Determination of Wave Form Afore the Ship Bow

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Afore the bodies with blunt forward ends, when Froude numbers are not very high, there are waves which may be assumed to be stationary. The fluid velocity on the crests of such waves may drop to zero. Therefore, the calculation of there waves is a complicated nonlinear problem [1]. The paper deals with the development of the method of such calculations and contains examples of its application. Stationary flow of ideal noncompressible fluid is considered. The velocity potential Φ is introduced which satisfy there relationships:

$$\Delta \Phi = 0; \quad (\nabla \Phi, N) = 0; \quad (\nabla \Phi, \nabla \Phi)+2*y*Fr = 1; \qquad (1)-(3)$$

Relationship (1) is valid inside the fluid. Relationship (2) is satisfied for all boundaries of the flow, and relationship (3), only for the free surface S. The Froude number is built on the draft of the streamlined body; $y \to 0$ and $\nabla \Phi \to \{1,0\}$ at $x \to -\infty$ .The form of S is not predetermined, it is sought for by means of modification of the iteration method [2]. Each iteration involves two steps. The first step consists in solution of the problem (1), (2) for the area with the boundary including the known surface S^* approximating the required surface S. As a result of solving (1),(2), $\nabla \Phi$ has been calculated, and the accuracy of satisfying the condition of (3) at S^* has been verified. If the discrepancy is fairly small, then S^* is identified with S, and the problem is solved. If it is not so, the second step begins, at which conditions of (2), (3) are quasi-linearized at S , as in [2,3]:

$$d\frac{\partial \varphi}{\partial N}+U\frac{\partial h}{\partial T}+hN_x\frac{\partial U}{\partial T}=0\,;\quad U\frac{\partial \varphi}{\partial T}=\frac{U^2-1}{2}-\frac{y}{F_r^2} \qquad (4),(5)$$

Here, $h=S^*$-abscissa changes from iteration to iteration; T and N=unit vectors of the tangent and normal to S^*; $U=-(\nabla \Phi, T)$; N_x= horizontal component of N; φ = disturbance of the potential Φ; $\alpha<1$ = relaxation factor. The bow wave height $y(0)=0.5Fr$ is fixed in calculations, and the wave length is to be found. Equation (5) is a singular integral equation in relation to q - density of φ, and $|q| \to \infty$ at $y \to 0$. The value of h(x) is determined from (4) on condition that h(0) = 0 after calculation of q from (5). The S^* - points ordinates will not change when the surface forms are clarified. Close to the wave crest $U \to 0$, and equations (4),(5) degenerate. Therefore, instead of these equations, asymptotic analytical formulae for q and h shall be used there [1]. For derivation of these formulae we shall consider the conformal mapping of the crest neighborhoods in the plane of the complex variable z = x+iy into the circle exterior

