SECOND-ORDER STEADY FORCE AND YAW MOMENT ON A SHIP ADVANCING IN WAVES

By Masashi KASHIWAGI and Makoto OHKUSU

Research Institute for Applied Mechanics,
Kyushu University, Fukuoka, Japan

Summary

A new analysis method is provided for the added resistance, steady sway force, and yaw moment acting on an advancing ship in oblique waves. The method is based on the principle of linear and angular momentum conservation, but instead of using the stationary-phase method, the method utilizes Parseval's theorem in the Fourier-transform theory. Calculation formulas for the steady sway force and yaw moment are obtained in a form involving only the Kochin function. With the unified slender-ship theory, numerical computations are also performed for the diffraction problem and compared with experiments.

Asymptotic form of the velocity potential and its Fourier transform

As shown in Fig. 1, we consider a ship advancing at constant forward velocity U into a plane progressive wave of amplitude a, circular frequency ω_0, and wavenumber $k_0 = \omega_0^2/g$, with g the gravitational acceleration. The angle of wave incidence is denoted by χ, with $\chi = 0$ corresponding to the following wave. Due to the incident wave, the ship performs sinusoidal oscillations about its mean position with the circular frequency of encounter $\omega = \omega_0 - k_0 U \cos\chi$.

The assumptions of the linearity and the inviscid flow with irrotational motion permits us to write the velocity potential in the form

$$\Phi(x,y,z,t) = -Ux + \phi(x,y,z,t) \tag{1}$$

$$\phi(x,y,z,t) = \mathrm{Re}\left[\frac{ga}{i\omega_0}\{\varphi_0(x,y,z) + \varphi(x,y,z)\}\, e^{i\omega t}\right] \tag{2}$$

$$\varphi_0 = e^{-k_0 z - ik_0(x\cos\chi + y\sin\chi)} \tag{3}$$

Eq.(3) is the incident-wave potential and φ in (2) the disturbance potential due to the presence of a ship, consisting of the scattered and radiation potentials.

Using Green's theorem and an asymptotic form of the Green function expressed in a form of the inverse Fourier transform, we can obtain the far-field approximation of φ valid at large distances from the x-axis:

$$\varphi(x,y,z) \sim \frac{i}{2\pi}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right] H^{\pm}(k)\frac{\nu}{\sqrt{\nu^2 - k^2}}\, e^{-\nu z \mp i\epsilon_k y\sqrt{\nu^2-k^2} - ikx}\, dk \tag{4}$$

where

$$\left.\begin{aligned} \nu &= (\omega + kU)^2/g = K + 2k\tau + k^2/K_0 \\ K &= \omega^2/g\,, \quad \tau = U\omega/g\,, \quad K_0 = g/U^2 \end{aligned}\right\} \tag{5}$$

$$k_{1,2} = -\frac{K_0}{2}\left[\,1 + 2\tau \pm \sqrt{1+4\tau}\,\right], \quad k_{3,4} = \frac{K_0}{2}\left[\,1 - 2\tau \mp \sqrt{1-4\tau}\,\right] \tag{6}$$

$$\epsilon_k = sgn(\omega + kU) = \begin{cases} -1 & \text{for } -\infty < k < k_1 \\ \;\;1 & \text{for } \;\; k_2 < k < \infty \end{cases} \tag{7}$$

Here $H^{\pm}(k)$ in (4) is the Kochin function and can be written as

$$H^{\pm}(k) = C(k) \pm i\,\epsilon_k\, S(k) \tag{8}$$

where

$$\left.\begin{matrix} C(k) \\ S(k) \end{matrix}\right\} = \iint_{S_H} \left(\frac{\partial \varphi}{\partial n} - \varphi \frac{\partial}{\partial n} \right) e^{-\nu\zeta + ik\xi} \left\{ \begin{matrix} \cos(\eta\sqrt{\nu^2 - k^2}) \\ \sin(\eta\sqrt{\nu^2 - k^2}) \end{matrix} \right\} dS \tag{9}$$

The upper or lower of the complex signs appearing in (4) and (8) is to be taken according as the sign of y is positive or negative, respectively.

