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Numerical Computation of Plunging Wave Impact Loads on a Vertical Wall

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1. Introduction

The physics of wave impact such as those of plunging breakers on ocean structures are little understood because of the presence of strong nonlinearities and complicated effects of trapped air. Chan & Melville (1988) present careful measurements of plunging wave pressure on a vertical wall and show that the peak impact pressure is very high ($O(3 \sim 10)\ \rho C^2$, where ρ is the density of water, and C is the characteristic phase velocity). The impact pressure and its oscillation frequency are significantly different between repeated experiments, and is also very sensitive to the wall location relative to the plunging wave. Despite the importance of impact loads, quantitative predictions based on direct computations are so far scarce, and in particular a useful scaling law has yet to be found. Vinje & Brevig (1981) performed some numerical calculations of impact pressure due to a breaking wave without trapped air. Dommermuth, Yue, *et al* (1988) performed a numerical simulation of a deep-water plunging breaker which compared extremely well to experiment.

Following these works, we focus here on the impact process itself. Our primary objective is to calculate the wave impact pressure with air trapping and compare it to the experiments of Chan & Melville (1988). In order to make a stable calculation, a robust computational method is now under development. In this abstract, we present the mathematical formulation and some new ideas in the numerical method.

2. Mathematical formulation

The physical problem is formulated as the irrotational flow of a homogeneous, incompressible, and inviscid fluid in a two dimensional rectangular tank with a piston wavemaker at one end. The sketch is shown in Fig.1. Here, dimensionless variables are used so that the depth of fluid in the tank, density of fluid, and gravitational acceleration are set to unity. We define the complex potential $\beta(z,t)=\phi(z,t)+i\psi(z,t)$, where $z=x+iy$, ϕ is the velocity potential and ψ the stream function. Since the velocity potential and stream function are solutions to Laplace's equation in the fluid domain, Cauchy's integral theorem can be applied. As in Dommermuth, *et al* (1988), mirror images with respect to the bottom are added to reduce the number of unknowns. The governing equation is:

$$2\pi i\beta(\zeta,t) = \int_C \left[\frac{\beta(z,t)}{z-\zeta} - \frac{\beta^*(z,t)}{z^*-\zeta-2i}\right] dz \qquad (1)$$

where $C = B_L \cup B_R \cup F$ and $()^*$ denotes complex conjugate. If we now let ζ approach C, Fredholm integral equations of the second kind for ϕ on F and ψ on B can be obtained and the system solved. Eq.(1) is also valid when β is replaced by any analytic function, such as $\partial\beta/\partial t$. In practice it is useful to solve the boundary-value problem for both β and $\partial\beta/\partial t$, because we can calculate impact pressure directly from the solution of $\partial\beta/\partial t$ and Bernoulli's equation.

On the wall and wavemaker, ψ and $\partial\psi/\partial t$ are specified as boundary conditions. We fix the stream function value on B_B and B_{R1}:

$$\psi = \dot{\psi} = 0 \qquad \text{on } B_B,\ B_{R1} \tag{2}$$

where $(\dot{\ })$ denotes time derivative. The stream function and its time derivative on the wavemaker are given by

$$\psi = U(t)(y+1) \qquad \text{on } B_L \tag{3}$$

$$\dot{\psi} = \dot{U}(t)(y+1) \qquad \text{on } B_L$$

where $U(t)$ is the prescribed velocity of the wavemaker. On B_{R2}, the stream function is given in a similar way.

On the free surface, ϕ and $\partial\phi/\partial t$ are given. The kinematic and dynamic boundary conditions are respectively

$$\frac{Dz}{Dt} = \frac{\partial\beta^*}{\partial z} \qquad \text{on } F_1 \text{ and } F_2 \tag{4}$$

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} - \left|\frac{\partial\beta}{\partial z}\right|^2 = \frac{1}{2}\left|\frac{\partial\beta}{\partial z}\right|^2 - y - p \qquad \text{on } F_1 \text{ and } F_2 \tag{5}$$

where D/Dt is the material derivative, and p is the atmospheric pressure on F_1 and the trapped air pressure on F_2. The velocity potential ϕ and the position of the free surface is calculated by integrating (4) and (5) in time. In order to get the new values of $\partial\phi/\partial t$, β must be solved first and $\partial\phi/\partial t$ is obtained upon substitution into (5).

For the mathematical model of the trapped air, either an aero-static or aero-dynamic model can be considered. For simplicity, we choose the static model as a first try. Such a model involves three simplifying assumptions: (i) the process of trapped air compression is adiabatic; (ii) there are no shock waves in the trapped air and the pressure is uniform (*i.e.*, $(\rho u^2)_{\text{air}} \ll (\rho u^2)_{\text{water}}$; and (iii) there is no leakage of air and no formation of foam. With these assumptions, the model of trapped air is described simply by a polytropic gas law: $pv^\gamma = constant$, where v is the volume of the trapped air and γ is a physical constant equal to 1.4 for air.

3. Numerical technique

For a successful simulation of the impact process, special treatments such as regridding and smoothing are indispensable to remove high-wavenumber instabilities and maintain appropriate panel sizes. Unfortunately, most of these treatments involve certain amount of trial and error to achieve optimal results. To reduce such trial and error and to maximize the robustness of the calculations, we develop a new adaptive automatic regridding and smoothing method. This method is constructed in two stages. The first stage is fitting and smoothing. The second stage is regridding.

