On the Impulsive Motion of a Wavemaker

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Introduction

In fluid mechanics, the problem of floating moving bodies in gravity waves has received much attention in the past few decades. In many cases both analysis and computations encounter difficulties near the intersection of a body with the free surface. In recent publications, nonlinear transient waves induced by the movement of a wall have been the subject of extensive study. It is known that in case of an impulsive start analytical solutions become singular at the intersection of the wall with the free surface. Mathematically, this singularity is due to a confluence of the boundary conditions. Physically, the singularity is plausible in view of the splashing which occurs in the slamming of a ship.

In this paper we focus our attention to the impulsive motion of a wavemaker and the fluid motion as a result of it. Usually, the problem is reduced to two dimensions and a velocity potential is introduced under the assumptions of potential flow theory. Analytical solutions can then be obtained by using small time expansions for both the velocity potential and the free surface elevation. Such approaches can be found in [3, 5, 7], revealing a logarithmic singularity in the free surface elevation. However, the lowest order solution for the potential does not match the initial condition. Another approach suggested in [4, 8] is to use small amplitude or small Froude number expansions. This leads to solutions showing (nonphysical) wiggles near the intersection point, which vanish if surface tension is added.

A computer program has been developed jointly by MARIN, Delft Hydraulics and the University of Twente for the numerical simulation of 2D nonlinear gravity waves. It is based on a 3D higher order panel method developed by Romate [9]. The program is capable of simulating highly nonlinear waves, overturning waves and interactions of waves with fixed structures. Numerical results on these cases have been described in [1, 2]. It is for the occurrence of the above mentioned singularities in relatively simple problems, that we have found it beneficial to test the program on these items first, before we proceed to the implementation of a numerical scheme for the 2D simulation of nonlinear wave-body interactions. Our results will be compared with those obtained by Lin [5] and with the analytical solutions from the small time expansions.

Mathematical Formulation

Under the assumption that the fluid is ideal, the irrotational fluid flow is determined by Laplace's equation for the velocity potential and by the nonlinear dynamic and kinematic boundary conditions at the free surface. The field equation is transformed to a boundary integral equation, based on Green's identity. Higher order approximations for the variables of interest are used, and each collocation point is situated in the middle of its panel. Substi-

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tuting the potential for Dirichlet boundaries (such as the free surface) and its normal derivative for Neumann boundaries (such as a wavemaker) yields a system of linear equations, which is solved by either Gaussian elimination or the Preconditioned Conjugate Gradients Squared Method. Finally, the time stepping is performed by either the classical fourth order Runge-Kutta method or a fourth order mixed Runge-Kutta/Taylor method.

Following the earlier work of Whitham [10], we have taken some effort in formulating the problem in terms of a variational principle for the associated Lagrangian functional L. The field equation and the free surface conditions are easily obtained from appropriate variations in L. In addition it was found that for well-posedness of the problem, a Lagrangian description should be used for the free surface.

Problem Description and Analytical Solutions

As shown in figure 1, the piston wavemaker is a vertical rigid boundary, and the origin of the coordinate system $\{x, y\}$ is initially located at the intersection of the wavemaker and the free surface. At time $t = 0$, the wavemaker starts to move impulsively in horizontal direction with a constant velocity $U = 1$ towards the fluid.

Due to the impulsive motion of the wavemaker, the free surface is changed from its undisturbed level at $y = 0$ to a new position at $y = \eta(x, t)$. The kinematic and dynamic boundary conditions on the free surface read:

$$\phi_y = \eta_t + \phi_x \eta_x \ , \qquad (1)$$

$$\phi_t = -\frac{1}{2}(\nabla\phi \cdot \nabla\phi) - g\eta \ . \qquad (2)$$

At the bottom the vertical velocity must vanish and at the vertical wavemaker the horizontal velocity is equal to 1.

