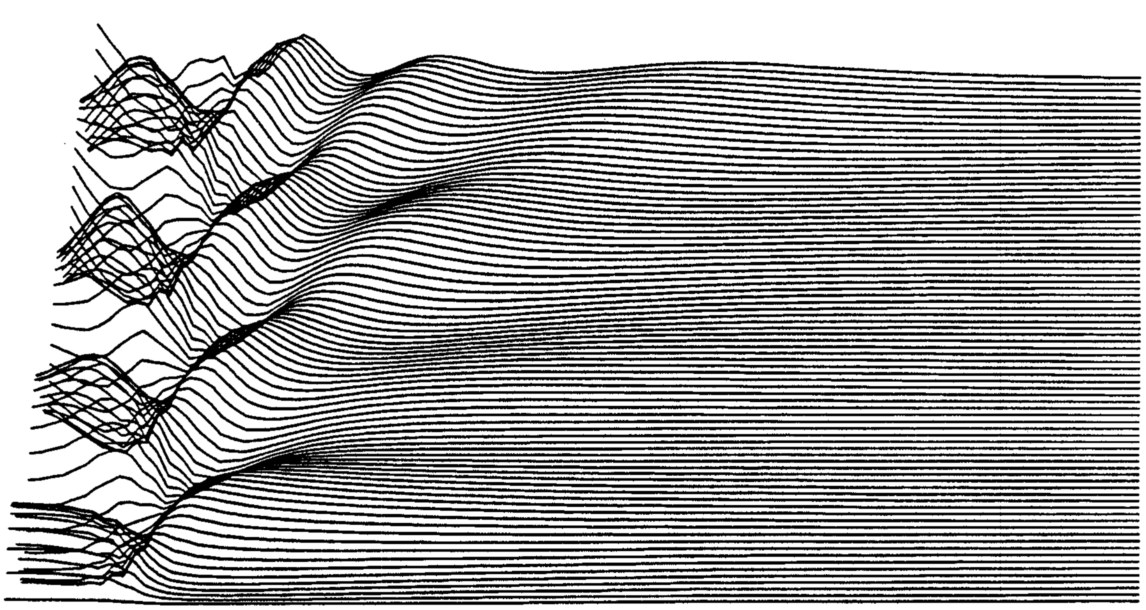

**Fig.2** Time evolution of free-surface elevation, in the right half domain, for $a/b = 0.1$, $d/b = 0.25$, $\omega\sqrt{b/g} = 2.09$, $Re = \sqrt{b^3 g}/\nu = 10^3$.

