Two-Dimensional Solitary Waves Generated by a Moving Disturbance

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This study compares shallow water waves generated by a distribution moving at transcritical speeds as predicted by a KdV equation and a fully two-dimensional algorithm.

A disturbance moving steadily with a transcritical velocity in shallow water with a flat bottom can generate a succession of solitary waves, advancing upstream of the disturbance. A train of weakly nonlinear and weakly dispersive waves develops downstream of a region of depressed water surface trailing just behind the disturbance. This phenomenon was first found numerically using a generalized Boussinesq model by Wu & Wu (1982) for two-dimensional long waves generated by a moving surface pressure or bottom topography distribution. This phenomenon was later experimentally confirmed by Ertekin, Webster & Wehausen (1986) and Lee, Yates & Wu (1989). A forced KdV equation (fKdV) with the additional assumption that the velocity of the disturbance is near the critical velocity can also be used to model the phenomenon (e.g. Akylas, 1984; Cole, 1985; Wu, 1987; and Lee, Yates & Wu, 1989). Dommermuth & Yue (1987) derived a high-order equation for this problem. Some attempts have been made to extend the two-dimensional formulations to three-dimensional waves with sidewalls (see, for example, Mei, 1986; Katsis & Akylas, 1987; Ertekin *et al*, 1986; and Pederson, 1988).

Choi *et al* (1990) used a finite element method based on Luke's variational principle for the problem of a ship moving in a channel. Grilli & Svendsen (1989) used the mixed Eulerian-Lagrangian time stepping formulation with a boundary integral technique for the runup and reflection of a two-dimensional solitary wave in a numerical tank. In their method, Grilli & Svendsen solved the Laplace equation numerically with the fully nonlinear free surface boundary conditions.

Our study of the two-dimensional shallow water waves uses two methods. First, the fKdV is solved by a finite difference scheme with a predictor-corrector algorithm to advance time (Wu, 1987 and Lee *et al*, 1989). Our fKdV calculations agree with those in Wu (1987) to graphical accuracy. In the second method, we use the same Eulerian-Lagrangian formulation as used by Grilli & Svendsen, but with a desingularized boundary integral technique (see Cao, Schultz & Beck, 1990). A fourth-order Runge-Kutta-Fehlberg algorithm is used to advance time. We use the same moving free surface pressure distribution as used in Wu (1987):

$$p(x,t) = \begin{cases} \frac{1}{2}P_m \left\{1 + cos[\frac{2\pi}{L}(x + F_n t)]\right\} & \text{for} \quad -\frac{L}{2} < (x + F_n t) < \frac{L}{2} \\ 0 & \text{elsewhere.} \end{cases}$$

where F_n is the depth Froude number and L the pressure extension. The variables are nondimensionalized by the water depth, the gravitational acceleration, and the fluid density. The motion is assumed to start impulsively from the rest with the initial free surface displacement $\eta(x,0) = -p(x,0)$.

As in our previous calculations for three-dimensional waves (Cao *et al*, 1990) and in the fKdV calculations (Lee *et al*, 1989), a computational window moving with the disturbance is used. The time step Δt and the spatial mesh size Δx are chosen such that $J = \frac{\Delta x}{F_n \Delta t}$ is an integer. The window is shifted every J time steps so the position of the disturbance in the window will not change. Each time the window is shifted, an old node point is dropped at the downstream side of the window and a new point emerges into the window from upstream. Because of the finite computational domain, a treatment for the open boundaries is necessary to reduce non-physical reflections from the boundaries. Wu & Wu (1982), Wu (1987) and Lee *et al* (1989) used $\eta_t = \pm\eta_x$ (appropriate for linear waves with unit phase speeds) at the upstream and downstream boundaries respectively. In our method, there is no special treatment for the downstream boundary since the last node is dropped—we have found minimal effect of various downstream window locations. For the upstream boundary, the position and potential of the incoming point must be specified. In our previous computation for three-dimensional waves (Cao *et al*, 1990), the incoming points were assumed undisturbed and we had no difficulties with upstream reflections. In this two-dimensional problem, however, we have found that we need a more refined technique. We use extra upstream nodes outside the computational window which are convected by the computed velocity field without contributing to the boundary integral equation. Of course, if the waves are generated by spatially periodic disturbances, periodic boundary conditions can be applied without any difficulty—we leave this for further study.

