Nonlinear propagation of bichromatic wave trains

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Introduction

Numerical techniques are now available to predict the low frequency motion of floating bodies (e.g., semisubmersible platforms or submarines close to the free surface). In order to compare the results of such computations to those of experiments, it is essential to first determine the wave kinematics in the laboratory tank, for both the fundamental components and the subharmonics. This is a very difficult task for irregular seas and, in order to get some insight into the problem, experiments were performed at the Bassin d'Essais des Carènes to study the propagation of bichromatic wave trains.

In order to analyze the low frequency components of the wave field, it was decided to use not only free surface elevation measurements but also pressure measurements at the bottom of the tank. A procedure was defined which permits the separation of "set-down" (the component locked to the fundamental components), spurious waves generated at the wavemaker, and long waves reflected at the beach end. The results of these pressure measurements and the description of the signal processing algorithms can be found in Boudet and Cointe (1991).

During the experiments, strong nonlinear phenomena were observed in the propagation of the bichromatic wave trains. These phenomena are not predicted by second-order theories and can be important, not only for the interpretation of laboratory experiments but also in connection with the models used to characterize an irregular sea state.

In this paper, we briefly describe the phenomena that were observed both experimentally and numerically (using a fully nonlinear simulation of the flow). We also compare these results to a simplified model based on the Schrödinger equation for the wave envelope.

Experimental results

The experiments were performed in the towing tank n° 2 of the Bassin d'Essais des Carènes. This facility is 150m long, 8m wide, and (for the present experiments) 1.8m deep. It is equipped at one end with a flap-type wavemaker and at the other end with a standard beach to avoid reflection.

The experiments were performed using 5 capacitive wave probes and 5 differential pressure transducers. The wave probes were disposed at the following distances from the wavemaker: 12m, 20.63m, 22.33m, 24.08m, 27.1m. A preliminary test campaign was performed with the closest wave probe at 42m from the wavemaker and a much smaller spacing between the probes. During this campaign, very strong (and somehow unexpected) nonlinear deformations were observed. The new configuration was chosen to be able to measure the initial phase of the development of the nonlinear deformations.

The wavemaker was given a bichromatic law of motion (angular frequencies ω_1 and ω_2). Several test conditions were chosen and, for each of them, at least three different steepnesses were used (with $ak \leq 0.1$ for each component). We show on figure 1 the free surface profile measured at 42m during the preliminary test campaign, corresponding to a case with $\omega_1 = 3.93$rad/s and $\omega_2 = 4.25$rad/s. The dissymmetry of the wave envelope is obvious. It increased with the wave steepness. Actually, in this case, the first wave of each wave group is already breaking at the location of the wave probe. A Fourier analysis of the wave elevation shows the apparition of two peaks at the frequencies $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$.

The amplitude of these parasitic peaks grows with the distance from the wavemaker, as shown in figure 2 where the amplitude spectra are shown at 20.6m and 27.1m from the wavemaker with $a_1k_1 = 0.077$ and $a_2k_2 = 0.091$[1], $\omega_1 = 3.93$rad/s and $\omega_2 = 4.13$rad/s.

Numerical simulations

In order to confirm these experimental results and to have access to more detailled informations, fully nonlinear simulations of the flow were performed using the Sindbad code[2]. This code assumes the flow to be potential and uses the Mixed Eulerian-Lagrangian method to approximate the solution. The tank was equipped with a piston-type wavemaker at one end and an absorbing zone at the other end. For the results presented here, the wavemaker was given a bichromatic law of motion ($\omega_1 = 3.93$rad/s and $\omega_2 = 4.13$rad/s, $a_1k_1 = 0.078$ and $a_2k_2 = 0.081$). The simulations were performed for two different tank lengths, 66m and 100m. As an example, for the 100m long tank, 600 nodes were distributed along the free surface, for a total number of 650. The simulations were performed over 150s (3000 time steps).

We show in figure 3 the amplitude spectra resulting from the simulation (with the 66m long tank) at 12m and 30m from the wavemaker. The trends observed during the experiments are well reproduced by the numerical simulation. Such a simulation has the interest of yielding access to much more information about the flow characteristics than the experiments. In particular, we have a direct access to the wave elevations at any point along the tank (vs. only five for the experiments).

