(Abstract for the 6th International Workshop on Water Waves & Floating Bodies, April 1991)

# THE HIGH-ORDER DIFFRACTION FORCES ON A SUBMERGED CIRCULAR CYLINDER

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## Introduction

When a circular cylinder is fixed under a free surface in the presence of waves, it experiences not only oscillatory forces but also mean steady forces because of nonlinear free surface effects. Based on the assumption of potential flow, Ogilvie (1963) obtained the first-order oscillatory forces and the mean second-order forces. He also showed that there was a mean uplift force but no mean horizontal force up to second order. Longuet-Higgins (1977) observed in his experiment that a submerged body experienced a drift force whose direction is opposite to that of the incoming waves, and suggested that this negative drift force can most likely be attributed to wave breaking. The experiments of Miyata, *et al* (1988) do not support all of Longuet-Higgins' predictions because they found that as the cylinder moved closer to the free surface leading to more intense breaking, the negative drift force was in fact reduced.

In this work, we apply a high-order spectral method, which is an extension of the method of Dommermuth & Yue (1987) for nonlinear wave-wave interactions, to calculate the high-order diffraction forces on a submerged circular cylinder. Specifically, the vertical and horizontal drift forces are obtained up to the fourth-order and the oscillatory forces up to frequencies which are third harmonics of the fundamental wave frequency. These results are compared to available experimental measurements.

## Mathematical formulation

A global Cartesian coordinate-system $(x, y)$ is located at the mean water level directly above the cylinder center with $y$ positive upward and $x$ positive in the direction of wave propagation. A local cylindrical coordinate-system $(r, \theta)$ is placed at the cylinder center which is at a depth $H$ below the mean water level. Thus $r^2 = x^2 + (y + H)^2$ and $\theta$ is measured counter-clockwise from positive $x$. For simplicity, the units of time $(t)$ and mass are chosen such that the gravitational acceleration and the fluid density are one. We assume that the flow is irrotational, and that the fluid is homogeneous, incompressible, and inviscid. A velocity potential $\Phi(x, y, t)$ is defined to describe the flow such that $\Phi$ satisfies Laplace's equation within the fluid. Since no flux can cross the cylinder, we have

$$\Phi_r(R, \theta, t) = 0 \quad \text{for} \quad 0 \le \theta < 2\pi \tag{1}$$

where, $R$ is the cylinder radius. We also define the surface potential

$$\Phi^s(x, t) = \Phi(x, \eta(x, t), t) \tag{2}$$

where $y = \eta(x, t)$ denotes the free surface, which we assume to be continuous and single-valued. In terms of $\Phi^s$, the kinematic and dynamic boundary conditions on the free surface are respectively

$$\eta_t + \Phi^s_x \eta_x - (1 + \eta_x^2)\Phi_y(x, \eta, t) = 0, \quad \text{and}$$

$$\Phi_t^s + \eta + \frac{1}{2}(\Phi_x^s)^2 - \frac{1}{2}(1+\eta_x^2)\Phi_y^2(x,\eta,t) = 0 \qquad (3)$$

for zero atmospheric pressure. As initial conditions, the surface potential $\Phi^s(x,0)$ and elevation $\eta(x,0)$ are prescribed in terms of exact Stokes waves. In addition, periodic boundary-conditions are imposed an integer number of wavelengths away from the origin for $-L \le x < L$, and $\nabla\Phi \to 0$ as $y \to -\infty$.

As in Dommermuth and Yue (1987), we assume that $\Phi$ and $\eta$ are $O(\varepsilon)$ quantities, where $\varepsilon$, a small parameter, is a measure of the wave steepness. We then expand $\Phi$ in a perturbation series in $\varepsilon$ up to order $M$:

$$\Phi(x,y,t) = \sum_{m=1}^{M} \Phi^{(m)}(x,y,t). \qquad (4)$$

Here and hereafter, $()^{(m)}$ denotes a quantity of $O(\varepsilon^m)$. We further expand each $\Phi^{(m)}$ evaluated on $y=\eta$ in a Taylor series about $y=0$, so that from (2) we have

$$\Phi^s(x,t) = \Phi(x,\eta,t) = \sum_{m=1}^{M}\sum_{k=0}^{M-m} \frac{\eta^k}{k!}\frac{\partial^k}{\partial y^k}\Phi^{(m)}(x,0,t). \qquad (5)$$

