SLOW-DRIFT SIMULATIONS OF A MULTI-LEG STRUCTURE IN A SHORT CRESTED SEA STATE

by

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BACKGROUND

The prediction of the slow-drift responses of marine structures in a sea state, wind and current is of central interest to the design of Floating Production Systems (FPS) operating in large water depths. Several aspects of the enviromnent (wind, waves and current) and components of the FPS (structure, tethers, mooring lines, risers) must be considered for the reliable prediction of the slow-drift oscillations. They typically arise in the surge-sway-yaw directions but may also be significant in the remaining three vertical directions when the excitation from the environment is significant and the corresponding restoring forces small.

In the present study we consider the coupled surge-sway slow-drift oscillations of a structure consisting of four vertical circular cylinders located at the corners of a square. Closed form expressions are derived for the slow-drift excitation and wave-drift damping across the frequency axis and the wave-heading angle and simulations of the slow-drift oscillations are obtained in short crested seas.

The Surge-Sway Slow-Drift Equations of Motion

Assume that the resonance of the slow-drift oscillation occurs at a period large relative to the modal period of the wave spectrum. The slow-drift second-order responses may then be decoupled from the linear responses and shown to obey the coupled system of equations

$$[M+A]\ddot{\mathbf{X}}(t)+[B](t)\dot{\mathbf{X}}(t)+[C]\mathbf{X}(t)=\mathbf{F}(t) \tag{1}$$

where $\mathbf{X}=(X_1,X_2)$ denotes the surge-sway slow-drift vector displacement, $[M+A]$ is the inertia and zero-frequency added-mass matrix of the structure and $[C]$ is the restoring coefficient matrix corresponding to the tether or mooring system. The slow-drift excitation vector $\mathbf{F}(\mathbf{t})$ is here obtained in terms of the sway-surge mean drift forces on the four leg structure for narrow-banded wave spectra. The slow-drift damping matrix $[B](t)$ is time-dependent and is obtained from the solution of the low-speed hydrodynamic problem outlined in the next paragraph.

The Slow-Drift Damping Coefficients

For a small slow-drift velocity $U = |\dot{\mathbf{X}}|$ the forward-speed free-surface flow around the structure may be linearized about the $O(1)$ zero-speed problem. Its solution may be obtained with standard methods and will supply the drift force necessary for the evaluation of the slow-drift excitation force $\mathbf{F}(t)$. The $O(U)$ correction obeys the inhomogeneous free-surface problem discussed by Sclavounos (1989) and its solution will supply the slow-drift damping coefficient matrix $[B]$.

For a single vertical circular cylinder of infinite draft, closed form expressions have been obtained for the drift damping coefficient in the frequency domain. The corresponding result for the drift force follows from the McCamy and Fuchs theory. Employing the Linton and Evans (1990) theory, explicit expressions were also derived for the drift damping and drift force of arrays of vertical circular cylinders.

Figure 1 illustrates the diffraction drift damping on a rectangular array of cylinders with diameter $d = 26.5m$ and spacing $D = 80m$. It is interesting to note that unlike the drift force, the drift damping coefficient comes negative over certain frequency segments. This suggests that in a monochromatic wave train and for an appropriate selection of the frequency, ideal wave effects may amplify rather that damp the slow-drift oscillation. The response of the structure in a polychromatic train is discussed next.

Slow-Drift Simulations in a Short Crested Sea

The solution of the system (1) was carried out in a Gaussian short crested sea state with $\cos^2(\beta - \pi/6)$ angular spreading combined with a Pierson-Moskowitz frequency spectrum for a wind speed of 40 knots. The $\beta = 0$ direction coincides with one of the axes of symmetry of the platform and a zero wave energy density is assumed outside the β range $(-\pi/3, 2\pi/3)$.

Here, the slow-drift excitation $\mathbf{F}(t)$ and drift damping coefficient $[B](t)$ are stochastic processes evaluated in terms of their frequency-domain values illustrated in Figure 1, using the Newman approximation for narrow-banded wave spectra. In order to illustrate the nature of the surge-sway slow-drift response, 30 frequencies were used in the time-series approximation of the ambient wave elevation and 7 wave headings for the resolution of the angular wave spreading. The slow-drift resonant period of the structure in surge or sway was taken to be $T = 150$ seconds.

Figure 2 illustates the slow-drift trajectory of the structure on the $x - y$ plane, obtained from the solution of the system (1) by a Runge-Kutta scheme with built-in step control. The corresponding radial deflection $R = \sqrt{X_1^2 + X_2^2}$ is illustrated in Figure 3 over a time period of 30,000 seconds, or about 200 resonant periods.

