The Fourier-Kochin-Galerkin approach to the calculation of flow about a ship advancing in waves

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Introduction

Prediction of 3D flow about a ship advancing in waves, within the frequency-domain approach, using a panel-method based on the Green function satisfying the linearized free-surface boundary-condition involves two significant difficulties: (i) the numerical evaluation of the Green function and its gradient and (ii) their subsequent integration over the panels and the segments approximating the hull-surface and the waterline, respectively. These basic difficulties -- which stem from the complex singularity of the Green function -- are addressed in different ways in Chang (1977), Guevel & Bougis (1982), Inglis & Price (1982), Ohkusu & Iwashita (1989), and Wu & Eatock Taylor (1989).

One way of avoiding the difficulties of the Green function associated with the linearized free-surface condition consists in using the considerably-simpler Rankine-source as a Green function. This approach indeed has become popular for predicting ship wave-resistance, and was recently extended to ship-motions by Sclavounos & Nakos (1990). However, a drawback of this approach is that it requires a significantly-greater number of panels, because panels must be distributed not only on the wetted-hull of the ship but on the free surface as well. Furthermore, the radiation condition is not easily satisfied, and indeed a way of satisfying this condition does not appear to have been devised yet for the "low-speed regime" when waves propagate both ahead and behind the ship. Rankine-singularities also are ill-adapted for representing the short waves in the wave-spectrum, and for computing the far-field wave-pattern (for which the Green function satisfying the linearized free-surface condition is in fact ideally suited).

The previously-mentioned basic numerical difficulties due to the Green function associated with the linearized free-surface condition can be avoided in an alternative way by adopting the approach used by Kochin (1937,1940) for the problems of steady flow about a ship and of diffraction-radiation at zero forward speed. Kochin's approach was recently extended to the problem of ship motions in Noblesse & Hendrix (1991). The approach essentially consists in a Fourier-representation of the flow; it differs from the previously-mentioned panel-methods in that the order in which integrations are performed in these panel-methods, namely "Fourier-integration" followed by "space-integration" (corresponding to numerical evaluation of the Green function and its gradient followed by their integration over the hull-panels and the waterline-segments) is reversed. The Fourier-Kochin flow-representation given in Noblesse & Hendrix (1991) is summarized below, and a Galerkin solution-procedure, in which the unknown velocity potential at the hull-surface is represented using a set of continuous basis-functions defined over the whole hull-surface, is formulated. This Fourier-Kochin-Galerkin (FKG) approach is shown to offer several advantageous features.

The Fourier-Kochin formulation

The potential $\phi(\xi)$ at any point $\xi = (\xi,\eta,\zeta)$ on the hull-surface is defined by the solution of the following integro-differential equation:

$$\phi(\xi) = \phi_S(\xi) + \phi_R(\xi) \quad (1)$$

The potential ϕ_S is given by the sum of a source-distribution and a dipole-distribution of simple Rankine-singularities over the hull-surface h, as follows :

$$4\pi\, \phi_S(\xi) = \int_h da\ [\ (1/r'-1/r)\ d\phi/dn - \phi\ d(1/r'-1/r)/dn\ ] \quad (2)$$

The potential ϕ_R in (1) accounts for free-suface effects and is defined by the double Fourier-integral

$$4\pi^2\, \phi_R(\xi) = \int_\alpha \int_\beta d\alpha\, d\beta\ \exp[\zeta(\alpha^2+\beta^2)^{1/2} - i(\xi\alpha+\eta\beta)]\ S(\alpha,\beta)\ /\ [\ (f+i0-F\alpha)^2-(\alpha^2+\beta^2)^{1/2}\ ] \quad (3)$$

where f and F are the nondimensional frequency-number and Froude-number defined as $f = \omega(L/g)^{1/2}$ and $F = U/(gL)^{1/2}$, with L & U = ship length & speed, ω = encounter-frequency, and g = acceleration of gravity; furthermore, $S(\alpha,\beta)$ is the spectrum-function defined by the sum of an integral over the hull-surface h and an integral along the ship-waterline w, as follows:

$$S(\alpha,\beta) = \int_h da \exp[(\alpha^2+\beta^2)^{1/2}z+i(\alpha x+\beta y)] \; [\, d\phi/dn - \{(\alpha^2+\beta^2)^{1/2}n_z+i(\alpha n_x+\beta n_y)\}\phi\,]$$
$$+ F \int_w dl\, t_y \exp[i(\alpha x+\beta y)] \; [\, Fn_x d\phi/dn + 2i(f-F\alpha/2)\phi + F(t_x\phi_t+s_x\phi_s)\,] \quad (4)$$

where $t = (t_x, t_y, 0)$ is the unit tangent-vector to the waterline w, $s = (s_x, s_y, s_z)$ a unit vector tangent to the hull h and directed downward, $n = (n_x, n_y, n_z) = t \times s$ the unit outward normal-vector to the hull-surface h, and $(d\phi/dn, \phi_t, \phi_s)$ the components of the velocity vector along the vectors (n, t, s).

The foregoing Fourier-Kochin formulation entails three basic computational tasks, namely (i) evaluation of the simple-singularity potential ϕ_S defined by (2), (ii) evaluation of the spectrum-function $S(\alpha,\beta)$ defined by (4), and (iii) evaluation of the double Fourier-integral (3). Tasks (i) and (ii) correspond to "space-integrations", and task (iii) is a "Fourier-integration". The "space-integrations" in the Fourier-Kochin formulation are relatively trivial tasks. In particular, the hull-integral and the waterline-integral defining the spectrum-function $S(\alpha,\beta)$ can be evaluated analytically if h and w are approximated by flat triangular-panels and straight segments, respectively, and the corresponding integration-formulae are in fact simple; this "space-integration" is incomparably simpler than integrating the Green function and its gradient over the hull-panels and the waterline-segments.

The "Fourier-integration" in the Fourier-Kochin formulation is also fundamentally easier than the corresponding "Fourier-integration" in the usual panel-methods, which consists in evaluating the Green function and its gradient, because the spectrum-function $S(\alpha,\beta)$ in the integrand of the Fourier-integral (3) vanishes for large values of $\alpha^2+\beta^2$. Indeed, the Green function associated with the linearized free-surface condition is defined by (3) in which the spectrum-function $S(\alpha,\beta)$ is equal to the function $\exp[(\alpha^2+\beta^2)^{1/2}z+i(\alpha x+\beta y)]$. It can then readily be seen from (3) that the Fourier-integral is divergent in the special case when we have $\zeta = 0 = z$, $\xi = x$, $\eta = y$, which obviously corresponds to the singularity of the Green function. This singularity is eliminated in the Fourier-Kochin formulation due to the "space-integration" carried out in (4) prior to the "Fourier-integration" (3).

Galerkin solution-procedure

The flux $d\phi/dn$ across the hull-surface in the foregoing integro-differential equation is presumed known but arbitrary as far as the present hydrodynamic problem is concerned. The well-known specific forms of the hull-flux $d\phi/dn$ corresponding to the six basic radiation-problems in the usual linearized theory of the motion of a rigid ship are important particular cases, but other hull-flux distributions must also be considered to account for elastic structural-responses of the ship, and for nonlinear large-amplitude ship-motions as is shown in Pawlowski, Bass & Grochowalski (1988). Following the latter study, a modal representation of the hull-flux distribution is used, i.e. it is assumed that any hull-flux distribution $d\phi/dn$ required for consideration can be represented in terms of a set of basis-functions v_n, as follows: $d\phi/dn = \Sigma_n c_n v_n$ where c_n are coefficients and Σ_n means summation over the integer n. The integro-differential equation must then be solved for the set of hull-flux distributions

$$d\phi/dn = v_n \quad \text{with } n = 1 \text{ to } N \quad (5)$$

Let ϕ_n represent the hull-potential distribution, given by the solution of the integro-differential equation, corresponding to the hull-flux distribution $d\phi/dn = v_n$. The Galerkin solution-procedure recently used in Wu & Eatock Taylor (1989) is adopted here. The hull-potential distribution ϕ_n thus is represented in terms of a set of basis-functions μ_m, as follows:

$$\phi_n = \Sigma_m C_{mn} \mu_m \quad \text{with } m = 1 \text{ to } M \quad (6)$$

where the coefficients C_{mn} are unknown. The Galerkin solution-procedure may be regarded as a generalization of the usual approach, in which the number of basis-functions M is equal to the number of panels and the basis-functions μ_m are discontinuous functions taking the values 1 on one panel and 0 on all the other panels. Upon substituting (5) & (6) into (1)-(4), multiplying (1) by μ_k, and integrating over the hull-surface, we may obtain the following system of M linear algebraic-equations for determining the M unknown coefficients C_{mn}:

$$\Sigma_m [\, I(\mu_k,\mu_m) + Q(\mu_k,\mu_m) + R(\mu_k,M_m)\,] C_{mn} = P(\mu_k,v_n) + R(\mu_k,N_n), \quad (7)$$

where m = 1 to M and k = 1 to M, and the "influence-coefficients" I, P, Q and R are defined below.

The "influence-coefficients" I, P, and Q are defined in terms of hull-surface integrals involving only simple

Rankine-singularities, as follows:

$$I(\mu_m,\mu_k) = \int_h da\ \mu_k\ \mu_m \tag{8a}$$

$$4\pi\ P(\nu_n,\mu_k) = \int_h da\ \mu_k\ [\ \int_h da\ \nu_n\ (1/r'-1/r)\ ] \tag{8b}$$

$$4\pi\ Q(\mu_m,\mu_k) = \int_h da\ \mu_k\ [\ \int_h da\ \mu_m\ d(1/r'-1/r)/dn\ ] \tag{8c}$$

The "influence-coefficients" I, P, and Q are independent of the frequency-number f and of the Froude-number F, and thus must be evaluated only once for a given hull-form.

The "influence-coefficients" $R(\mu_k,M_m)$ and $R(\mu_k,N_n)$ are given by the Fourier-integral

$$4\pi^2\ R(\mu_k,S) = \int_\alpha\int_\beta d\alpha d\beta\ M_h{}^*(\alpha,\beta;\mu_k)\ S(\alpha,\beta)\ /\ [(f+i0-F\alpha)^2-(\alpha^2+\beta^2)^{1/2}] \tag{9}$$

where the spectrum-function $S(\alpha,\beta)$ is equal to either the function $N_n(\alpha,\beta)$ or the function $M_m(\alpha,\beta)$ defined as

$$N_n(\alpha,\beta) = N_h(\alpha,\beta;\nu_n) + F^2\ N_w(\alpha,\beta;\nu_n) \tag{10a}$$

$$M_m(\alpha,\beta) = M_h(\alpha,\beta;\mu_m) - 2iF(f-F\alpha/2)\ M_w(\alpha,\beta;\mu_m) - F^2\ M_w{}'(\alpha,\beta;\mu_m) \tag{10b}$$

The spectrum-function $M_h{}^*(\alpha,\beta;\mu_k)$ in the Fourier-integral (9), and the five spectrum-functions $N_h(\alpha,\beta;\nu_n)$, $N_w(\alpha,\beta;\nu_n)$, $M_h(\alpha,\beta;\mu_m)$, $M_w(\alpha,\beta;\mu_m)$, $M_w{}'(\alpha,\beta;\mu_m)$ in (10a,b) are defined in terms of integrals over the hull-surface h and the waterline w, as follows:

$$N_h(\alpha,\beta;\nu) = \int_h da\ \nu\ \exp[(\alpha^2+\beta^2)^{1/2}z+i(\alpha x+\beta y)] \tag{11a}$$

$$N_w(\alpha,\beta;\nu) = \int_w dl\ \nu\ n_x\ t_y\ \exp[i(\alpha x+\beta y)] \tag{11b}$$

$$M_h(\alpha,\beta;\mu) = \int_h da\ \mu\ [(\alpha^2+\beta^2)^{1/2}n_z+i(\alpha n_x+\beta n_y)]\ \exp[(\alpha^2+\beta^2)^{1/2}z+i(\alpha x+\beta y)] \tag{11c}$$

