Proposed Abstract for Woods Hole Workshop, April 1991

The Cauchy-Poisson problem for finite depth in three dimensions

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The Cauchy-Poisson problem treats the waves generated by an instantaneous localised splash. Waves spread out from the disturbance, the long waves travelling faster than the short waves. The corresponding velocity potential is readily expressed as an integral. We are concerned with the behaviour near the wave front $r = t(gh)^{\frac{1}{2}}$ for large t, which we may therefore expect to be able to treat by some modification of the Method of Steepest Descents. This has recently been discussed by J.N. Newman, who finds a leading term involving the square of an Airy function whereas the leading term in two dimensions involves just the Airy function itself. During the past year I have tried to find a more complete expression but my attempt has met with only very limited success. My asymptotic expansion was found to involve modified Airy integrals like

$$\int \frac{\exp(\frac{1}{3}iu^3 + izu)}{u^{\frac{1}{2}}} du,$$

but this did not imply a contradiction with Newman's result because it could be shown that this integral is in fact the square of an Airy function. My asymptotic expansion was found to be valid only in a narrow zone near the wave front whereas the corresponding result in two dimensions has a wide zone of validity. One reason is, that in two dimensions the integrand has two nearly coincident saddle points whereas in three dimensions the integrand has two nearly coincident saddle points together with a branch point.

Mei: Around 1960, K. Kajiura of Tokyo worked out the asymptotic behavior of the leading wave of a tsunami due to sudden drop or sudden tilt of a circular disk on the sea bed. His results are for the leading wave only. Have you checked yours with his?
Ref: References list and Chapter 2 (for a brief description) of "Appl. Dyn. of Ocean Surface Waves" by C. C. Mei.

Ursell: I am grateful for this reference.