Of significant interest in practice is the knowledge of the extreme statistics of the slow-drift response. They may be obtained from a theoretical study of the statistical properties of the system (1) of from the simulation illustarted in Figure 3, where the envelope of the slow-drift response may be easily detected.

The effects of viscous damping, wind and current as well as the extereme statistics will be the subject of a future investigation.

References

Linton, C.M. & Evans, D.V., 1990, The Interaction of Waves with Arrays of Vertical Circular Cylinders, *J. Fluid Mech.*

Sclavounos, P.D., 1989, The Slow-Drift Wave Damping of Floating Bodies, *Fourth International Workshop on Water Waves and Floating Bodies, Øystese, Norway.*

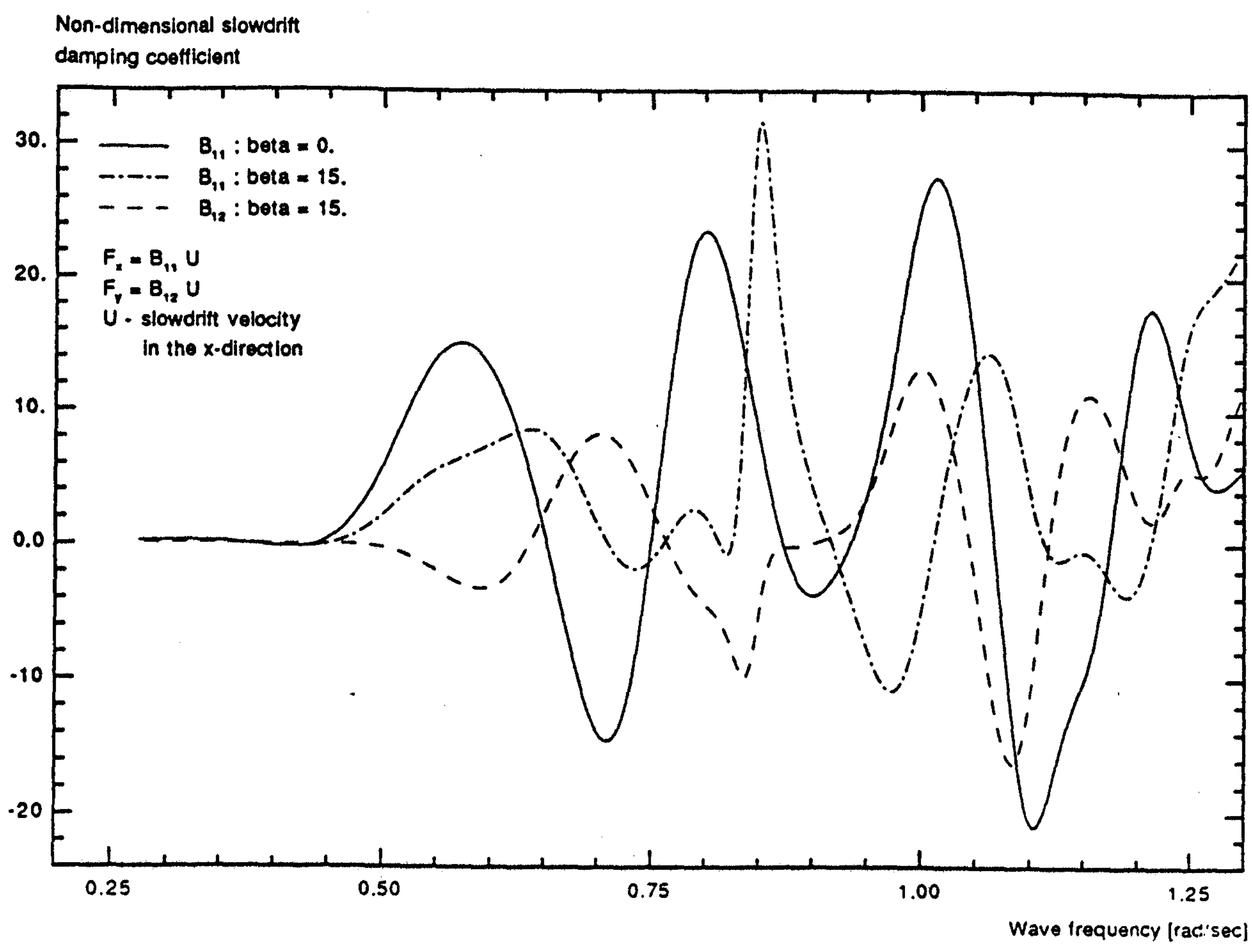

Figure 1 Slowdrift damping coefficients B_{11} and B_{21} for two different wave headings. The coefficients are made non dimensional by $\rho A^2 R\omega_0$, where ρ is the water density, A is the incident wave amplitude, R is the cylinder radius and ω_0 is the wave frequency (zero-speed).

