

Water Wave Impact:  
Computations, Theory and a Comparison with Measurements.

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1) Introduction

This work concerns the enormous and short-lived pressures exerted by a breaking wave when it meets a solid surface. The pressures may endure for only a millisecond but are usually 10 or 100 times greater than the hydrostatic pressure component. We have investigated the simplest problem: wave impact on a fixed vertical wall, though the work may give insight into wave-body impact, such as the slamming of ship hulls in heavy seas.

At the last Workshop we showed preliminary results of boundary-integral computations of irrotational waves meeting a vertical wall (Peregrine and Cooker, 1990). We found that if the wave face has a vertical tangent when it reaches the wall then the fluid, in the neighbourhood of the waterline, can achieve *accelerations exceeding 1000g*. There are *correspondingly large vertical pressure gradients and pressures*, just below the waterline. These high transient accelerations appear to be the key to understanding the "shock" pressures measured by experimenters since the pioneering work of Bagnold (1939).

Here we make a comparison between the computations and recent laboratory measurements by Arami and Hattori (1989) (abbreviated to A&H) who examined waves about 10cm high. For those examples where the fluid motion is too violent to compute accurately, we supplement the numerical results with pressure impulse theory, a model first presented at the last Workshop (see Cooker and Peregrine, 1990). The pressure impulse is defined by

$$P(\underline{x}) = \int_{t_b}^{t_a} p(\underline{x}, t) dt, \quad (1)$$

where  $p$  is the pressure;  $t_b$ ,  $t_a$  are the times just before and after impact. For an incompressible fluid,  $P$  is harmonic and  $\nabla P = \rho(\underline{u}_b - \underline{u}_a)$ , where  $\underline{u}_b$  and  $\underline{u}_a$  are the fluid velocity fields just before and after impact. We assume that  $p$  is a triangular hat function in time between the instants  $t_b$  and  $t_a$ , and we define the impact time  $\Delta t = t_a - t_b$ , so that from (1) the peak pressure is given by

$$p_{pk}(\underline{x}) \approx 2P(\underline{x})/\Delta t. \quad (2)$$

## 2) Results

Figure 1 shows boundary-integral computations of a wave of height 3.21h moving against a vertical wall on a still water depth, h. (Units are chosen so that  $h = g = \rho = 1$ .) The wave height matches that recorded on video by A&H. The waterline accelerates vertically with a speed  $v_y = 20 (\sqrt{gh})$  and an acceleration of 4000 (g). Large pressures accompany this acceleration as can be seen from figure 2. In this example the numerical method could not resolve the violent surface motion, and the computations halted. At the last time computed the maximum pressure is 44.5 ( $\rho gh$ ) and still rising.

We now use the computed horizontal speed, wave height, and waterline position as "initial" conditions (at time  $t_b$ ) for pressure impulse theory. Figure 3a shows how  $\Delta t$  is equated with the time it would take the waterline to rise to the same level as the top of the wave (with initial speed  $v_y$  and under constant vertical acceleration  $a_y$ ). Figure 3b is the idealized boundary-value problem for the pressure impulse and its Fourier series solution is given in the caption. The maximum peak pressures found from three calculations and three measurements of A&H, are shown in Table 1. The spread of values is similar and shows the sensitivity of results to initial conditions (in this case the initial wall-wave distance and initial wave slope).

TABLE 1. Impact time  $\Delta t$  and peak pressure  $p_{pk}$  compared. H is the total height of water at the wall,  $U_o$  is impact speed and  $\mu$  is the fraction of wall struck.

.....Computations.....						----Experiments (A&H)--	
Case	H(m)	$U_o$ (m/s)	$\mu$	$\Delta t$ (ms)	$p_{pk}$ (kPa)	$\Delta t$ (ms)	$p_{pk}$ (kPa)
IV	0.126	1.52	0.300	1.344	48.58	1.5	36.7
XI	0.126	1.39	0.169	1.227	28.62	3.2	27.2
XIV	0.126	1.52	0.181	1.329	28.9	3.0	14.5

Further comparisons between theory, computations and experiments will be presented. A full preliminary report is in Passoni, Cooker, Peregrine (1990).

