A Hydro-Aero-Elastic Model for Slamming of a 2-D Ship Hull Section

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Introduction

For a two-dimensional ship hull with a flat bottom or small deadrise angles, the bottom slamming will happen when it is dropped on to a water surface. According to Chuang's experiments[1], the slamming pressure reaches its maximum value before the bottom touches the water surface. This implies that the maximum pressure on the bottom will be developed due to the compression of the air between the free surface and the bottom of the ship hull. When slamming occurs, the bottom is quite close to the free surface. The compressed air flow will depend on the geometric boundary at each time instant. Therefore, the deformation of both the flexible bottom and the free surface should be considered. Since the physical model, as shown in Fig.1, is affected by "hydro-aero-elastic" interactions, all three factors should also be included in the mathematical model.

There are two methods in modelling the compressed air flow in the slamming problem. The first approach is that one takes into account the free surface deformation and treats the unsteady air flow as a one-dimensional flow and the time variable term in the free surface potential is neglected. The other is that a two-dimensional air flow is considered and the free surface is treated as a flat rigid surface. None of the two methods considers the elastic effect of the flexible bottom. According to the published theoretical and experimental results, the difference between experimental data and numerical results could be due to the deformation of the bottom. The one-dimensional air flow could be a good approach because the air layer between the bottom and the free surface is very thin.

Formulation of the Problem

In our study, we assume that the air is a perfect gas and that the flow is isentropic. The governing equations of the unsteady flow of compressible air are:

$$[\frac{\partial}{\partial t}+(u+c)\frac{\partial}{\partial x}](u+\frac{2c}{\gamma-1})+\frac{c}{h}(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x})=0, \quad (1)$$

$$[\frac{\partial}{\partial t}+(u-c)\frac{\partial}{\partial x}](u-\frac{2c}{\gamma-1})-\frac{c}{h}(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x})=0, \quad (2)$$

where c is the velocity of sound, h is the height between the free surface and the bottom, u is the velocity of the air flow and the gas constant γ is 1.4. An additional equation is required to solve the above three unknowns, u, c and h. It is formulated according to the deformation of the free surface and the ship bottom as follows:

$$\frac{\partial h}{\partial t}=\frac{\partial d}{\partial t}-\frac{\partial \eta}{\partial t}+\frac{\partial w}{\partial t}, \quad (3)$$

where $d(t,x)$ is the vertical distance between the rigid bottom and the undisturbed free surface, η is the free surface elevation and w is the deflection of the bottom. Together with the initial and boundary conditions, the above problem can be solved by the characteristics method [4]. The characteristic equations are:

$$du+\frac{2}{\gamma-1}dc+\frac{c}{h}(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x})dt=0, \quad (4)$$

$$du-\frac{2}{\gamma-1}dc-\frac{c}{h}(\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x})dt=0, \quad (5)$$

which should be solved along the following characteristic directions:

$$\frac{dx}{dt}=u+c, \quad (6)$$

$$\frac{dx}{dt} = u - c. \tag{7}$$

The slamming pressure is obtained from:

$$\frac{p}{p_0} = \left(\frac{c}{c_0}\right)^{\frac{2\gamma}{\gamma-1}}, \tag{8}$$

where c_0 is speed of sound in undisturbed air amd p_0 is the atmospheric pressure.

The unsteady free surface flow is induced by the slamming pressure on the free surface. The fluid is treated as inviscid and incompressible, and the flow is irrotational. The free surface elevation is :

$$\eta(x,t) = -\frac{1}{g}\frac{\partial\phi(x,0,t)}{\partial t} - \frac{p(x,0,t)}{\rho}, \tag{9}$$

where ϕ is the potential function of the free surface flow, ρ is the fluid density and p is the pressure on the free surface. The potential function has been given by Wehausen and Laitone [5],

$$\phi(x,y,t) = -\frac{1}{\pi\rho}\int_{-\infty}^{\infty} d\xi \int_0^t p(\xi,0,t)d\tau \int_0^{\infty} cos[\sqrt{gk}(t-\tau)]cos[k(x-\xi)]e^{ky}dk. \tag{10}$$