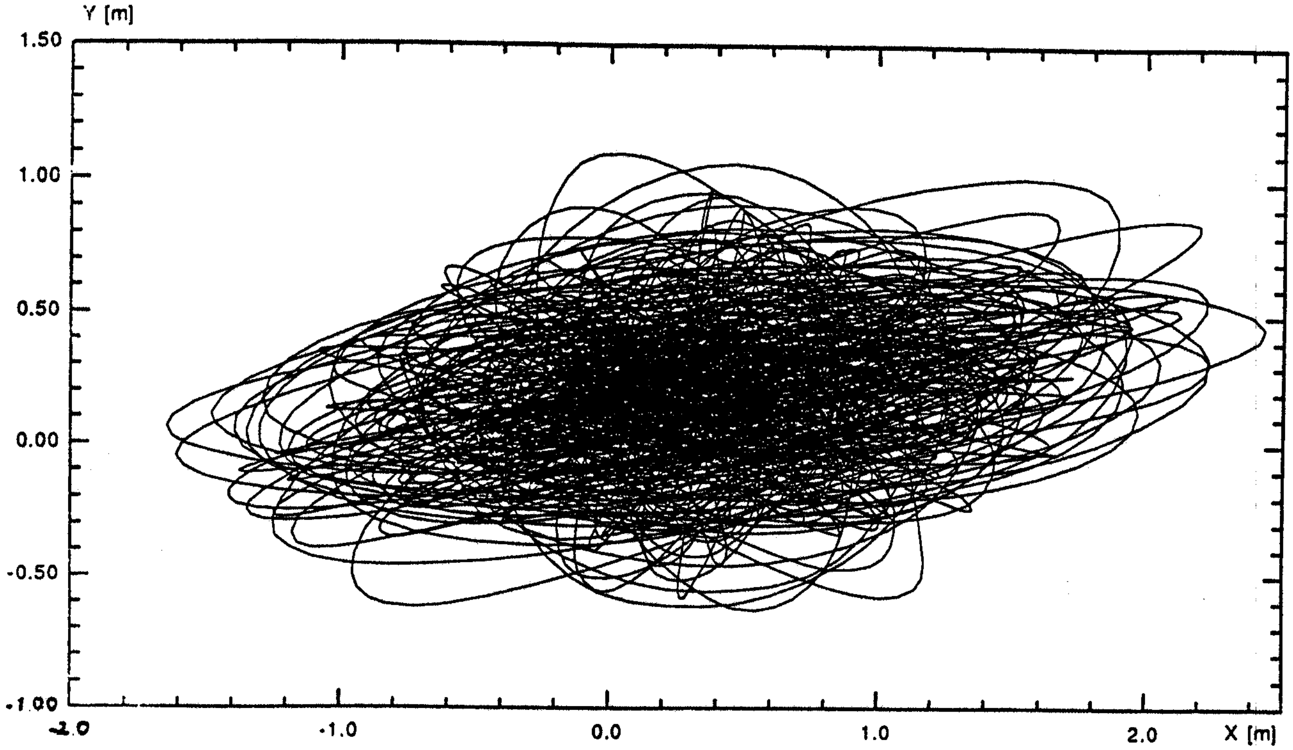

Figure 2 Trajectory of the simulated slow-drift motion in the $x - y$-plane.