From (4), we can readily obtain the Fourier transform of the disturbance potential in the following form, with $u(\nu^2 - k^2)$ being the unit step function:

$$F\{\varphi(x,y,z)\} = i\epsilon_k H^{\pm}(k)\, u(\nu^2 - k^2) \frac{\nu}{\sqrt{\nu^2 - k^2}} e^{-\nu z \mp i\epsilon_k y\sqrt{\nu^2 - k^2}} \tag{10}$$

The Fourier transform of the incident-wave potential φ_0 takes the form

$$F\{\varphi_0(x,y,z)\} = 2\pi\delta(k - k_0\cos\chi)\, e^{-k_0 z - ik_0 y \sin\chi} \tag{11}$$

where $\delta(k - k_0\cos\chi)$ is Dirac's delta function, thus contributing only for $k = k_0\cos\chi$.

Calculation formulas for the steady force and moment

Let us consider the rate of change of linear and angular momentum within the fluid domain bounded by the ship's hull S_H, the free surface S_F, and a control surface S_C at a large distance from the ship. Taking account of that there is no flux across S_H and S_F, the pressure is zero on S_F, and the fluid motion is periodic, the steady force in the x-y plane, $\overline{\mathbf{F}}$ and the moment about the z-axis, $\overline{M}_z$ are given by

$$\overline{\mathbf{F}} = -\overline{\iint_{S_C} [\, p\,\mathbf{n} + \rho\nabla\phi\,(\nabla\phi\cdot\mathbf{n} - Un_x)]\, dS} \tag{12}$$

$$\overline{M}_z = -\overline{\iint_{S_C} [\, p(\mathbf{r}\times\mathbf{n})_z + \rho\{(\mathbf{r}\times\nabla\phi)_z + yU\}(\nabla\phi\cdot\mathbf{n} - Un_x)]\, dS} \tag{13}$$

where p is the fluid pressure given by Bernoulli's equation, ρ the fluid density, $\mathbf{n}$ the normal vector pointing out of the fluid domain, $\mathbf{r}$ the position vector, and the subscript x or z denotes the x- or z-component of vector quantities, respectively. The overbar in (12) and (13) means taking time average.

Instead of a circular cylinder of large radius about the z-axis, we take two flat plates as the control surface, which are, as shown in Fig. 1, located at $y = \pm Y$ and extend from $x = -\infty$ to $x = +\infty$ and from the instantaneous free surface down to $z = +\infty$. It should be emphasized that all the disturbance waves radiating away from the x-axis are precisely included in the asymptotic form of φ, (4). Thus neglected are only the contributions from the local waves near the x-axis; these will become zero at $x = \pm\infty$ in the 3-D case.

Note that $n_x = 0$ on the present control surface. Then considering the added resistance as an example, we have from (12) and (2)

$$\overline{R} = \frac{\rho g a^2}{2k_0} \mathrm{Re} \int_0^{\infty} dz \int_{-\infty}^{\infty} dx \left[\frac{\partial\varphi}{\partial x}\frac{\partial\varphi^*}{\partial y} + \frac{\partial\varphi}{\partial x}\frac{\partial\varphi_0^*}{\partial y} + \frac{\partial\varphi_0^*}{\partial x}\frac{\partial\varphi}{\partial y} \right]_{-Y}^{Y} + O(\phi^3) \tag{14}$$

where the asterisk denotes the complex conjugate and $[\quad]_{-Y}^{Y}$ means the difference between values of the quantity in brackets at $y = Y$ and $y = -Y$.

The integrations with respect to x in (14) can be easily performed with Fourier transforms, (10) and (11), and Parseval's theorem expressed by

$$\int_{-\infty}^{\infty} f(x)g^*(x)\,dx = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(k)G^*(k)\,dk \tag{15}$$

The z-integrations in (14) will be analytically carried out, because φ and φ_0 have simple dependences on the coordinate z as seen in (10) and (11).