Comparisons of some preliminary results for the depth Froude number $F_n = 1$ are shown in Fig. 1-3 with $L = 2.0$. Fig. 1 compares the results for a weak forcing $P_m = 0.02$ (one tenth of that used in Wu, 1987). The agreement between the methods is good although the transient trailing waves from the fKdV equation depart from the disturbance faster downstream and our method predicts steeper waves. For stronger forcing $P_m = 0.1$, both methods predict upstream runaway solitons as shown in Fig. 2. Again, our method predicts steeper and faster solitons. This becomes more obvious when the forcing becomes stronger. As shown in Fig. 3 for $P_m = 0.15$, the two-dimensional computation predicts a steeper front wave that breaks at time $t = 24.2$. For $P_m = 0.2$, which is used in Wu (1987), the program stops sooner. While not shown in the figures, we also found the the runaway solitons for Froude numbers near 1 (e.g. 0.85 and 1.1 with $P_m = 0.1$) using both methods. For the supercritical velocity $F_n = 1.5$ with $P_m = 0.1$, shown in Fig. 4 in the preprint of the abstract for this workshop, both methods reveal the same flow characteristics: the downstream transient waves travel at slower speeds than the disturbance and eventually one single soliton travels with the forcing. The height of this soliton is smaller than those generated by the disturbances near the critical velocity as expected, since the flow is not in resonance.

In the fKdV model, the waves generated by a pressure disturbance are the same as the waves by a topography disturbance if the two disturbances have an same nondimensional distribution. This is due to the fact that the fKdV model assumes small ϵ and uses the vertical average of the flow velocity over the water layer, thus ignoring the two-dimensional effect of the vertical variation to a certain extent. We examine the differences in the waves generated by the free surface pressure and the bottom topography of identical distribution using the fully nonlinear model. Fig. 4 shows the waves generated by a free surface pressure and a bottom topography with a semi-elliptical distribution. The extension L and maximum P_m of the elliptical distribution are 2.0 and 0.1 respectively. The differences in the waves by the two different disturbances are very significant. The free surface pressure behaves as a stronger forcing than the bottom topography. The waves by the pressure starts to break at about $t = 55$. The breaking occurs at the first wave peak downstream side of the disturbance.

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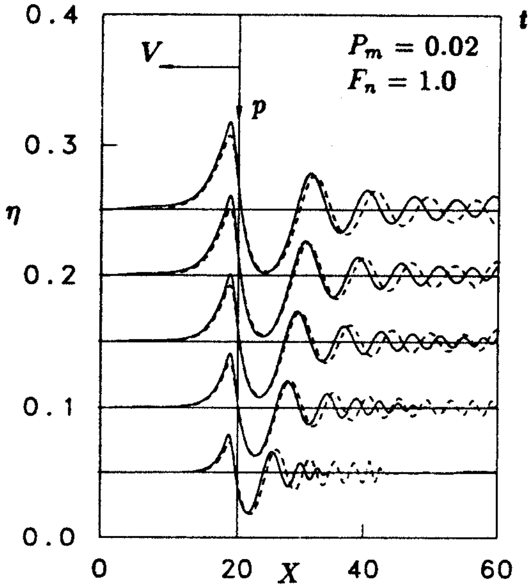