Next, we expand the velocity potential and the vertical deviation of the free surface in power series of t:

$$\phi(x, y, t) = \sum_{n=0}^{\infty} \phi_n(x, y)\, t^n \ , \qquad (3)$$

$$\eta(x, t) = \sum_{n=0}^{\infty} \eta_n(x)\, t^n \ . \qquad (4)$$

The leading-order solution $\phi_0(x, y)$ is obtained straightforward by the Fourier-series method (see [5]). Substituting $x = 0$, we obtain an expression for the potential along the wavemaker at $t = 0^+$:

$$\begin{aligned} \phi_0(0, y) &= \frac{4}{\pi^2} \int_0^{\pi y/2} \log\left[\tan\left(\frac{\pi}{4} + \frac{v}{2}\right)\right] dv \\ &\quad - \frac{8G}{\pi^2} \ , \end{aligned} \qquad (5)$$

where G is Catalan's constant. The lowest order nontrivial term in the free surface elevation is given by (see [5]).

$$\eta_1(x) = -\frac{2}{\pi} \log\left[\tanh\left(\frac{\pi x}{4}\right)\right] \ . \qquad (6)$$

Numerical Results

As indicated before, the wavemaker starts to move from a state of rest with a constant horizontal velocity $U = 1$. At $t = 0^+$, the velocity is a step function and as a consequence the acceleration is infinite. The length of the tank is 10, the water depth is 1 and the time is from 0 to 0.2. Comparative tests with a tank length of 20 show that during this time interval there is no reflection from the vertical wall downstream. The boundary integral equation is solved using a wide range of elements on the boundary ($N = 66, 132, 264$). The number of panels on the wavemaker in these tests is 8, 16 and 32 respectively. Cosine spacing is used in order to obtain a dense grid near the intersection point.

Note that in our method no collocation point is situated at the intersection.

First we compare our results for the distribution of the potential on the wavemaker at $t = 0^+$ with the analytical solution (5) and with the results obtained by Lin's model according to Vinje and Brevig [5]. Figure 2a shows the results obtained by Lin with constant and cosine spacing, where the intersection point (which is a collocation point in his model) satisfies the wavemaker condition only. It is clear that even with cosine spacing there is still a significant error near the intersection point. Figure 2b shows the improvement obtained if the intersection point is forced to satisfy both the wavemaker and the free surface conditions. Figure 3a shows our results, where the position of the intersection point is calculated from extrapolations from the collocation points on the adjacent boundaries. From figure 3b, showing the results close to the intersection point, it is clear that even with the smallest number of elements ($N = 8$) on the wavemaker, excellent agreement with the analytical solution is obtained (a straight line connects the results for $N = 8$ in this plot).

Figures 4a and 4b show the quality of our method in predicting the free surface elevation due to the impulsive start of the wavemaker. Altogether these results give us a firm confidence in the future capability of our programs (2D and 3D) in simulating ship motions in nonlinear gravity waves.

Acknowledgements

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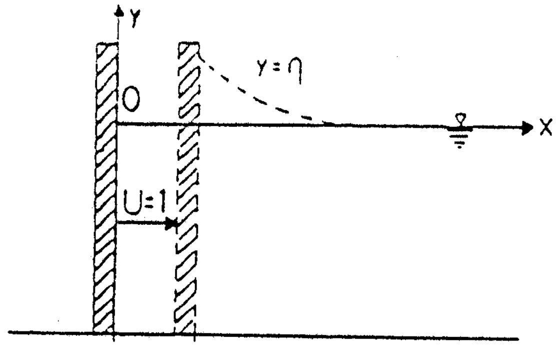

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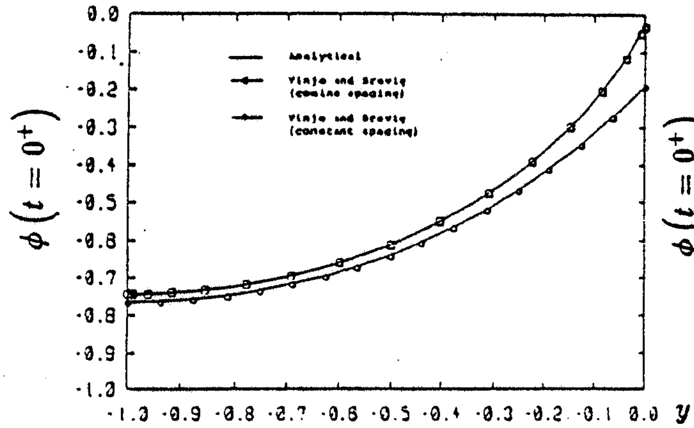