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### References

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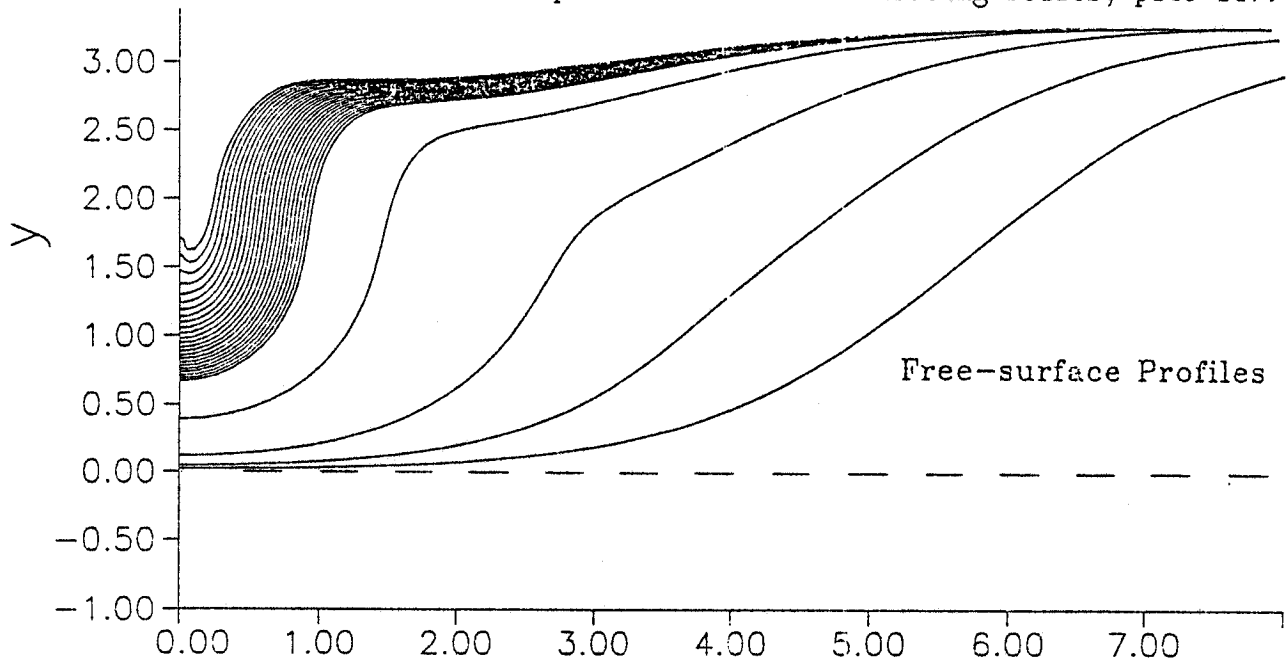


Figure 1): Computed free-surface profiles for a wave of height  $3.21h$  meeting a vertical wall at  $x = 0$ . Times 0 [0.5] 1.5, 1.9 [0.01] 1.94 . SWL is at  $y = 0$ , with uniform depth,  $h = 1$ . Waterline acceleration is  $4017g$  and vertical velocity is  $20\sqrt{gh}$  at the last computed time, (still increasing). At  $t = 0$  the wave has approximately a tanh profile centred at  $x = 5.4$  with a uniform flow to the left at  $x = +\infty$ . In the absence of the wall the wave overturns.