At a given instant of time, we may consider $\Phi^s$ and $\eta$ known, so that (5) is a Dirichlet boundary condition for the unknown $\Phi$. Thus, expanding (5) and collecting terms at each order, we obtain a sequence of Dirichlet boundary conditions for the unknown $\Phi^{(m)}$'s on $y=0$. These boundary conditions, in addition to $\Phi^{(m)}$ being periodic for $-L \le x < L$, $\Phi_r^{(m)}(R,\theta,t) = 0$ for $0 \le \theta < 2\pi$, and $\nabla\Phi^{(m)} \to 0$ as $y \to -\infty$, define a sequence of boundary-value problems for $\Phi^{(m)}, m = 1,2,\cdots,M$, in the domain $y \le 0$.

By distributing dipoles on the mean water line and sources on the cylinder's boundary, we can derive a complete set of eigenfunctions that are used to solve these boundary-value problems for $\Phi^{(m)}$. Finally, we can obtain the pressure distribution and force on the body surface using Bernoulli equation.

## Numerical Results

All the computations are performed in deep water using periodic boundary conditions. For simplicity, the computational domain is fixed to be $-\pi \le x \le \pi$. The initial conditions are right-going Stokes waves which we prescribe from exact (14 decimals of accuracy) results. The fourth-order Runge-Kutta scheme is used for the time integration of the free surface conditions (3).

Systematic convergence tests are performed and the numerical errors are less than 1% for all the computations shown. The steady state of the forces on the cylinder is approached after a simulation of typically $3 \sim 4$ wave periods. In the present study, the computation is stopped after $7 \sim 8$ wave periods before the forces are influenced by the (periodic boundary) images of the body. The amplitudes of the harmonic forces are obtained by using Fourier transform of the limit-cycle time histories of the forces.

Figure 1 shows the comparison of the first harmonic force between our high-order ($M=4$) numerical results and Ogilvie's linear potential solution as well as Chaplin (1984)'s experimental measurements as a function of the Keulegen-Carpenter number $K_c$. For our purpose, $K_c$ is defined by $\pi a e^{-kH}/R$ where $a$ is the (first-order) incident wave amplitude. For $K_c \le 0.6$, both numerical results and experimental measurements agree well with the linear solution. This is not surprising since for small wave slopes, neither nonlinearity nor flow separation significantly affects the wave-body interaction.

As $K_c$ increases, Chaplin's measurements show that the first harmonic force is substantially reduced from the linear solution which he claims is due to the effect of clockwise circulations around the cylinder. Our numerical results show a similar reduction due to high-order diffraction effects but the magnitude is small compared to that due to circulation.

Interestingly, the effect of circulation does not seem to affect the higher harmonic forces. This is shown in Figure 2 which compares our numerical predictions to Chaplin's experimental results. The comparisons are very good up to $K_c \sim 1$ beyond which the effect of wave breaking probably cannot be neglected. From the computed data, it is also observed that the second- and third-harmonic forces are second and third order in the Keulegen-Carpenter number $K_c$ as expected.

Figure 3 shows the dependence of the horizontal drift force on the body submergence. The horizontal drift force is negative and, as expected, its magnitude increases as the body submergence decreases. For other than the very shallow submergence case, ($kH >\sim 1.75kR$), the computed results agree well with the measurements of Miyata, *et al* (1988). Since our computations do not allow for wave breaking, this suggests that higher-order diffraction effects rather than wave breaking is the dominant cause of the negative drift forces in these cases. For shallower submergence, for which wave breaking is observed, the magnitude of the measured drift force increases less rapidly than the numerical prediction indicating that the presence of wave breaking may in fact *reduce* the horizontal drift force on the cylinder.

To indicate convergence, results for $M$ =2, 3 and 4 are also shown. These show that the horizontal drift force is due largely to interactions up to second-order only. Specifically, we find that the main contributions are due to quadratic self-interactions of the zeroth- and of the second-harmonic second-order potentials.

Figure 4 compares the vertical drift force among our numerical results, Ogilvie's second-order potential solution, and Miyata's experimental measurements. The agreement is reasonably good. The second-order predictions are quite adequate and the effect of wave breaking (for the smaller submergence) appears to be less significant.

