On non-linear reflection from a submerged, circular cylinder

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Abstract

At the last workshop it was shown by Friis (1990) that for a submerged circular cylinder with axis parallel to the crests of the incident wave, the second order reflection coefficient is exact zero. The same result has, independently, been derived by McIver and McIver (1990) and Wu (1990). The result is an extension of the classical papers by Dean (1948) and Ursell (1950) where it is proved that the first-order reflection coefficient is zero.

Here these results will be further generalized. We consider the reflection of the Fourier mode with frequency $m\omega$ (the m-harmonic mode) where m is an arbitrary integer and ω the frequency of the incident wave. This mode consists of terms of order $m, m+2, m+4$ etc. The lowest order term, i.e. the term of order m, is usually the dominating one. We shall prove that the reflection of this term is zero, so that the reflection of the m-harmonic mode is at most of order $m+2$.

We also consider incident bichromatic waves and show that the reflection coefficient for the "sum-frequencies" is zero.

Furthermore, it will be shown that the oscillatory forces in the x- and y-direction due to the m-harmonic mode of order m have identical amplitudes and a phase-difference $\pi/2$.

The proves are based on Green's theorem, using an appropriate Green function. An essential point in the proves is the following: The integral equation for the first order motion is

$$-\int_C \varphi_1 \frac{\partial G}{\partial n} ds + \pi\varphi_1 = 2\pi\varphi_0 \qquad (x,y)\in C$$

where C denotes the circular contour, φ_1 the total first order potential, G the Green function and s the arc-length. φ_0 is the velocity potential for the incident wave. If the right-hand side is generalized slightly, the equation is also valid for higher order terms and still we may prove that there is no reflection.

For $m = 2$ and 3 our results are confirmed by numerical and experimental results obtained by others.

References

Dean, W.R. 1948, On the reflection of surface waves by a circular cylinder. *Proc. Camb. Phil. Soc. 44, 483.*

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McIver, M., and McIver, P. 1990, Second-order wave diffraction by a submerged circular cylinder. *I. Fluid Mech. 219, 519.*

Ursell, F. 1950, Surface waves on deep water in the presence of a submerged circular cylinder. *Proc. Camb. Phil, Soc. 46, 141.*

Wu, G. W. 1990, On the second order wave reflection and transmission by a horizontal cylinder. *Appl. Ocean Res. (to appear).*

Evans: Your result helps to explain the fact that when a submerged cylinder makes small circular motions the waves generated radiate away from the cylinder in one direction only. This appears to be true even when higher harmonics are present in the waves. Since you have shown that the force coefficient amplitudes are equal in heave and sway, and are $\pi/2$ out of phase, the same is true of the radiated wave amplitudes (by the Haskind relations). So in the case of circular motion, that being a combination of equal amplitudes but $\pi/2$ out of phase in heave and sway, wave cancellation occurs.

Palm: It is a very interesting remark.