Approximate model

The free surface elevation is written in terms of the complex envelope as:

$$\eta = Re\{A(x,t)e^{j(kx-\omega t)}\} \tag{1}$$

where ω is a mean frequency, $j^2 = -1$, and k is the wavenumber related to ω by the dispersion relation.

The following Schrödinger equation is written to describe the evolution of the envelope:

$$j\left[A_t + \frac{\partial\omega}{\partial k}A_x\right] + \frac{1}{2}\frac{\partial^2\omega}{\partial k^2}A_{xx} + \beta AA^*A = 0 \tag{2}$$

where A^* is the complex conjugate of A and β is given by:

$$\beta = \frac{-\omega k^2}{16\sinh^4 kH}(\cosh 4kH + 8 - 2\tanh^2 kH) + \frac{\omega}{2\sinh^2 2kH}\frac{(2\omega\cosh^2 kH + k\frac{\partial\omega}{\partial k})^2}{gk - (\frac{\partial\omega}{\partial k})^2} \tag{3}$$

In the special case of a bichromatic wave train, this equation yields two equations for the amplitudes of the components at frequency $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$ when it is assumed that the fundamental components do not evolve during the propagation. A solution to these equations can readily be found by performing a series expansion of the unknowns.

We show in figure 4 the comparison between this approximate model and the numerical simulation (for both the 66m long and the 100m long tanks) using Sindbad[3]. For the simulation in the 66m long tank, the results are in good agreement.

In the other case, the initial growth is accurately predicted, but not the long-term evolution. In fact, the analysis of low frequency second order components reveals that the absorbing zone

[1]These values are measured at 12m from the wavemaker.

[2]This code was developped at the IFP with a partial support from DGA/DRET.

[3]The comparison with the experiments can only be performed over a much shorter length due to the small number of wave probes.

used in the Sindbad code is not very efficient for long waves. In this case, the amplitudes of free long waves (generated at the wavemaker and reflected at the beach end) are much larger than the amplitude of the second order locked wave. Consequently, the spurious components also interact with the fundamental components and modify the wave kinematics in the tank. This phenomenon is likely to explain why the results for the 100m long tank are not in good agreement with those of the approximate model.

Conclusion

Strong nonlinear deformations of bichromatic wave trains have been exhibited both experimentally and numerically. Unlike the deformations of regular waves that only appear in deep water after a rather long distance of propagation, these deformations appears at a very short distance from the wavemaker (they are already noticeable at one group length). The initial growth of the parasitic components was recovered using a simplified model based on the Schrödinger equation for the wave envelope.

These phenomena have several consequences. For experiments concerning low frequency motions without forward speed (e.g., semisubmersible platforms), it seems judicious to perform the tests relatively close from the wavemaker. In the forward speed case (e.g. submarines), the non-homogeneity of the wave field should probably be considered. The consequences for the characterization of an irregular sea state might also have to be investigated since it is very doubtfull that the phase independance hypothesis can be made (at least in laboratory conditions).

Finally, for fully nonlinear simulations in numerical wave tanks, damping zones should be able to damp out long waves (the tank natural modes) as well as the short waves. We are trying to improve the efficiency of the absorbing zone used in the Sindbad code.

Acknowledgements

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References

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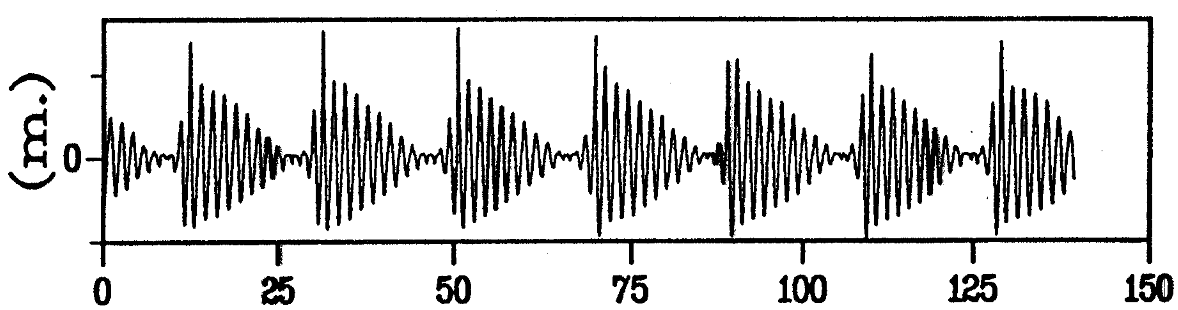