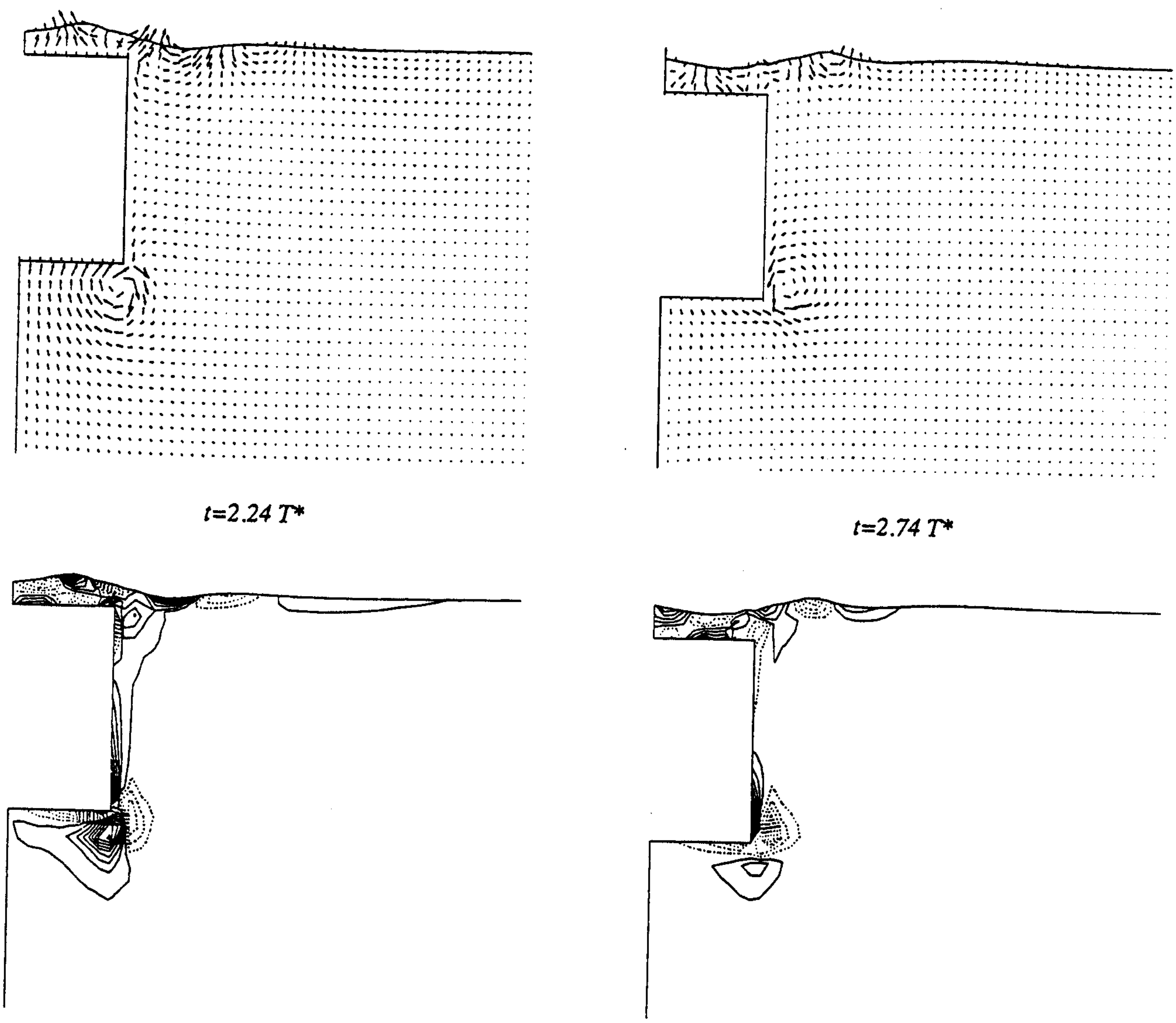


Fig.3 Velocity-vector and vorticity-contour plots at time $t = 2.24T^*$ and $2.74T^*$, where $T^*$ is the period of body oscillation: $a/b = 0.1$, $d/b = 0.25$, $\omega\sqrt{b/g} = 2.09$, $Re = \sqrt{b^3 g}/\nu = 10^3$. Solid lines in the vorticity-contour plots denote clockwise vorticity and the dotted lines the counter-clockwise vorticity.

## Discussions

**K. Mori:** I can understand your method for the grid generation is quite general. But it may be so much time consuming in case of free-surface flow computations where the transformation is necessary at every time step.

As I understand your grid size seems very uniform for all the domain. The grid size must be much smaller for the flow in the boundary layer. It must be the function of the wave length at the same time; the wave does not propagate well when the grid size is not enough small to the wave length. It must be important especially for unsteady flow problems. A study on the grid dependency should be carried out as one of the future tasks.

**P. Ananthakrishnan & R. W. Yeung:** The grid equations are solved iteratively using mixed over-under relaxation technique. With the known grid values from previous time step taken as the initial guess, the number of iterations required for convergence is reduced significantly; for the results reported here, the number of iterations required at each time step is $< 10$. If the problem is unsteady and the domain is changing, the grid naturally has to change to conform with the free-surface and the body movement. This is to be expected.

The flow plots given in Fig. 3 corresponds only to the near field. Using the reference-space based grid-generation procedure, we were able to discretize the fluid domain efficiently with fine grid spacings in the near field, where the flow gradient is large, compared to that in the far field. Using the present solution method, we were able to solve accurately other free-surface flow problems in which the generation of free-surface shear layer plays a crucial role in the flow evolution; see, for example, [3] or *Proc.* $19^{th}$ Symp. on Naval Hydrodynamics, Seoul, Korea, 1992. We have also analyzed the convergence properties of our numerical method with respect to mesh and time-step sizes; these results are also reported in the above two works.