$$M_w(\alpha,\beta;\mu) = \int_w dl\ \mu\ t_y\ \exp[i(\alpha x+\beta y)] \tag{11d}$$

$$M_w{}'(\alpha,\beta;\mu) = \int_w dl\ (\mu_t t_x + \mu_s s_x)\ t_y\ \exp[i(\alpha x+\beta y)] \tag{11e}$$

$$M_h{}^*(\alpha,\beta;\mu) = \int_h da\ \mu\ \exp[(\alpha^2+\beta^2)^{1/2}z-i(\alpha x+\beta y)] \tag{11f}$$

The spectrum-function $M_h{}^*$ stems from the additional integration over the hull-surface required by the Galerkin solution-procedure. Within the framework of the Fourier-Kochin formulation, this "space-integration" is a relatively trivial task as was already noted, and indeed the "Galerkin-integration" of the potential $\phi_R(\xi)$ in (1) merely requires the calculation of one additional spectrum-function; i.e. a sixth spectrum-function must be evaluated in addition to the five spectrum-functions already required in (10a,b). A more general version of the foregoing Galerkin solution-procedure is presented in Noblesse (1991), where the hull-surface is divided into a number of patches within which a Galerkin-repesentation is used.

Modal-representation of the spectrum-functions

Let each of the six spectrum-functions be represented in terms of a set of basis-functions $S_p(\alpha,\beta)$, e.g.

$$N_h(\alpha,\beta;\nu_n) = \Sigma_p\ c_{pn}\ S_p(\alpha,\beta) \quad \text{with } p = 1 \text{ to } P \tag{12}$$

where c_{pn} are coefficients, which can be determined by solving the following system of P linear algebraic-equations

$$\Sigma_p\ [\ \int_\alpha\int_\beta d\alpha d\beta\ S_p(\alpha,\beta)\ S_q(\alpha,\beta)\ ]\ c_{pn} = \int_\alpha\int_\beta d\alpha d\beta\ N_h(\alpha,\beta;\nu_n)\ S_q(\alpha,\beta) \tag{13}$$

and similarly for the spectrum-functions $N_w, M_h, M_w, M_w{}', M_h{}^*$. The six spectrum-functions defined by (11a-f), the two Fourier-integrals in (13) and the coefficients c_{pn} in (12) and (13) are independent of the frequency-number f and the Foude-number F, and thus must be evaluated only once for a given hull-form.

Upon using the modal-representation (12) for the six spectrum-functions $N_h, N_w, M_h, M_w, M_w{}', M_h{}^*$ in (9) and (10a,b), we may define the "influence-coefficient" R given by (9) in terms of the coefficients R_{pq} defined by the following Fourier-integrals:

$$R_{pq}(f,F) = \int_\alpha\int_\beta d\alpha d\beta\ S_p(f^2\alpha,f^2\beta)\ S_q(f^2\alpha,f^2\beta)\ /\ [(1+i0-\tau\alpha)^2-(\alpha^2+\beta^2)^{1/2}] \tag{14}$$

where $\tau = f\ F = U\omega/g$. If a single set of basis-functions $S_p(\alpha,\beta)$ can be used to represent the six spectrum-functions $N_h, N_w, M_h, M_w, M_w{}', M_h{}^*$ for any hull-form, the coefficients R_{pq} defined by (14) are independent of the hull-form and must then be computed only once for a broad set of values of the parameters f & F, and used for predicting the flow due to any ship.

Conclusion

In summary, the approach summarized in the foregoing is based upon three main ideas: (i) the Fourier-Kochin formulation (1)-(4), (ii) the use of a Galerkin solution-procedure, and (iii) the modal-representation (12) for the spectrum-functions. This last idea makes it possible to break up the Fourier-integration (9), which depends on both the hull-form and the parameters f & F, into the Fourier-integrations defined in (13) which are independent of the parameters f & F, and the Fourier-integration (14) which does not depend on hull-form.