The results obtained in the above manner can be summarized as follows:

$$\begin{aligned}\frac{\overline{R}}{\rho g a^2} =& \frac{1}{4\pi k_0}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right]\left\{|C(k)|^2 + |S(k)|^2\right\}\frac{\nu}{\sqrt{\nu^2 - k^2}}\,k\,dk \\ &- \frac{1}{2}\,\mathrm{Im}\,[H(k_0,\chi)]\cos\chi\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\overline{F}_y}{\rho g a^2} =& -\frac{1}{4\pi k_0}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right]\mathrm{Im}\,\{2C(k)S^*(k)\}\,\nu\,dk \\ &+ \frac{1}{2}\,\mathrm{Im}\,[H(k_0,\chi)]\sin\chi\end{aligned} \tag{17}$$

$$\begin{aligned}\frac{\overline{M}_z}{\rho g a^2} =& \frac{1}{4\pi k_0}\left[-\int_{-\infty}^{k_1} + \int_{k_2}^{k_3} + \int_{k_4}^{\infty}\right]\mathrm{Re}\,\{C'(k)S^*(k) - C^*(k)S'(k)\}\,\nu\,dk \\ &- \frac{1}{2}\,\mathrm{Re}\left[C'(k_0,\chi) + i\,S'(k_0,\chi) + \frac{1}{k_0}\left(\tau + \frac{k_0\cos\chi}{K_0}\right)H(k_0,\chi)\right]\sin\chi\end{aligned} \tag{18}$$

Here $H(k_0,\chi)$ is the function obtained after substituting $k = k_0\cos\chi$ and $\pm\epsilon_k\sqrt{\nu^2 - k^2} = k_0\sin\chi$ into the Kochin function $H^{\pm}(k)$, $C'(k)$ and $S'(k)$ may be given by analytical differentiations of (9), and $C'(k_0,\chi) + i\,S'(k_0,\chi)$ is defined in the same way as $H(k_0,\chi)$.

Applying the foregoing procedure to the energy-conservation principle in order to recast the second term on the right-hand side of (16), we can show with relative ease that eq. (16) is identical to Maruo's added-resistance formula [1]. Eqs. (17) and (18) are the results obtained for the first time by the present analysis. It is noteworthy that deriving (18) may be intractable as long as we follow Maruo's analysis using the stationary-phase method. In the limit of vanishing forward speed, the above formulas recover Newman's results [3] on the drift force and moment.

Numerical examples

Computations were performed for the diffraction problem, using the unified slender-ship theory [2] to determine the scattered potential φ and the Kochin function. Fig. 2 shows the steady sway force on a half-immersed prolate spheroid of beam-to-length ratio $B/L = 1/6$, in oblique waves of $\chi = 135°$. At $U = 0$, the unified-theory results agree well with independent results by a 3-D panel method. The results for $Fn = 0.15$ indicate that forward-speed effects are small; but this is not the case for the added resistance and steady yaw moment. Fig. 3 is a comparison of computed and measured profile of diffraction wave at $y = 0.4L$, generated by a prolate spheroid of $B/L = 1/5$ moving in head waves ($\lambda/L = 1.0$, $Fn = 0.2$); the bow of ship model is located at $x = 0$. Except near the ship, good agreement can be observed, implying that the Kochin function and thus the steady force and yaw moment predicted by the unified slender-ship theory will be relatively accurate. Further computations are now in progress and their results will be presented in the foreseeable future.

References

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[2] Sclavounos, P. D. : The Diffraction of Free Surface Waves, J. Ship Res., Vol. 28, No. 1, (1984) pp. 29-47

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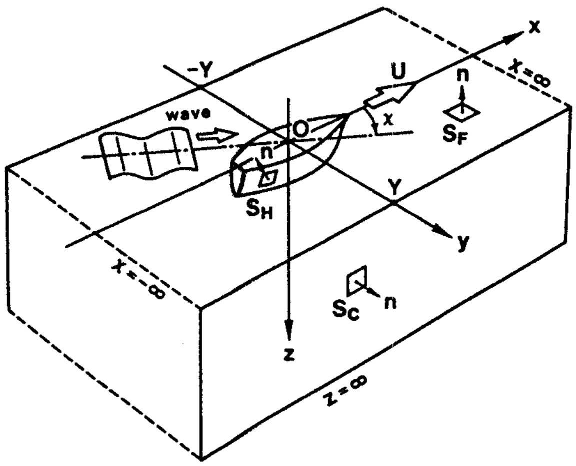