Figure 1

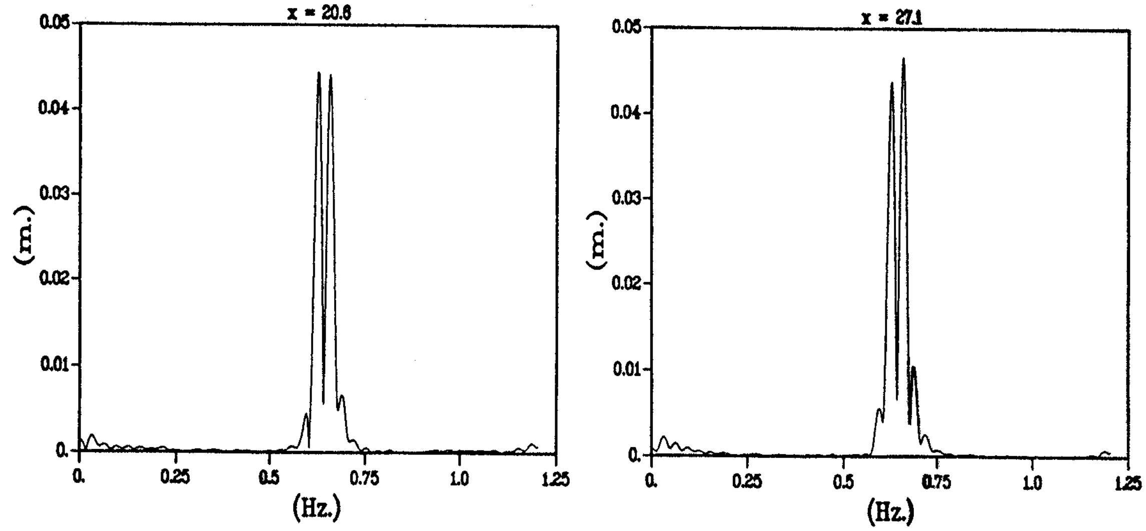

Figure 2

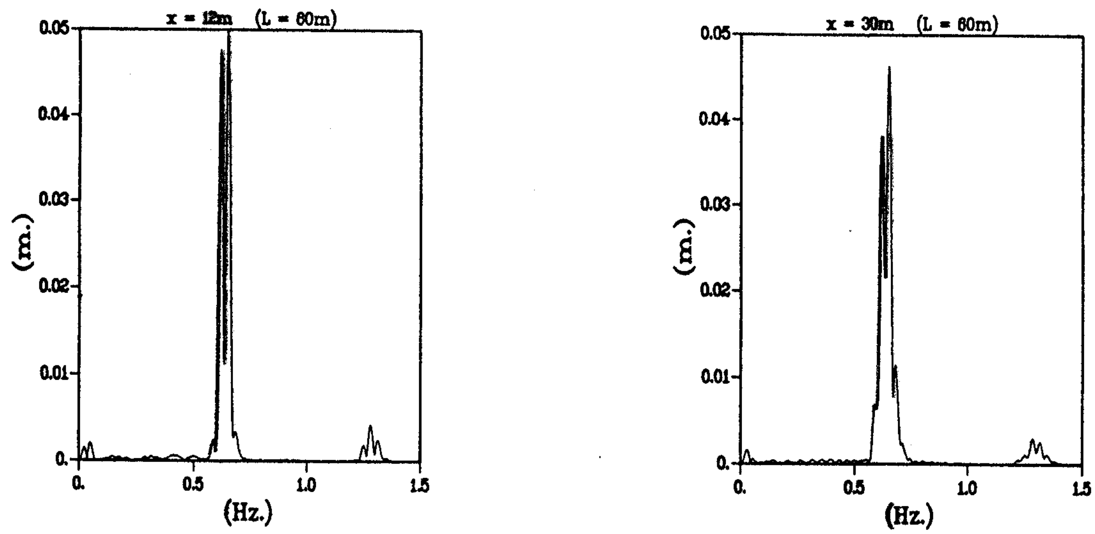

Figure 3

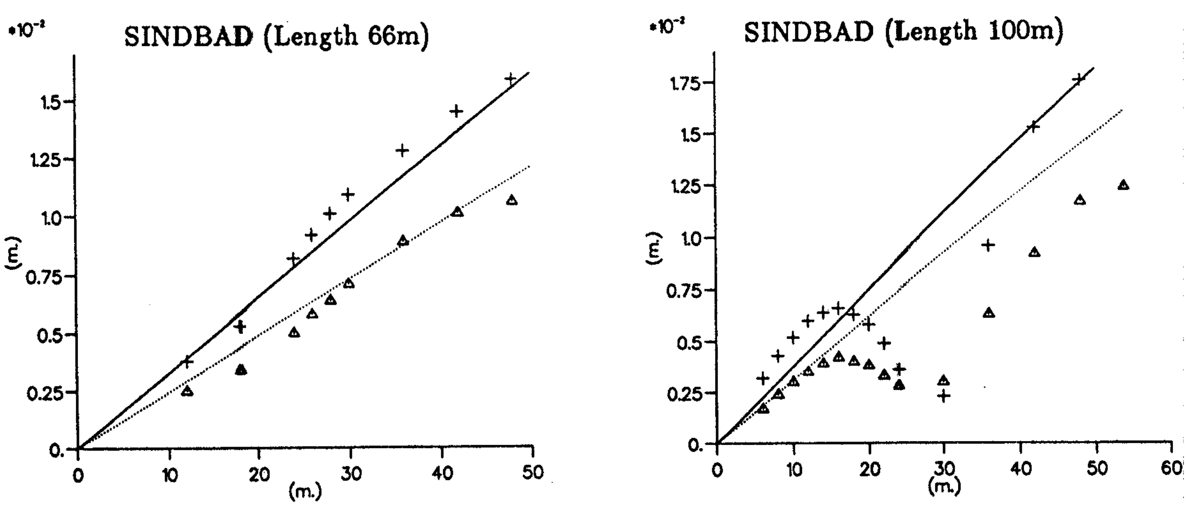

Figure 4 : Sindbad $+ : 2\omega_2 - \omega_1$, $\Delta : 2\omega_1 - \omega_2$
Model ——— : $2\omega_2 - \omega_1$, ... : $2\omega_1 - \omega_2$

Mei: The reported phenomenon was noted by G. Keller around 1982 in Edinburgh, and theoretically confirmed by Lo & Mei (1984, J. Fluid Mech.). The theory makes use of the fourth order extension of Schrodinger equation due to Dysthe (1979) and predicts the asymmetric growth and decay of side bands. The cubic Schrodinger equation is inadequate.

Boudet, Cointe & Molin: We thank you for this comment. We did not know of this work because our reference was your book edited in 1983. Our model predicts (approximately) the asymmetric growth of the resonant components because a fourth order term is kept in the equations. In fact, our main purpose for this part of the study was to confirm the experimental observations (somehow unexpected) and the results of SINDBAD simulations by another method.

Tulin: It is possible to calculate the evolution of these resonant disturbances in a more direct and less restrictive manner than is possible with the Schrodinger equations, and this method also allows the calculation of the waveform deformations. Did the experiments and SINDBAD calculations show a systematic deformation of the wave form with the evolution of the disturbances, and what were the nature of the deformations?

Boudet, Cointe & Molin: During the experiments, only the wave elevations at several points along the tank were measured and we had no direct access to the spatial waveform. SINDBAD could give such information, but we did not look carefully at these deformations. We thank you for your suggestion.