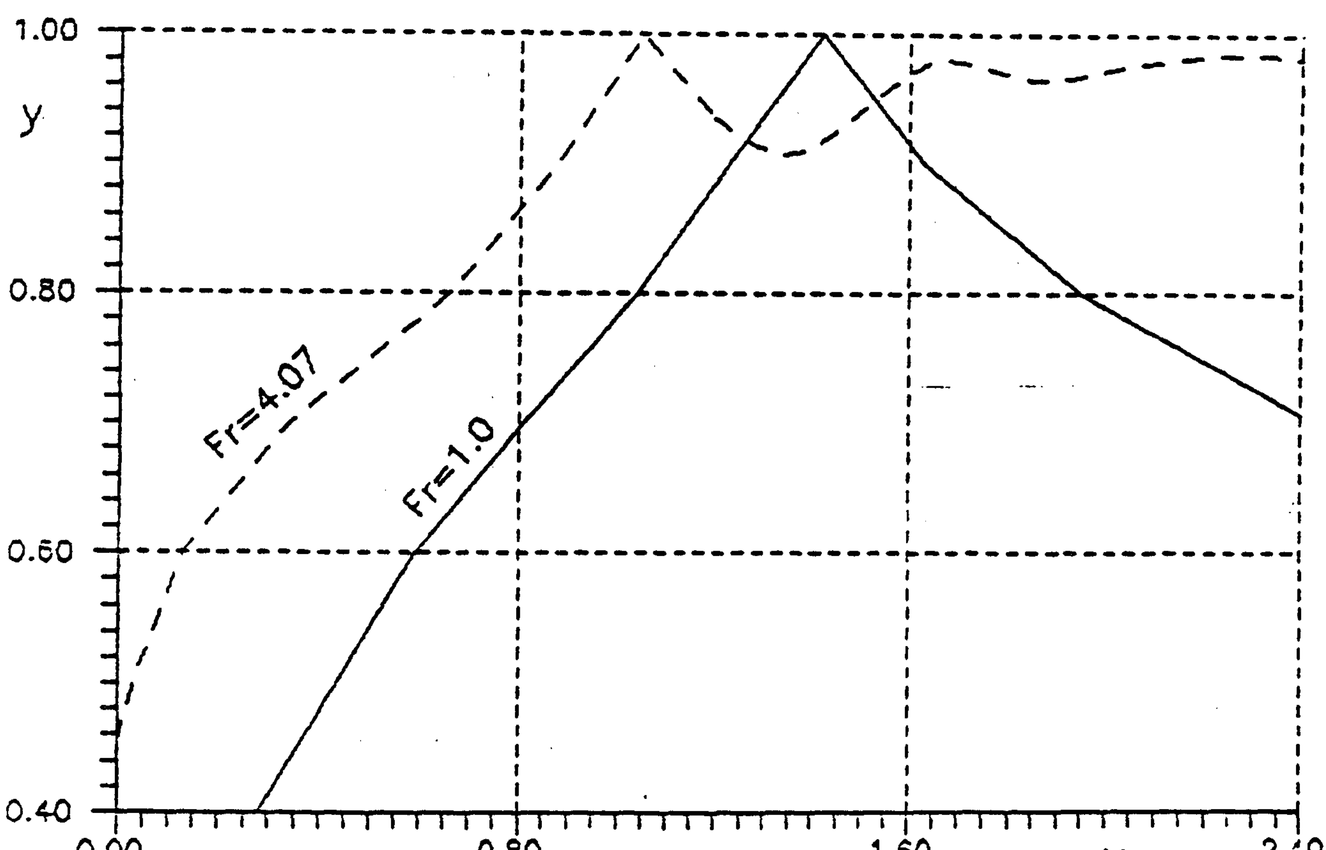

Fig. 1

in the canonical plane of the variable ζ. At $x \to 0$ $h(x)=C_1|x|^{\beta}$, $q(x) = C_2|x|^{3\beta/2-1}$, C_1 =const, C_2 =const are used in the calculations. $\beta=2\pi/\alpha-2$, α is the angle of the crest point. For the vertical stem β =2.

For perfection of the numerical method the problems of axially symmetric [2] and plane [4] waves over the isolated flow have been solved. The existence of two types of waves with zero speed at the crest has been found. At small Fr the waves with flat crests resembling those calculated in [5] have been plotted (see Fig.1; figures near the curves denote the values of Fr ; the solid lines refer to the plane flow, and the dashed eines lines, to the axially symmetric flow). For these flows in the narrow range of high Froude numbers the Stokes waves have been plotted. The upper limit of this range in the plane flow at Fr =1.27 is in good agreement with the predictions [5]. Fig. 2 presents the calculation results for the waves afore the semi-infinite plane rectangular bow. It has been possible to obtain the waves of the displayed type in this flow only at Fr $\leqslant$ 1.1 In conclusion, it is interesting to note that, if in derivation of the formulae for the wave crest the angle vertex is mapped into the critical point of the circle, then $\alpha=4\pi/3$ at β =1.