Fig. 1 Coordinate system and notations

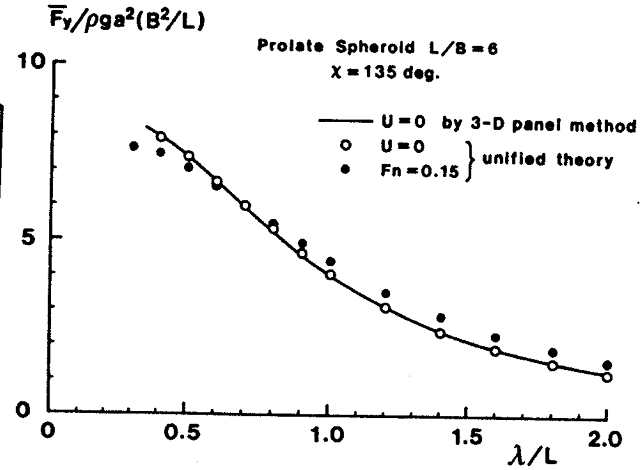

Fig. 2 Steady sway force on a half-immersed prolate spheroid of $L/B = 6$

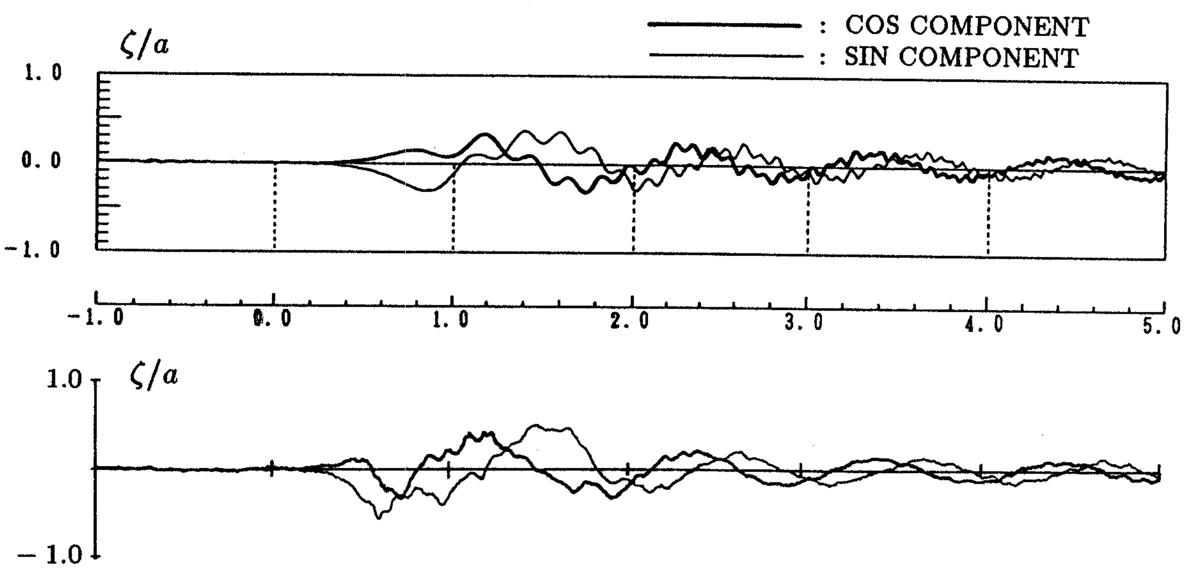

Fig. 3 Comparison of diffraction wave between computed and measured (prolate spheroid of $L/B = 5$, $Fn = 0.2$, $\lambda/L = 1.0$, $y/L = 0.4$) Upper: computed, Lower: measured

Sebastiani: Did you try to apply your formulation to the vertical plane forces which are of great interest for high speed catamarans and SWATH vessels?

Kashiwagi: No I didn't, but I think that it is possible. However, we should note that the basic equation for the vertical steady force will be different in form from eq.(12). Even if we restrict ourselves to the wave-induced linear forces and resulting motions of a catamaran with forward velocity, we have no reliable and reasonable theory. Thus, before proceeding to the steady vertical force on a catamaran, there are several linear problems which need to be resolved.