NONLINEAR OSCILLATIONS, BIFURCATIONS AND CHAOS IN WAVE-STRUCTURE INTERACTION SYSTEMS

Oded Gottlieb
Ocean Engineering Program
Oregon State University

INTRODUCTION

Complex nonlinear and chaotic responses have been recently observed in various compliant ocean systems[1-3]. These systems are characterized by a nonlinear restoring force and a coupled wave induced exciting force. The restoring force includes material and geometric nonlinearities whereas the exciting force consists of a quadratic wave-structure drag component and a wave induced inertial force. Excitation of semi-submersibles and similar multi-component structures, is characterized by wave kinematics that cannot be evaluated at the structures center of gravity. Consequently, the exciting force is further complicated by an additional parametric excitation coupling both displacement and velocity of the structure. While weakly nonlinear systems have been studied extensively from both classical and modern approaches[4,5], complex single equilibrium point systems are limited in their scope of analysis. Examples of such systems are the hardening Duffing equation[6] and the motions of a wind loaded structure[7]. This paper describes the theoretical nonlinear analysis of a compliant ocean system and reveals the complex dynamics recently uncovered numerically.

SYSTEM MODEL AND GLOBAL ATTRACTION

In past analysis, complex mooring system restoring force was investigated with an equivalently linearized drag force[1] or by analysis of a wave excited linear system[2]. Another example is the analysis of a quintic polynomial derived for the restoring moment of a rolling ship where the quadratic damping moment was approximated by a mixed linear-cubic model[8]. In order to investigate the nonlinear coupling effect of the wave-structure interaction, the exact quadratic drag component (F_D) is retained and a symmetric multi-point mooring system with a integrable (Hamiltonian) restoring force (R) is chosen (Fig.1). The exciting force is derived from linear wave kinematics for deep water ($kh > \pi/10$) slender body motion in the vertical plane where convective acceleration terms contribute additional nonlinear parametric terms (F_I). Normalization of the equations of motion ($x = kX$, $\theta = \omega t$) results with the following autonomous first order system (surge).

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -R(x) - \gamma y + F_D(x,y,\theta) + F_I(x,y,\theta) \\ \dot{\theta} &= \omega \end{aligned} \qquad (1)$$

where $R(x) = \Sigma\alpha_n x^n$, $n = 1,3,..,N$; $F_D(x,y,\theta) = \mu\delta(u-y)|u-y|/\omega^2$; $F_I(x,y,\theta) = \mu[1-(u-y)/\omega]u'$ and $u(x,\theta) = \omega f\cos(x-\theta)$; $u'(x,\theta) = \omega^2 f\sin(x-\theta)$. Note that $\mu \geq 1$ (buoyancy), $f < 1$ ($ka < \pi/7$ limiting wave steepness), $\gamma < \delta < 1$ (structural damping and hydrodynamic viscous drag respectively).

STABILITY ANALYSIS AND THE POINCARE' MAP

Global stability of the system reveals that the structurally damped Hamiltonian system has a strong Lyapunov function resulting in an asymptotically stable hyperbolic fixed point (sink) at the origin. With the addition of wave excitation, the sink becomes a hyperbolic closed orbit (limit cycle) which loses the circularity of the sink but is anticipated by the invariant manifold theorem to retain its stable characteristics[5]. This result ensures that solution remain bounded for small excitation ($|F_D|, |F_I| << 1$). The stability analysis of the averaged system[9] results in the bifurcation set of the approximate Poincare' map of the ocean system. Consequently, the fixed points and closed orbits of the averaged equations determine the stability of the periodic solutions of the original system (Fig.2). Application of Melnikov's method[5] to the perturbed averaged subharmonic system provide a criterion for the existence of transverse homoclinic orbits resulting in chaotic dynamics. The criterion is sensitive to the high frequency of the averaged system [θ/ϵ where ϵ is a measure of smallness of the parameter set: α_n ($n>1$), γ, δ, f] and only estimates for the separatrix splitting of the rapidly forced system can be obtained[10].

EXISTENCE OF PERIOD DOUBLING

Perturbation of an approximate periodic solution [$x(t)=x_0(t)+\xi(t)$; $y(t)=y_0(t)+\eta(t)$], results in a complete variational equation. Analysis of the linearized variational equation leads to a general Hill's equation which exhibits the existence of a period doubled solution through a stability loss in the unstable regions of the subharmonic response[11].

$$\begin{aligned} \dot{\xi} &= \eta \\ \dot{\eta} &= H_1(\theta)\,\xi + H_2(\theta)\,\eta \end{aligned} \qquad (2)$$

where $H_{1,2}[(x_0(t), y_0(t)] = H_{1,2}[x_0(t+mT), y_0(t+mT)]$ and m is the order of subharmonic. Application of Floquet theory[12] yields two forms [$Z(t)=Z(t+mT)$, $Z(t)=Z(t+2mT)$] to the particular solution of the variational equation [$\xi=\exp(\nu t)Z(t)$]. The boundary of the unstable region can then be determined [$\Delta(\omega^2)<0$ for $\nu>0$] resulting with a criterion based on the intersection of the frequency response and the local stability curves. This criterion coincides with that obtained for the approximate Poincare' map for a weakly nonlinear parameter set (Fig.2).

