Nonlinear Wave Diffraction over Submerged Obstacles

by

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Introduction

Recently there has been considerable interest in simulation of nonlinear waves diffracting over submerged obstacles. For bodies which are shallowly submerged, strong nonlinear behaviour is observed in the immediate vicinity of the body. Another characteristic is the generation of higher harmonics in the reflected and transmitted waves [1]. A sizeable number of past investigations have been devoted to interactions of solitary waves with bottom topography, e.g. [2]. Our present work focuses more on the effects of periodic incident waves; specifically, our goals include the determination of the magnitude of the shorter waves, the calculation of the forces and simulation of wave breaking, should that occur.

In the last workshop, a finite difference scheme using a new boundary-fitted coordinate method was discussed [3]. Using this particular technique, which is well suited for solving problems with boundaries of complex geometry, such as those that appear in nonlinear free-surface flows, we simulate, in time domain, flows over submerged bodies. Two different geometries are presently being studied - i) a submerged circular cylinder and ii) a bottom mounted semi-circular cylinder (sandbar).

Numerical Procedure

The model we are using consists of a rectangular wave tank, with a piston type wavemaker at the left end. The wavemaker generates sinusoidal waves which in turn interact with the submerged body. The water is initially is taken to be at rest. The amplitudes of the wave components upstream and downstream are computed using the Fast Fourier transform technique.

Since the mixed Eulerian-Lagrangian formulation [5] is used to advance the free-surface, even breaking waves can be simulated using the particular grid generation scheme of [3].

Typically nondimensionalization is carried out with the radius of the body and the acceleration due to gravity being set to 1. Simulation is carried on for 4 to 8 periods, which takes about 400 time steps. This is long enough for the diffracted waves to appear. The body is placed at a distance of at least 1 to 2 wavelengths away from the wavemaker. This is especially important for the second case, where standing wave modes between the wavemaker and the cylinder could be excited. For the submerged cylinder, the cases being studied correspond to the ones for which experimental data are available [4].

Results and Discussions

In Fig 1., the velocity vector plot for the case of flow over a submerged cylinder after 8 periods is shown. The wavelength (λ) and submergence of the center of the cylinder (h), non-dimensionalized with respect to the radius of the cylinder (a) are 7.45 and 1.5 respectively. This corresponds to a ka of 0.84, where k is the wave number. Fig 2. and Fig. 3 show the time evolution of the free surface elevation and forces on the cylinder respectively, for the same case. Though second harmonics can be observed in the transmitted wave, they are not very prominent. The amplitude of the forces and the phase lag of the vertical force is close to that predicted by linear theory [6].

In Fig 4., we present a case for a circular cylinder where the experimentally obtained transmission co-efficient is only 0.74. Here a number of short waves can be seen both upstream of and downstream from the cylinder. This is one of the cases where the results deviate substantially from linear and second order theories and numerical simulations help in capturing the nonlinear phenomena.

Transmission coefficients for four different cases are shown in Fig. 5 and are compared with experimental results of [4]. For these simulations, in the vicinity of the cylinder, we used a grid spacing of about 0.24 a . In spite of the coarse grid used, good agreement is obtained. It was observed that coarser grids can produce reflections upstream of the cylinder.

Fig 6. and 7. show the velocity vector plot and force spectrum for the case of wave diffraction over a sandbar. The non-dimensional length and frequency of the wave are 4 and 1.238 respectively. The second-order forces are quite prominent. Due to the unavailability of experimental measurements at the time of the presentation, more runs were not made.

References

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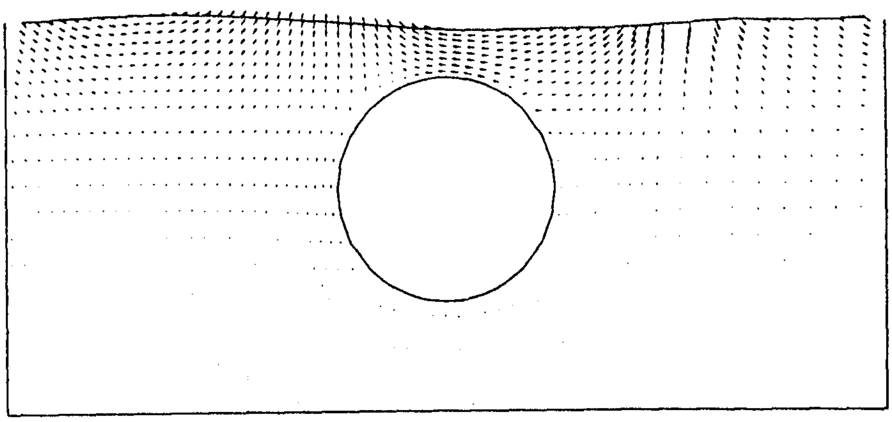