## References


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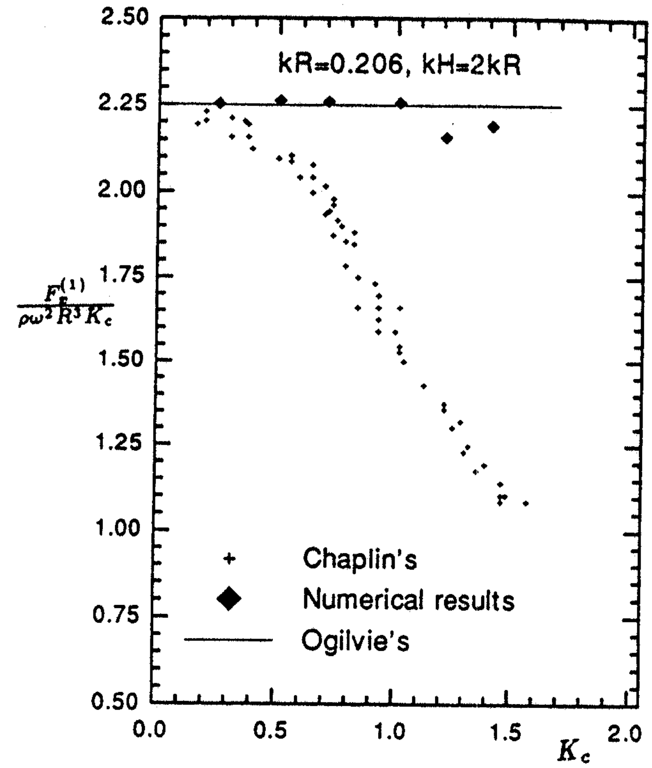