BIFURCATIONS AND ROUTES TO CHAOS

Within this boundary, further analysis of the 1/2 subharmonic ($m=2$) shows the existence of a period 4T solution. Thus, the Hill's equation suggests the possible cascade of period doubling bifurcations. An infinite period doubling sequence with a finite accumulation point results in chaotic motion. The period doubling route to chaos is continuous and can be observed with the appearance of even harmonics. The evolution of a subharmonic solution in parameter space results with contraction of the mT limit cycle. The abrupt change in size can lead to a chaotic attractor which becomes transient before settling to a regular motion. This route occurs near the local subharmonic tangent bifurcation. Furthermore, the parametric excitation coupling the system velocity causes a competition between coexisting attractors which results in intermittency. Numerical integration of the system verify the existence of a period doubled cascade (Fig.3) and the evolution of an attractor via a tangent bifurcation (Fig.4). The results are portrayed with phase plane diagrams (x,y) and Poincare' maps (X_p,Y_p) where the mT subharmonic repeats after m intervals and the chaotic attractor does not, consequently generating a fractal map.

SUMMARY AND CONCLUSIONS

Stability of approximate low-order periodic solutions enables the analysis of the nonlinearities governing the complex response of a compliant ocean system. Local and global bifurcations determine the possible existence of complex nonlinear and chaotic motions which cannot be obtained through evaluation of an equivalently linearized system. Period doubling and loss of stability of transverse homoclinic and subharmonic response may in turn lead to global bifurcations and the onset of chaotic motion. Thus, stability analysis of approximate solutions of this nonlinear ocean system, subjected to combined parametric and external excitation, reveals the complex dynamics recently uncovered numerically. Routes to chaos are identified and associated with the nonlinear mechanisms generating the instabilities.

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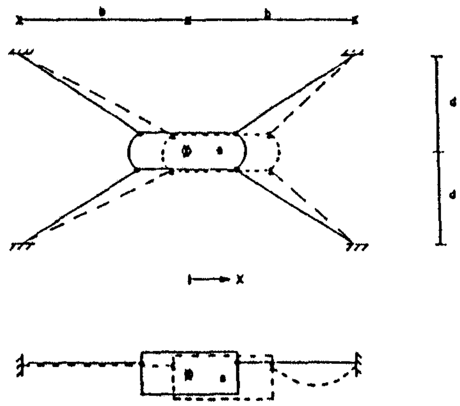


Fig.1 Mooring assembly

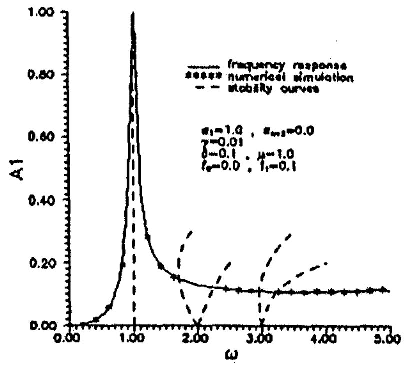
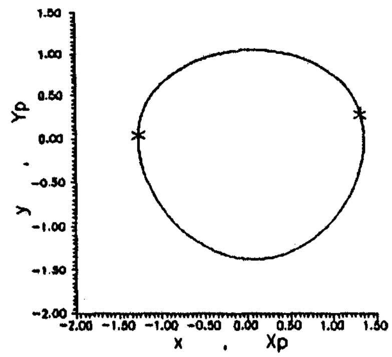


Fig.2 Stability diagram

(a)



(b)

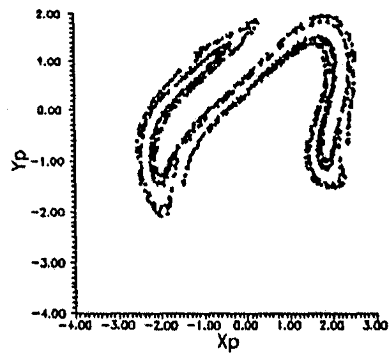
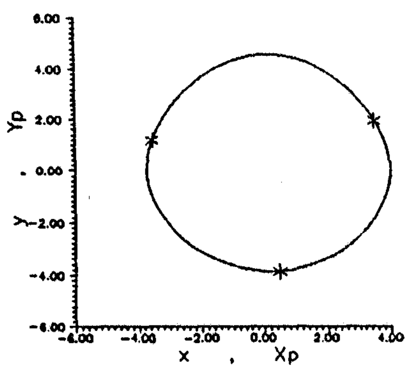


Fig.3 Evolution of an attractor via period doubling
[(a) 2 T (ω=1.9) ; (b) 2^{∞}T (ω=1.65)]

(a)



(b)

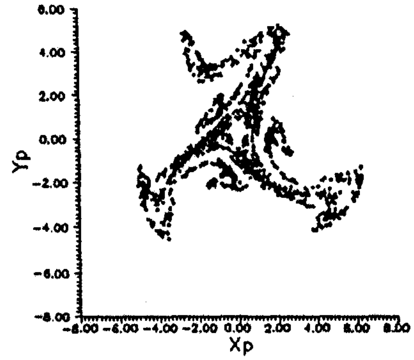


Fig.4 Evolution of an attractor via a tangent bifurcation
[(a) 3 T (ω=3.15), (b) Chaos (ω=3.1)]

Martin: Does one see chaotic motions experimentally of realistic moored ocean structures?

Gottlieb: A large number of nonlinear phenomena have been observed in various configurations of moored ships and ocean structures, but to date (to the author's knowledge) a chaotic ocean experiment has not yet been recorded. The experimental models have mainly revealed subharmonic motions (nT/m : $n=1$, $m>1$) [eq. Fujino & Sagara 1990 ($m=3,4$), Lean 1971 ($m=3$), Thompson et al 1984 ($m=2$)]. While the mathematical models (analytical, numerical) have revealed a variety of instabilities and sensitivity to initial conditions, the difficulties of generating and recording a chaotic fluid-structure interaction experiment in a large scale model are greater than controlling a desktop experiment (*eg* Chaotic Toys: Moon 1987) where the domains of attraction are easily defined and measured.

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