The flexible bottom of the ship hull section is simplified as a long plate with two built-in edges. Because the bottom is statically indeterminate, the bending moments at the two ends are solved by the conjugate-beam method. Under the unsteady slamming load action, the plate will dynamically responds to the load with a deflection. For simplicity, we solve the following differential equation of plate deflection at each time step:

$$\frac{Eb^3}{12(1-\nu^2)}\frac{d^2w(x,t)}{dx^2} = -M(x,t), \tag{11}$$

subjected to the boundary conditions:

$$w(-l,t) = 0, \qquad \text{and} \qquad \frac{dw(-l,t)}{dx} = 0, \tag{12}$$

where E is Young's modulus of the bottom material, ν is the Poisson's ratio, b is the thickness of plate, l is the half-breadth of plate and M is the bending moment. The following results can be obtained:

$$M(x,t) = m(x,t) - n(x,t) - M_0, \tag{13}$$

$$m(x,t) = \int_{-l}^{x}(x+l)p(x,0,t)dx, \tag{14}$$

$$n(x,t) = (x+l)\left[\int_{-l}^{x} p(x,0,t)dx - \int_{-l}^{l} p(x,t)dx\right], \tag{15}$$

and the bending moment at each edge:

$$M_0 = \frac{1}{2l}\int_{-l}^{l}[m(x,t) - n(x,t)]dx. \tag{16}$$

Numerical Examples and Discussions

In the numerical example, a bottom model with $l = 0.2m$, drop height$= 0.4m$, and mass$= 20kg$ is chosen as in Ref. [2]. Only the upper limits of the experimental results in Ref. [2] are used for comparison with the numerical results.

The characteristic equations (4) and (5) are solved by the characteristics method of specified time interval with a linear finite difference algorithm. The grids and the characteristic curves in numerical computation are shown in Fig.2.

Fig.3 gives the slamming pressure at the center point of the bottom. The experimental results are for the rigid bottom so that a rigid bottom model is used in computation. The boundary conditions at two edges of the bottom is $p = p_0$. According to (9), the pressure distribution should be continuous because the free surface elevation is continuous. Therefore, the pressures at two edges have to be smoothed in computation in order to obtain reasonable results. Further studies are needed to provide better modelling of the air jet flow for a more realistic pressure distribution at the exits of the bottom edges. The slamming pressure, with the consideration of the rigid free surface and the flexible bottom, is given in Fig. 4. A relatively rigid aluminum plate of $b = 2cm$ and $l = 20cm$ is used for the bottom in the numerical computation. However, the elastic bottom effect can still be identified from Fig. 4. It can be seen that the rigid bottom and rigid free surface air flow model gives much higher slamming pressure. Fig. 5 shows the slamming pressure with both the free surface and elastic bottom effects. It is found that the effect of elastic bottom would reduce the maximum slamming pressure. Also, the air flow model, with the consideration of the rigid bottom and free surface effects will overpredict the maximum slamming pressure. For a real ship bottom, a plate with stiffeners should be used. In order to obtain higher accuracy, a second order process should be applied to the characteristics method.

References

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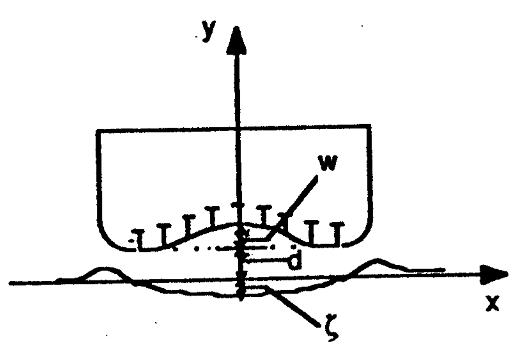