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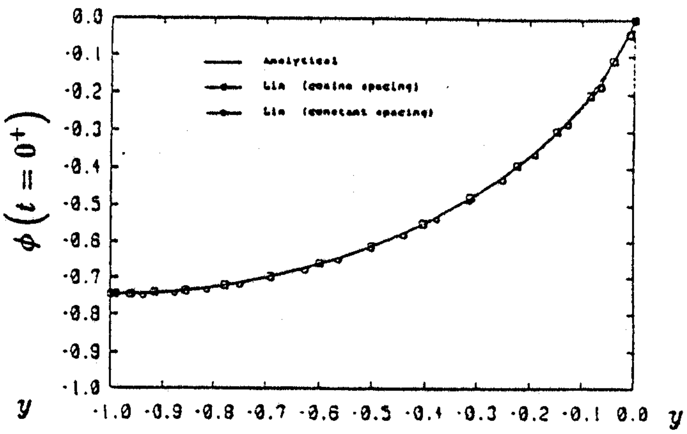

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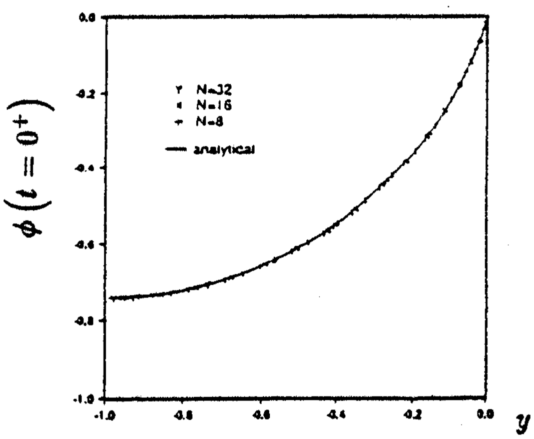

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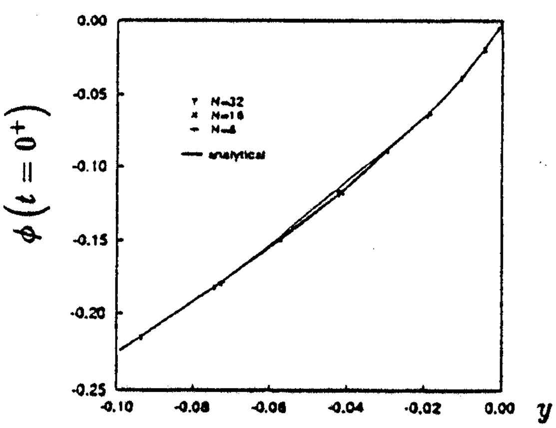

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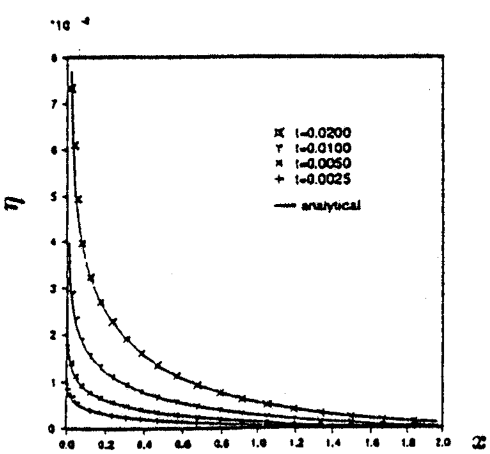

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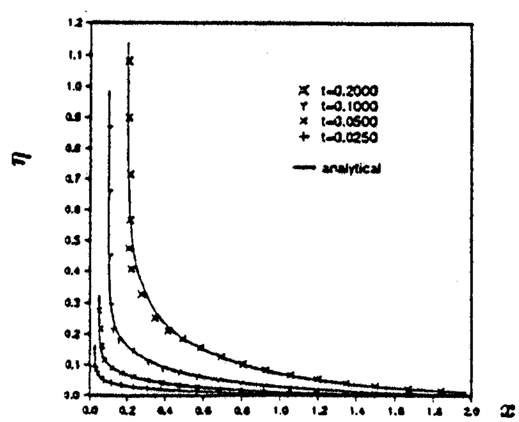

4b

Cao: Would you say something about the preconditioning technique that you used?

van Daalen & Huijsmans: I assume that you refer to the Preconditioned Gradients Squared Method we used as a matrix system solver. This is a method developed by Kaasschieter (Delft University, The Netherlands) and it is especially convenient for sparse systems. In our case, where we have a full system matrix, the method still works very well. I am not familiar with the details of the method, but I know that it is described in a journal on numerical algorithms. A briefer description can be found in Romate's Ph.D. Thesis (1989) from the University of Twente.