As a fitting and smoothing function, we use smoothing cubic splines. These functions act as both fitting and smoothing functions at the same time. Let a knot sequence $X = \{x_0, x_1, ..., x_n\}$ and real numbers $y_0, y_1, ..., y_n$ be given. The basic idea of the smoothing spline is to find the fitting function f which minimize the functional

$$K_\lambda(f) = J(f) + E_\lambda(f) \tag{6}$$

with weights $\lambda = [\lambda_0, \lambda_1, ..., \lambda_n]$, where $E_\lambda(f)$ is sum of square error and $J(f)$ is the total strain-energy of the spline function:

$$E_\lambda(f) = \sum_{j=0}^{n} \lambda_j (f(x_j) - y_j)^2 \tag{7}$$

$$J(f) = \int_{x_0}^{x_n} [f''(x)]^2 dx \tag{8}$$

The key point is finding the optimal λ for our purpose.

As a regridding function, we can use the idea of a mesh function introduced by Hyman & Naughton (1985). The mesh function is defined as that which measures the local goodness of the discrete approximation on the mesh. If a curve fitting function is obtained, we can define the mesh function as a function of arc length, curvature, *etc.* Fig.2 shows an example of the generated mesh on a spiral curve. In this example, curvature is utilized as a mesh function.

4. Results

Preliminary results using adaptive automatic regridding and smoothing will be presented. Application to the plunging wave slamming problem and comparisons to the experiments of Chan & Melville (1988) will be discussed.

References

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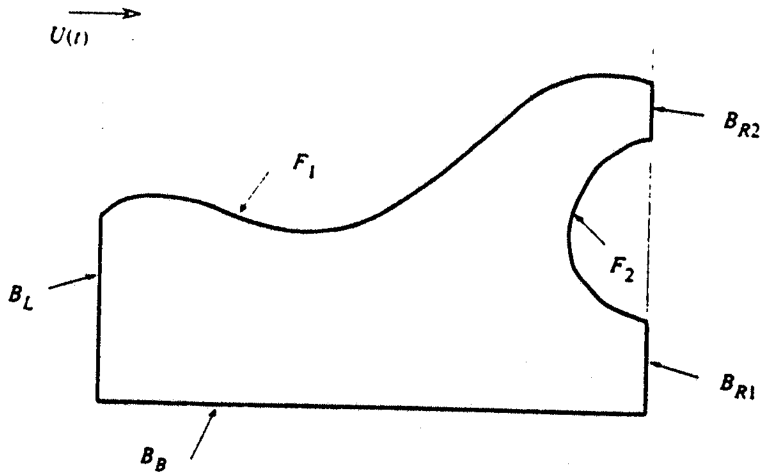

Fig.1 Sketch of wave impact.

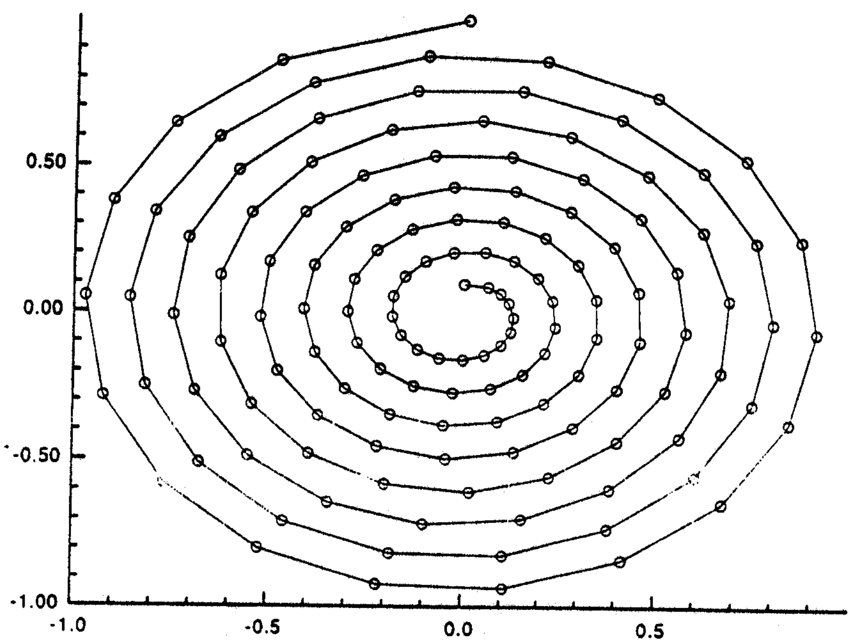

Fig.2 An example of generated grid.

R.W. Yeung: At the moment the first fluid particle comes into contact with the wall, this fluid particle has only a single value of velocity. Can you explain in more detail how you "split" this velocity up into two separate values, one upwards and the other downwards, corresponding to those of two fluid particles?

Tanizawa & Yue: At the first instant a single Lagrangian point crosses the wall, a truncation is performed and two new points on the wall are obtained by interpolation. Since dynamic time stepping is employed for the entire simulation (typically $\Delta t < .1\ell_{min}/V_{max}$, where ℓ_{min} is the size of the smallest panel, and V_{max} the maximum velocity of the Lagrangian points), the portions which are truncated are constrained to be a small fraction of the local panel dimensions.

Clement: (1) In your simulations, you may reach situations where the free-surface and the vertical wall make a very sharp angle. Did you check the precision of your numerical method in such cases by simulating numerically a known given potential? (2) What is the criteria you adopt to cut (or not to cut) the jet along the wall?

Tanizawa & Yue: (1) Yes, we did. By considering the solution in a triangular domain of height 1 and decreasing the base width down to 10^{-6}, we were able to confirm that the error in the 'vertical velocity' at the apex is less than about 1% even for the worst case. (2) We adopt a criterion in terms of the minimum jet thickness relative to local panel sizes (which are kept approximately constant). A typical value for cutting the splash tip is 0.2% of panel size. This criterion gives us excellent resolution of the details of the splash and the free surface near the impact zone. For such thin spray jets, however, the stable evaluation of the wall pressure near the tips becomes a difficult task.