Such solution is possible only at the nonzero velocity

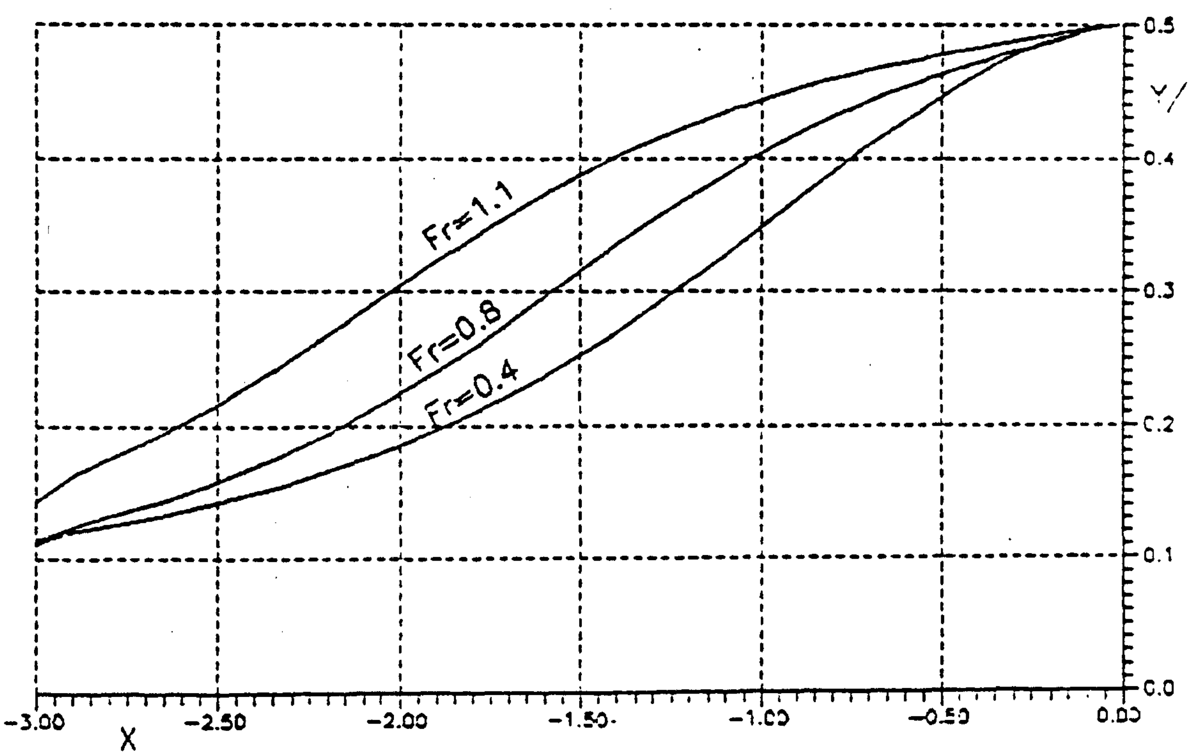

Fig. 2

circulation around the angle (as in the airfoil theory) and only for internal waves at the boundary of fluids with different densities. On one side of the boundary the Stokes waves will occur, and on the other side, the wave-vortex pairs.

References

1. G.Dagan, M.P. Tulin. J. Fl. Mech. 1972, v 51, part 3.
2. E.L.Amromin, M.A.Basin, V.A.Bushkovsky. Prik. Matematika i Mekhanika. 1990, N 1 (in Russian).
3. L.Larsson et al. 5th Conf. Num. Ship. Hydr., Hirashima, 1989.
4. E.L.Amromin, V.A.Bushkovsky, D.Y.Sadovnikov. Mekhanika Zhid. i Gaza. 1991, N 6 (in Russian).
5. L.K.Forbes, L.W. Schwartz. J. Fl. Mech. , 1982, v114.