Fig. 1 The hydro-aero-elastic model

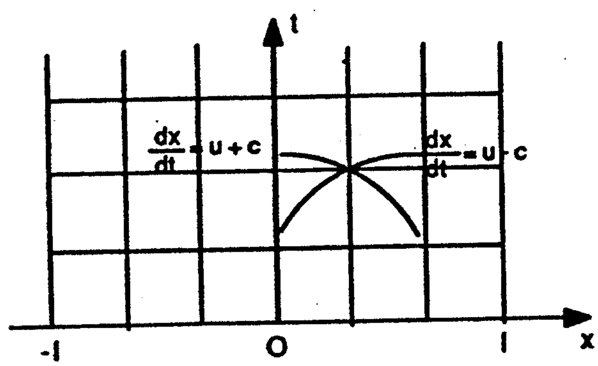

Fig. 2 The characteristic curves

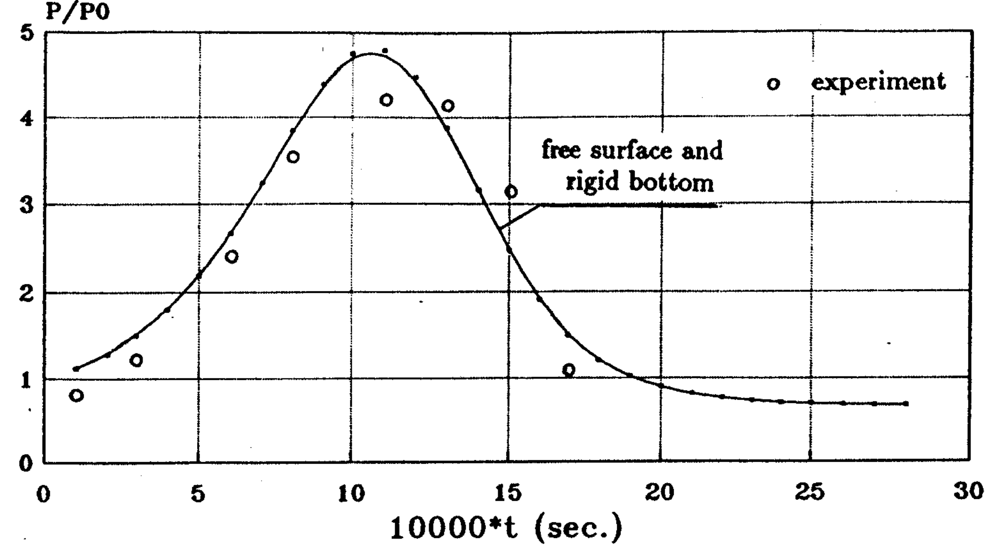

Fig. 3 Slamming pressure with free surface effect

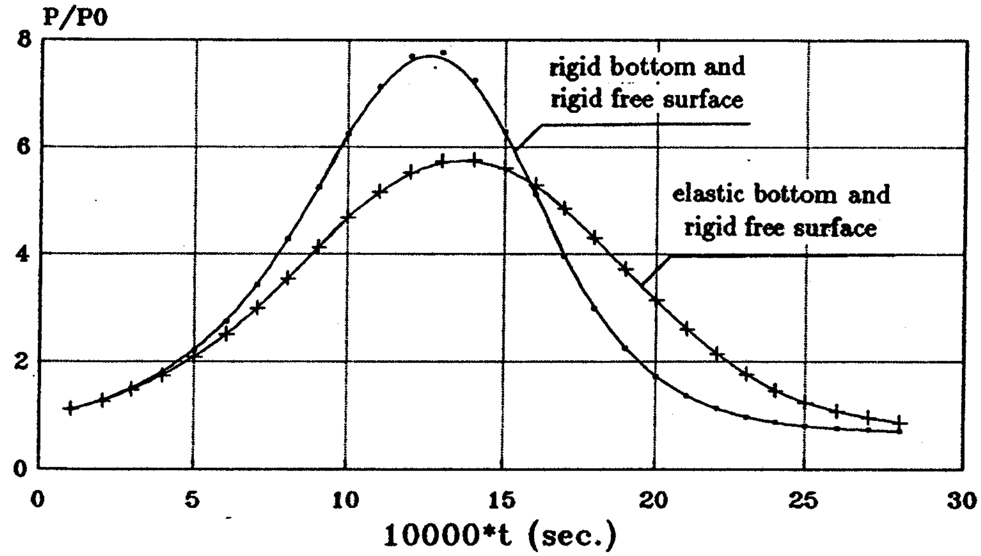

Fig. 4 Slamming pressure with elastic bottom effect

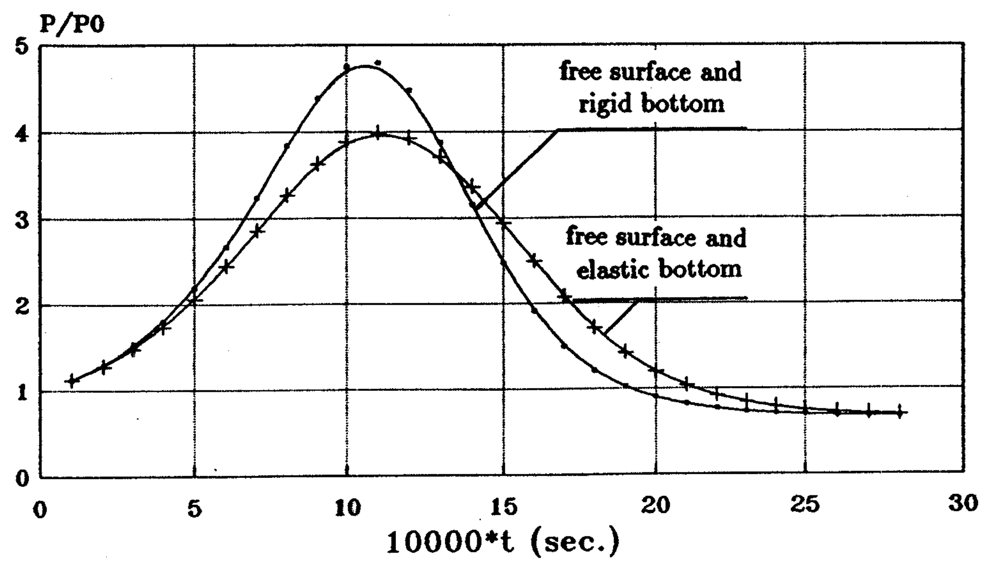

Fig. 5 Slamming pressure with free surface and elastic bottom effects

Tanizawa: What boundary condition do you use for your air flow calculation at the edge of the falling body? And what choking criteria do you assume? If you use a local speed of sound, your calculation is basically identical to the work of Verhagen. According to our experiments and similar computations, this criteria is not sufficient. At the edge, the formation of a mixed region of air and water may not be ignored.

Huang & Hsiung: For the subsonic flow $p = p_0$ is used; when the flow is supersonic, $u = c$ is used as the boundary condition. It is assumed that the air flow moves past the edges before the hull bottom touches the water, so the air-water mixed region is not modelled. Our work is not identical to the work by Verhagen. Verhagen did not consider the elastic effect of the hull bottom. And also, in our work an artificially smoothed pressure at the edges is used rather than the pressure determined by the local air flow.

Yeung: The M.S. Thesis of a former U.C. Berkeley student, Sam Ando, who is now at NRC of Canada was published in Journal of Ship Research on this subject. He had a treatment very similar to what is being carried out here. You may like to look into it as an additional reference.

Huang & Hsiung: Thank you for your information! We will pay attention to that paper.

Söding: I would have expected that for certain relations between the natural period of the bottom and the slam duration, the bottom deformations might increase the maximum pressures.

Huang & Hsiung: The slamming only lasts for a very short time interval, say 5 milliseconds in our computation. The dynamic effect of the elastic bottom may be neglected during such a short time interval.