ON A METHOD FOR REMOVING THE IRREGULAR FREQUENCIES FROM INTEGRAL EQUATIONS FOR WATER-WAVE RADIATION PROBLEMS

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Consider a body floating on the free surface of deep water and performing forced harmonic oscillations of small amplitude at a radian frequency ω .Under the usual assumptions of an inviscid, irrotational flow the problem may be described by a velocity potential ϕ satisfying the Laplace equation in the fluid domain, the linearized free surface condition, the body boundary condition and appropriate conditions at infinity. The most popular method for solving this problem is to apply Green's theorem to ϕ and an appropriate Green function G producing a Fredholm integral equation of the second kind for the values of the potential on the body surface S. Letting P denote the field point and Q denote the source point, we may write:

$$2\pi\phi(P) + \int_S\int \phi(Q)\frac{\partial G(P,Q)}{\partial n_Q}dS_Q = \int_S\int V(Q)\,G(P,Q)\,dS_Q \qquad (1)$$

where V(Q) is the prescribed velocity of the body and G(P,Q) is the usual wave-source potential.

As is well known, the integral equation (1) does not possess a unique solution at a discrete set of frequencies-the so-called irregular frequencies-where the corresponding homogeneous equation has a non-trivial solution. The existence of the irregular frequencies represents the most serious drawback of the boundary integral equation method and several methods for removing them have been proposed ex. [1],[2],[3],[4].

A different approach to solve the boundary value problem for ϕ was proposed in [5]. A point in the interior of the body is selected and designated as the origin. Then the Green function G may be expanded as:

$$G(P,Q) = \sum_{m=0}^{\infty}\sum_{j=0}^{\infty}\sum_{\sigma=1}^{2} a^{\sigma}_{jm}(P)\ \phi^{\sigma}_{jm}(Q) \qquad (2)$$

where the coefficients $a^{\sigma}_{jm}(P)$ and the water wave multipoles $\phi^{\sigma}_{jm}(Q)$ are defined in [5]. This approach leads to an infinite set of moment-like equations called the null-field equations for water waves:

$$\int_S\int[\phi(Q)\frac{\partial\phi^{\sigma}{}_{jm}(Q)}{\partial n_Q} - V(Q)\phi^{\sigma}{}_{jm}(Q)]\,dS_Q = 0 \qquad \sigma = 1,2 \quad m,j = 0,1,2... \qquad (3)$$

It is possible to show that the null-field equations possess a unique solution at all frequencies ([5],[6]). Furthermore the null-field equations have been used to solve radiation problems in two-dimensions involving bodies of simple geometry such as a heaving ellipse and a pair of circular cylinders.

In the present work, we seek to solve the integral equation (1) subject to the conditions (3). Equation (1) is solved numerically using a panel method. In the usual manner, the body surface is approximated by an ensemble of N plane quadrilateral elements of constant potential strength. A collocation method is used resulting in an algebraic system of N equations for the N unknown potential strengths. In order to eliminate the influence of the irregular frequencies, these equations are supplemented by the null field equations (3). If the infinite set of equations (3) is truncated after m=j=M, the result is an overdetermined system which is solved by a least squares procedure. Using arguments similar to [7] where an analogous procedure was adopted for the solution of exterior acoustics problems, it can be shown that the interior potential ϕ_i and its first M derivatives vanish at the origin. As a result, the solution is unique for wave numbers k < k_{M+2} (k_{M+2} is the M+2 th irregular wavenumber). The present method is related to the combined boundary integral equation method proposed in [4] where the solution of the integral equation was supplemented by the requirement that the interior potential be zero at certain interior points. The effectiveness of the present method stems from the fact that it removes the arbitrariness in selecting the number and location of these interior points. Instead, given an estimate of the location of the irregular frequencies, it provides a definite rule for the number of extra equations that must be used to guarantee uniqueness. Especially in the high frequency range where the number of nodes in the interior eigensolution increase,the present method is believed to have a clear advantage.

Preliminary results have been obtained for the hydrodynamic coefficients of a cylinder a rectangle and a sphere with very little additional computational effort compared to the original boundary integral method. In all cases considered, the performance of the method has been very good.

References

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Lee: (1) How do you solve the over-determined system and what is the additional computational effort required? (2) How much variation of the solution did you observe when you added the constraints, and when you increased the number of constraints?

Liapis: (1) Define by $[e]$ the N+M vector of error, then the task is to minimize $[e*]^T[e]$ with respect to $\phi_1, \phi_2 ... \phi_N$ (*denotes the complex conjugate and ^{T} the transpose). For the number of panels used so far (< 100) the additional computational cost is very small. (2) For the cases examined so far, away from an irregular frequency, the solution to the boundary integral equation also satisfies the nul field equations, so adding the constraints has little effect.

Grue: In solving water wave problems for bodies, for which one does not necessarily know the irregular frequencies, a tool which gives precise and efficient answers for all frequencies is desired. Is your model predicting the forces, etc. with good accuracy for frequencies far away from the irregular frequencies, and what is the extra cost with your method compared to the conventional panel method?

Liapis: For the cases examined thus far (that is a floating hemisphere, a right circular cylinder and a variety of 2-D cases which I did not have time to present) the present method gives accurate results for all frequencies. As a result, given an estimate of the $(M+2)^{th}$ irregular wavenumber K_{M+2}, the present method will give accurate results for $K < K_{M+2}$. The extra cost comes from solving a system of $N + M$ equations with N unknowns as opposed to a $N \times M$ system and evaluating NM extra influence coefficients. Although it is still very early to give a precise estimate of the extra CPU time required, the CPU for the modified method should not exceed $(N + M)/N$ as a percentage of the original method.

Martin: Have you tried incorporating your extra constraints (M, say) by using M Lagrange multiplies (rather than least squares)? This method has been used in acoustics for Schenck's method (as used in [4]) by A.F. Seybert & T.K. Rengarajan, 'The use of CHIEF to obtain unique solutions for acoustic radiation using boundary integral equations,' J. Acoust. Soc. Amer. 81 (1987) 1299-1306. These authors claim that the method is superior to least squares.

Liapis: If you use M Lagrange multiplies you will augment your original system to an $(N+M) \times (N+M)$ system. On the other hand, if you use least squares you will still have an $N \times N$ system but the coefficients must be changed using formulas of the form $\sum_{k=1}^{N+M} A_{ki}^* A_{kj}$ $i, j = 1$ to N. It might be true that the Lagrange multipliers method has a slight edge, but this is going to be significant only if a large number of panels and constraints is used.

Newman: In the discussion following equation (3) it seems surprising that $M + 1$ irregular frequencies are suppressed by M extra equations. Is this explainable?

Liapis: You have corrected an oversight in my presentation where I should have used $j + m = 0, 1, ...M$ that is $M + 1$ extra equations. In practice the method can perform much better because, if say you use only one extra equation, you enforce $\phi_i = 0$ at the origin (ϕ_i is the interior potential). Therefore in addition to the first irregular frequency for heave you also suppress all the irregular frequencies for which $\phi_i \neq 0$ at the origin. A similar argument can be used for more additional equations where you also force the partial derivatives of ϕ_i to vanish at the origin.

Wu: To me, Equation (3) is a Galerkin method. Any complete series can be used to replace $\phi_{jn}^{\sigma}(Q)$.

Liapis: Equation (3) is not a Galerkin method but an infinite set of moment-like equations which are derived using the series expansion (2) for the Green function. If you are not content with the present multipole set $\phi_{jm}^{\sigma}(Q)$ you may choose another complete set of harmonic potentials satisfying the free-surface and radiation conditions but I don't see any reason for doing this.