Given a set of basis-functions $S_p(\alpha, \beta)$ that can be used to represent the six spectrum-functions N_h, N_w, M_h, M_w, $M_w{}'$, $M_h{}^*$ for any hull-form, and the corresponding coefficients R_{pq}, the FKG approach involves three basic computational-tasks, namely evaluation of (i) the influence-coefficients I, P & Q defined by (8a,b,c), (ii) the spectrum-functions defined by (11a-f), and (iii) the related Fourier-integrals in (13). Each of these three tasks is independent of the frequency-number f & the Froude-number F, and thus must be performed only once for a given hull-form. Furthermore, the first two of these three tasks are relatively trivial and can be performed in an efficient and reliable manner; in particular, the cost of numerically-evaluating the "influence-coefficients" I, P & Q defined by (8a-c) and the six spectrum-functions defined by (11a-f) is practically independent of the number N & M of basis-functions v_n & μ_m, respectively, and the number of operations required to evaluate the spectrum-functions is proportional to the number of panels used for approximating the hull-surface. Furthermore, the dimensions of the M X M matrix & the P X P matrix in the systems of linear algebraic-equations (7) & (13), respectively, are expected to be of the order of 10^2, i.e. much smaller than the dimension —equal to the number of panels (of the order of 10^{3-4})— of the matrix of "influence-coefficients" in usual panel-methods.

The FKG approach however depends critically on the selection of appropriate sets of basis-functions for representing the potential-distribution on the hull-surface and the spectrum-functions, and on the evaluation of the coefficients Rpq defined by the Fourier-integral (14). The last of these important basic tasks, i.e. the Fourier-integration (14), is examined in Noblesse (1991).

Acknowledgments This work is supported by the Office of Naval Research and the Independent Research program of the David Taylor Research Center. I am indebted to Dr. Jacek Pawlowski at the National Research Council of CANADA for helpful discussions about the Galerkin solution-procedure.

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Kashiwagi: I found your paper to be quite impressive. I have some experience in calculating the wave profile on the free surface using slender-ship theory, which may interest you. In slender-ship theory, both z and ζ must be set equal to zero at which point, as you suggest, we encounter a singularity, when using the traditional procedure. My approach was to calculate first the Kochin function and then performe the integrations with respect to the Fourier-transformed variables. By following this procedure, I avoided singularities and got reasonable results.

Noblesse: Thank you Dr. Kashiwagi for bringing to my attention that you have previously used an approach similar to the Fourier-Kochin formulation. The use of a Galerkin solution-procedure can further simplify the Fourier integration beyond that achieved by using the Fourier-Kochin formulation.

Ohkusu: I have presented a single integral expression of the Green function. This expression has limits of integration which are dependent on the field point coordinates, but it is still possible to integrate analytically the effect on each panel, as long as we assume constant source strengths. I have hope that this expression for the Green function will work for your purpose. My colleague, Dr. Iwashita, is also working along these lines and he will present his results in the foreseeable future.

Noblesse: I would be most interested in using your single-integral expression for the Green function, which could result in very significant savings in computing time.

Martin: Galerkin methods are well known to have desirable properties for the numerical solution of integral equations. However, in practice, they are usually dismissed as they require double surface integrals over the body. Please comment.

Noblesse: In the present case, the additional hull-surface integration required in the Galerkin solution procedure amounts to integrating an exponential function, which can be performed analytically for flat hull panels. This Galerkin integration leads to the additional spectrum function given by $(11f)$.

Wu: To use your method, do I have to use constant elements?

Noblesse: In theory, flat panels are not required. In practice, however, the exponential function can only be integrated analytically for flat panels.

Clarisse: When considering the spatial integration of ∇G (which is necessary when considering the integral equation for example,) do you feel comfortable with the change of the order of integrations between the Fourier and the spatial integration? From the singular behavior of G, one can show that this interchange is indeed valid for G; however, when dealing with ∇G this is not obvious: the singularity of ∇G is not integrable in a Riemann sense.

Noblesse: I share your concern about the validity of interchanging the order of integration when integrating ∇G and I feel "comfortable" about doing it only to the extent that it greatly simplifies the required numerical calculations.