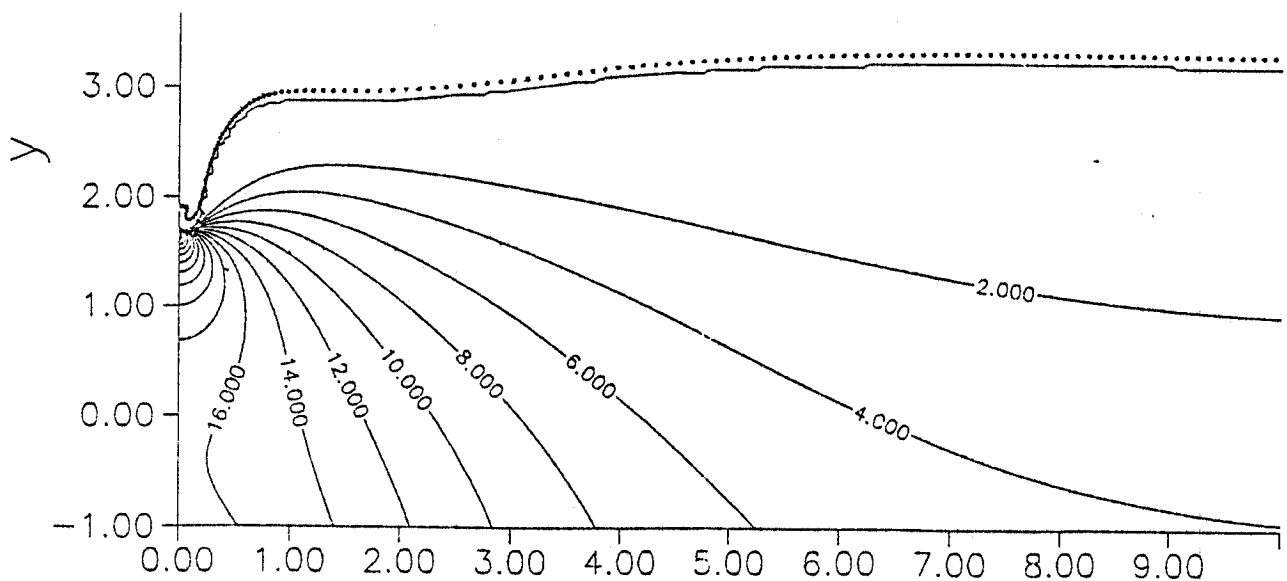


Figure 2): Instantaneous pressure contours at  $t = 1.94$  for the wave of figure 1. The maximum pressure is  $44.5 \rho gh$ , located just below the waterline.

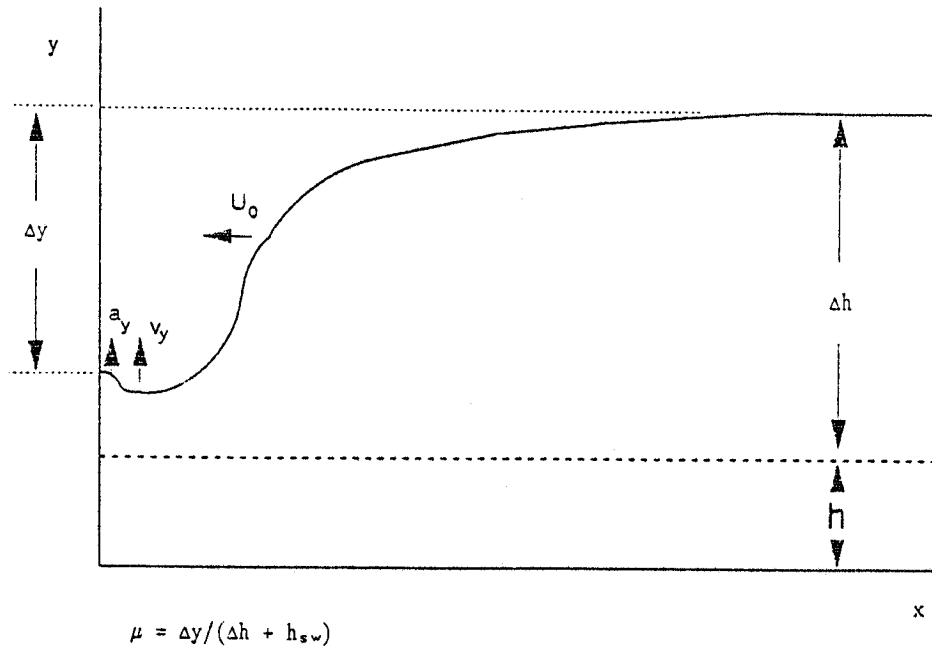


Figure 3a): Sketch of numerical solution at last time computed. These are the "before impact" conditions ( $t = t_b$ ) for pressure impulse theory. Impact time  $\Delta t$  = time it would take the waterline to rise to the level of the wave height ( $3.21h$ ) with the initial speed and constant acceleration given by the numerical solution.  $U_0$  is the horizontal impact speed,  $\mu$  the fraction of wall struck.