Figure 1. First-harmonic force

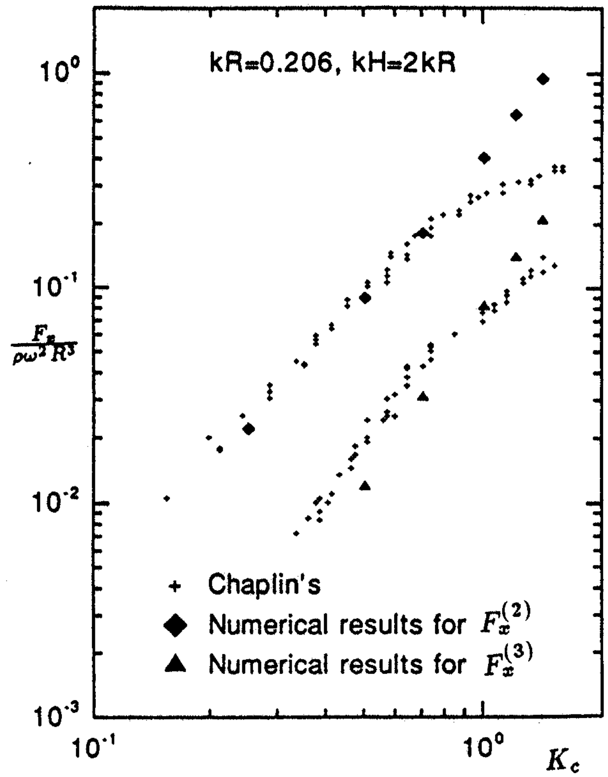


Figure 2. Second- and third- harmonic forces

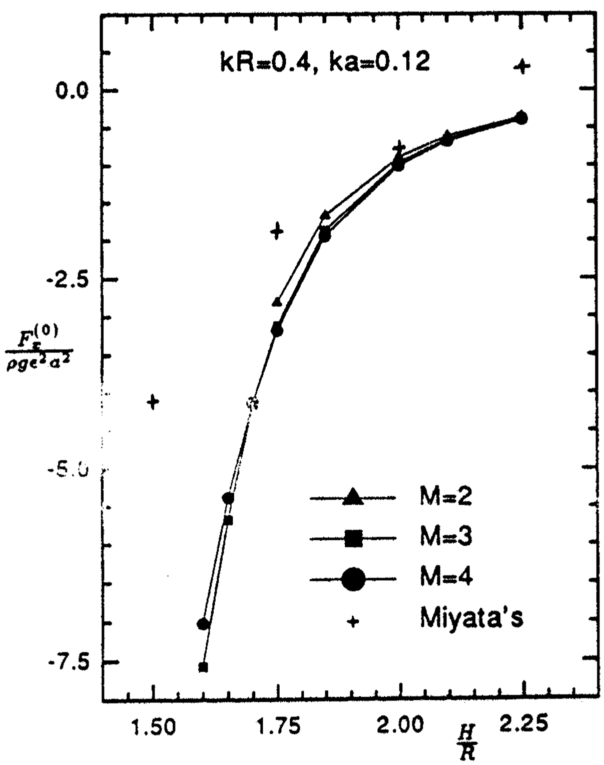


Figure 3. Horizontal drift force.

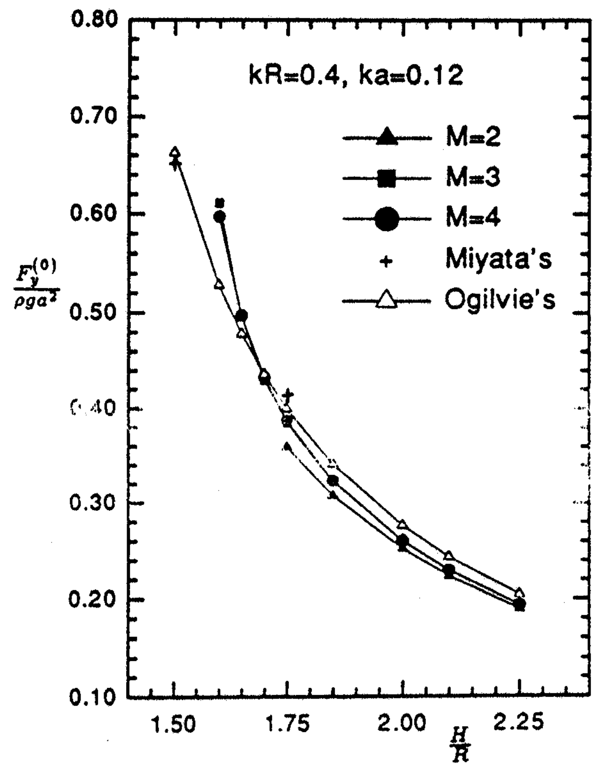


Figure 4. Vertical drift force.

**Grue:** (A) Did you compute the third harmonic wave amplitude downstream? And, what is the magnitude? This wave is hardly observed in the experiments relevant to your cases. (B) When the case you consider is close to the point where breaking is observed above the body, would it then be more appropriate to apply $M = 10$ or 20, say, than $M = 4$?

**Liu & Yue:** (A) We were able to compute the third harmonic waves for most cases. For deeper submergence and large incident wave slope, the third harmonic wave amplitude can be as large as that of the second harmonic wave. (B) Our computational experience with Stokes waves indicates that spectral resolution can be achieved for up to approximately 80% of Stokes' limiting steepness ($ka \sim 0.35$). For these cases, typical values of $M \sim O(10)$ are employed.

**Palm:** If the cylinder is deeply submerged, I think it is relatively easy to take into account the effect of the velocity circulation observed by Chaplin in your first order solution. It seems reasonable that this will give a better agreement between Chaplins experiments and your theory for second and third order oscillatory forces for $K_c$ of order unity.

**Liu & Yue:** We agree with you. In the present study, however, we have focussed primarily on the steady (fourth-order) drift force on the body.

**S. Liao:** Your method is based on perturbation theory and, therefore, a small parameter is needed. Maybe, you could overcome this disadvantage by using a continuous mapping technique. If you analyze the constructed continuous mapping in the mapping domain by using Taylor's series, you would find that the supposition of a"small parameter" is not needed.

**Liu & Yue:** The performance of the high-order spectral method for nonlinear waves is now well documented (see, *e.g.*, ref. [3]). The validity and convergence of our perturbation is limited by the radius of convergence (from $z = 0$) of $\Phi$, which cannot extend beyond the first singularity in the analytic continuation of $\Phi$ above $z = \eta$. It seems unlikely that the continuous mapping technique can exceed that limit. Indeed, from the computational data available for both methods, the high-order spectral method appears to be more effective for steep waves.