Fig. 1

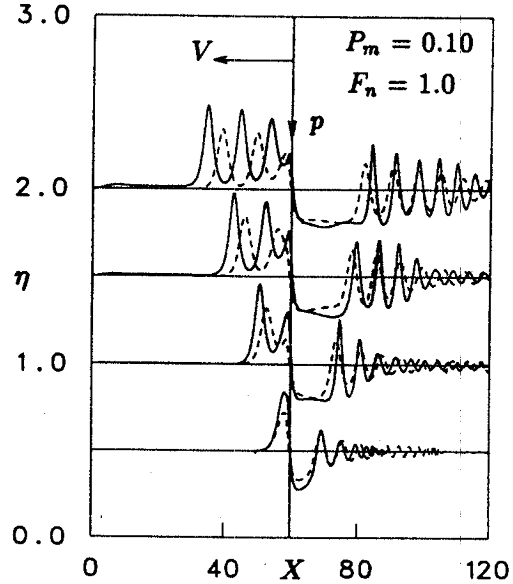

Fig. 2

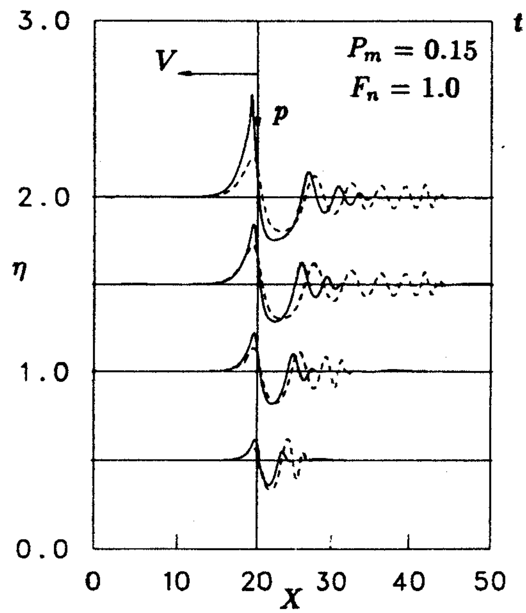

Fig. 3

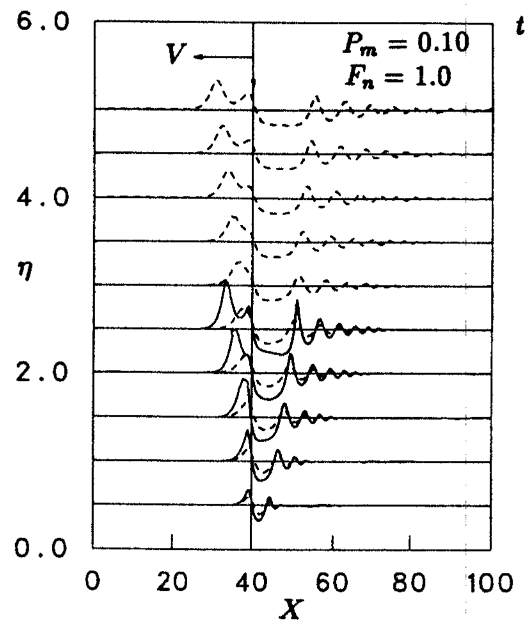

Fig. 4

Figures 1 to 3: Computed waves generated by a moving free surface distribution. (————: fully nonlinear calculations); (- - - - - - - - - - : fKdV calculations). The vertical lines indicate the centers of the pressure distributions moving in the $-X$ direction. The left vertical axes indicate the wave elevations η with a time offset given by the right vertical axes. $X = x + F_n$.

Fig. 4: Comparison of waves by free surface pressure and bottom topography. (————: bottom topography); (- - - - - - - - - - : free surface pressure).

Clement: Is there a major advantage to solving the boundary integral problem with a desingularized method instead of a standard boundary element method?

Cao: There are several advantages of the desingularized method over the standard method: 1) More accurate solutions may be obtained by a desingularized boundary integral method for a given truncation.
2) The kernels are nonsingular, so special care is not required to integrate the singular contribution. Simple numerical quadrature greatly reduces the computational effort by avoiding transcendental functions.
3) There is more flexibility since higher-order fundamental solutions can be more easily incorporated.
The indirect desingularized boundary integral method has two more advantages when compared to the direct one: 1) Integrals can be replaced by a summation if the desingularization distance is sufficiently large. This makes the computation even simpler. 2) The indirect method may result in smaller errors due to truncation of an infinite boundary, since the method requires no integration over the problem boundary.

van Daalen: (1) Can you explain why you didn't run into problems in 3-D computation with the open boundary conditions upstream (incoming points undisturbed), whereas you did encounter difficulties in 2-D computation? (2) Can you elaborate somewhat on the open boundary condition upstream? To be more specific, how does the convection mechanism work in this algorithm?

Cao: (1) In our 3-D computation, there were no waves traveling upstream since the computations were for deep water and a non-oscillating disturbance. The fluid upstream (far enough from the disturbance) is undisturbed. In the 2-D problem, because of the shallow water, there are solitons running upstream from the disturbance moving near the critical speed. The fluid upstream will eventually be disturbed. One would expect difficulties if the upstream incoming points are homogeneous. (2) In our computation, a fixed coordinate system is used, so the equations do not contain the explicit convection terms. The convection mechanism of the flow is implemented by using the moving computational window. Details are given in the paper.

Yue: You have emphasized the possible effect of nonlinearity in the comparison between MEL/BIEL vs. fKdV results. I would just like to point out that for the range of disturbance length parameter, L, you considered, two-dimensional effects are important and may play a dominant role.

Cao: We agree that the differences are primarily due to two-dimensional effects. This is also shown in the comparison of the waves produced by the free surface pressure and the bottom topography of the identical distributions.

Tulin: Comment to Cao, Schultz & Beck: Your list of previous authors should include Susan Cole. She has discovered the phenomenon of forward shooting solitons formations near the critical speed independently of other workers and prior to 1982.

Cao: We would like to thank the discussers for their interests in our paper. We would also like to thank Professor Tulin for directing our attention to S.L. Cole's work on this subject.