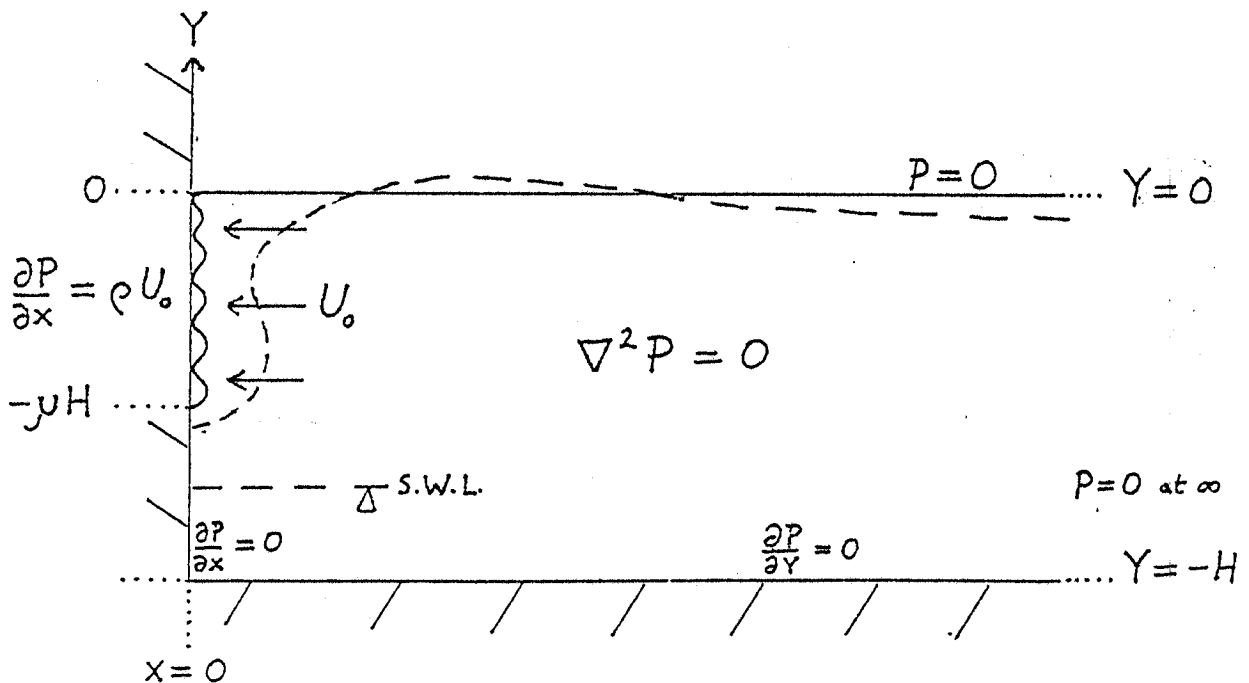


Figure 3b): Boundary-value problem for pressure impulse,  $P$ , corresponding to figure 3a.  $P(x, Y) = 2\rho U_0 H \sum_{n=1, \infty} \lambda_n^{-2} (\cos \mu \lambda_n - 1) \sin (\lambda_n Y / H) \exp (-\lambda_n x / H)$ , where  $\lambda_n = (n - \frac{1}{2})\pi$ , and  $x \geq 0$ ,  $-H < Y < 0$ .

**Yue:** I have a comment regarding the experimental result from Chan & Melville (1988) that you showed. They actually reported oscillations in the pressure signal which they were able to correlate with the reverberation time of the air pocket volume they estimated; thus providing evidence for the importance of trapped air. (They had intended to, and were successful in, obtaining plunging breaking waves with a fairly distinct overhang of the wave jet.)

**Cooker:** Pressure oscillations may well be due to the presence of the pressure of an air pocket trapped against the wall, or due to the vibration of bubbles formed by the air pocket once the pocket has broken up. I think that an air pocket will reduce the peak pressure. Recent work by Arami & Hattori (1989) reports experiments in which the highest peak pressures were brought about the least air entrapment against the wall. The message from this paper is that "impact" pressures can be generated without air entrapment and without even any direct collision between the free surface and the wall.

**Grue:** The question regards the integrated pressure on the wall. How much larger is the integrated impact pressure than the corresponding result obtained by linearized theory?

**Cooker:** Using our boundary integral method we have computed horizontal forces on the wall which exceed  $20\rho gh^2$ , where  $h$  is the still water depth. This compares with a typical linear wave theory force of  $\frac{1}{2}\rho gh^2$ . So impact forces are many times greater than hydrostatic pressure loads. Pressure impulse theory (for an appropriate choice of impact duration) also gives a horizontal thrust on the wall which is many times greater than hydrostatic.