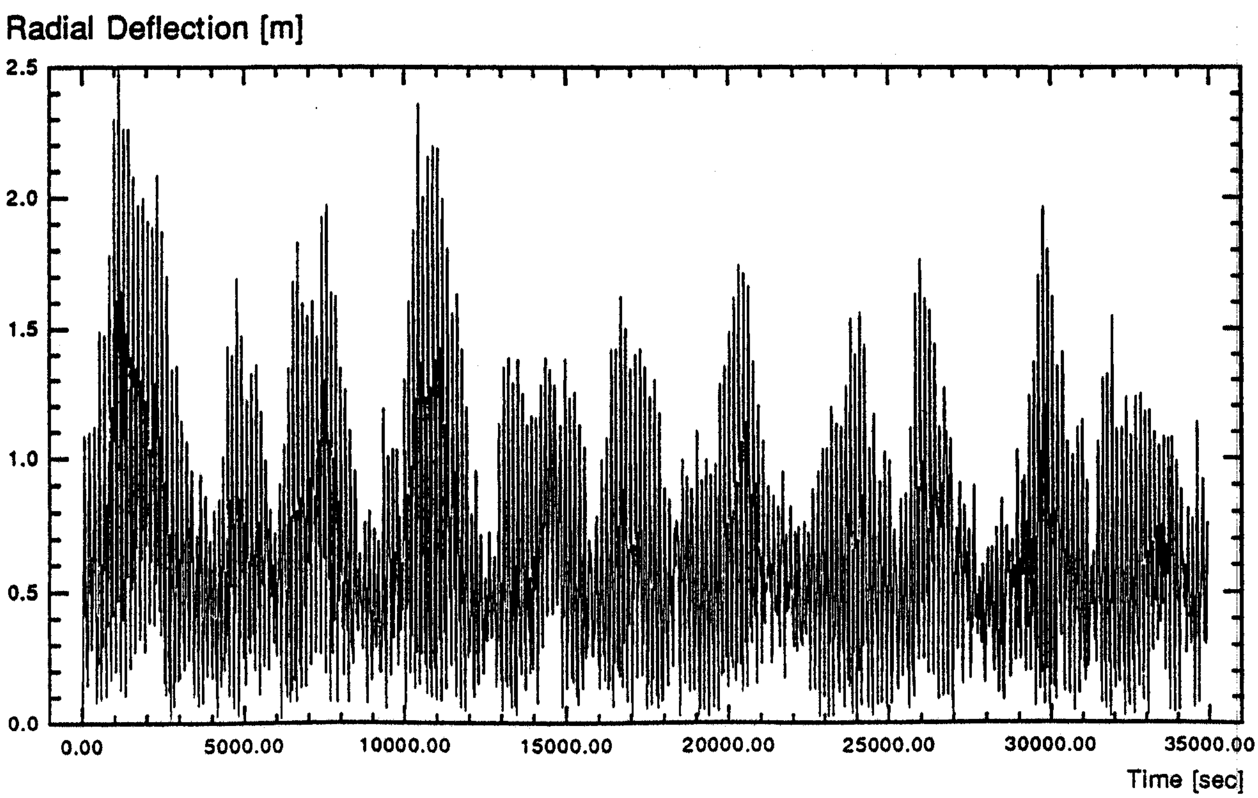

Figure 3 Radial deflection of the simulated slow-drift motion.

Kim: If Newman's approximation is used to simulate slowly-varying wave loads, the probability density functions in both uni- and multi-directional seas can be obtained in closed form (Kim & Yue, AOR (1989)). It is worth comparing your histograms with them. Even if the 2nd-order slowly-varying wave excitations are not Gaussian, the corresponding responses are very close to a Gaussian distribution (according to your results). Do you have a good explanation for this?

Emmerhoff & Sclavounos: We are aware of the closed form solutions for the slow-drift excitation PDF. We have elected instead to compute the histogram in order to allow the eventual treatment of nonlinearities in the slow-drift response. In response to your second question, it is indeed true that the slow-drift response, free of viscous or restoring nonlinearities appears to be nearly Gaussian, we attribute that to the small damping of the system but intend to investigate this property more carefully in the future.

Zhao: The results you presented for the wave drift force damping of a circular cylinder seem to be different from our numerical results (for $kA > 0.5 - 1.0$). Have you compared your results with our numerical results?

Emmerhoff & Sclavounos: In the mathematical formulation of the slow drift problem, we have considered the frequency of encounter $\omega = \omega_0 - kU \cos\beta$ to be a "perturbation" in the slow drift velocity U, and neglected all terms of $O(U^2)$ in the boundary conditions. From the documentation of your program, it seems like ω is a parameter and not a function of U. The difference is apparent for the range you mentioned, but as k becomes smaller and $\omega \to \omega_0$, the forces agree well.

Gottieb: 1. The surge-sway slow drift system excited by a monochromatic wave train numerically exhibited amplification of response. This behavior can be analyzed by application of Floquet theory (*eg* Nayfeh & Mook, 1979; Ioos & Jospeh, 1981) resulting in analytical bifurcation diagrams for this combined parametrically and externally excited system.

2. The system excited by both monochromatic and polychromatic wave trains can further analyzed by nonlinear deterministic and stochastic mapping techniques [*eg* Wiggins, 1990 (deterministic/chaotic systems), Kapitaniak, 1988 (random/chaotic systems)].

Ioos, G. & Joseph, D.D., 1981. Elementry Stability and Bifurcation Theory. Springer Verlag.

Kapitaniak, T., 1988. Chaos in Systems with Noise. World Scientific.

Nahfeh, A.H. & Mook, D.T., 1979. Nonlinear Oscillations. Wiley.

Wiggins, S., 1990. Introduction to Applied Nonlinear Dynamical Systems and Chaos. Springer-Verlag.

Sebastian: Have you investigated the possibility of extending the Kac-Siegert method to your slow-drift model in order to handle the extreme statistics?

Emmerhoff & Sclavounos: No, but we want to consider this method.