Fig 1. Velocity Vectors for Submerged Circular Cylinder case; ka = 0.84, h/a = 1.5

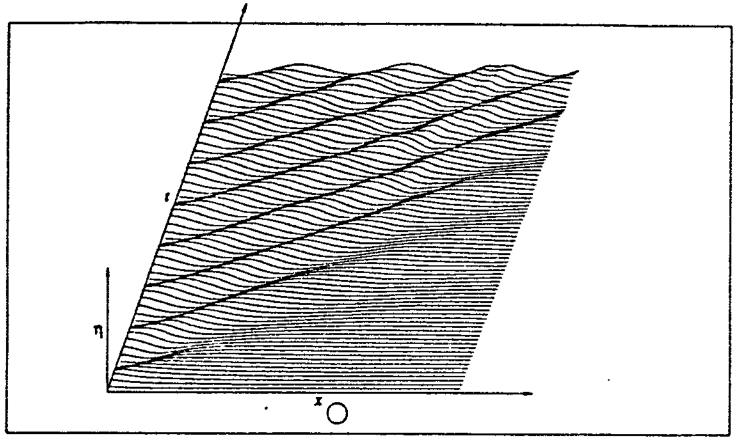

Fig 2. Free Surface Elevations for Submerged Circular Cylinder case; ka = 0.84, h/a = 1.5

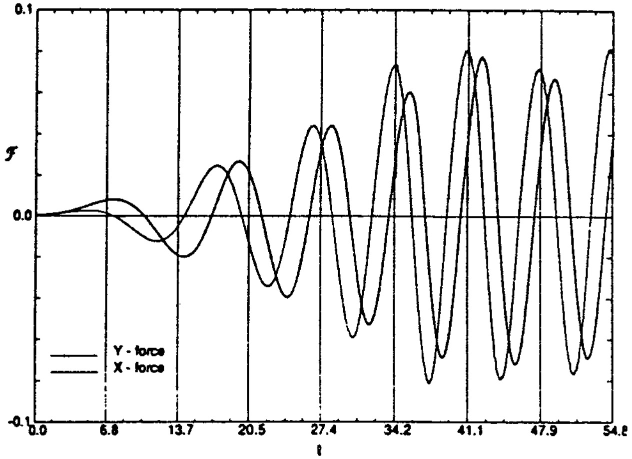

Fig 3. Forces acting on Submerged Circular Cylinder; ka = 0.84, h/a = 1.5

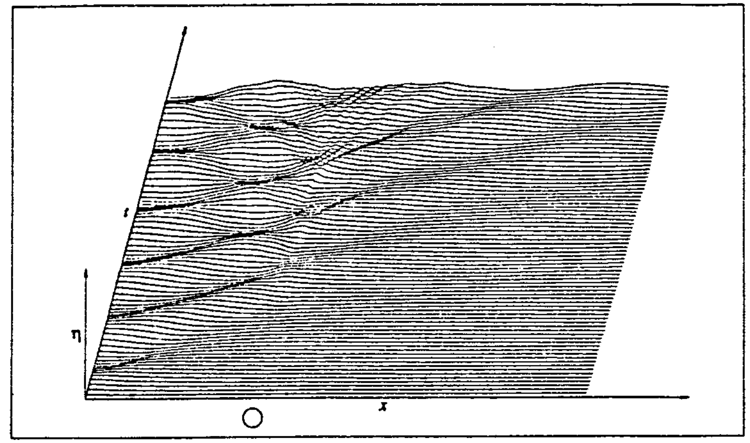

Fig 4. Free Surface Elevations for Submerged Circular Cylinder case; ka = 0.39, h/a = 1.275

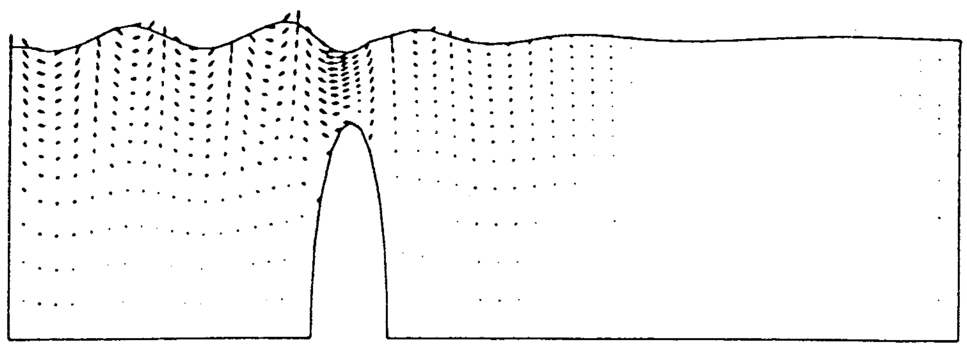

Fig 6. Velocity Vectors for flow over a Submerged Semicircular Mound.

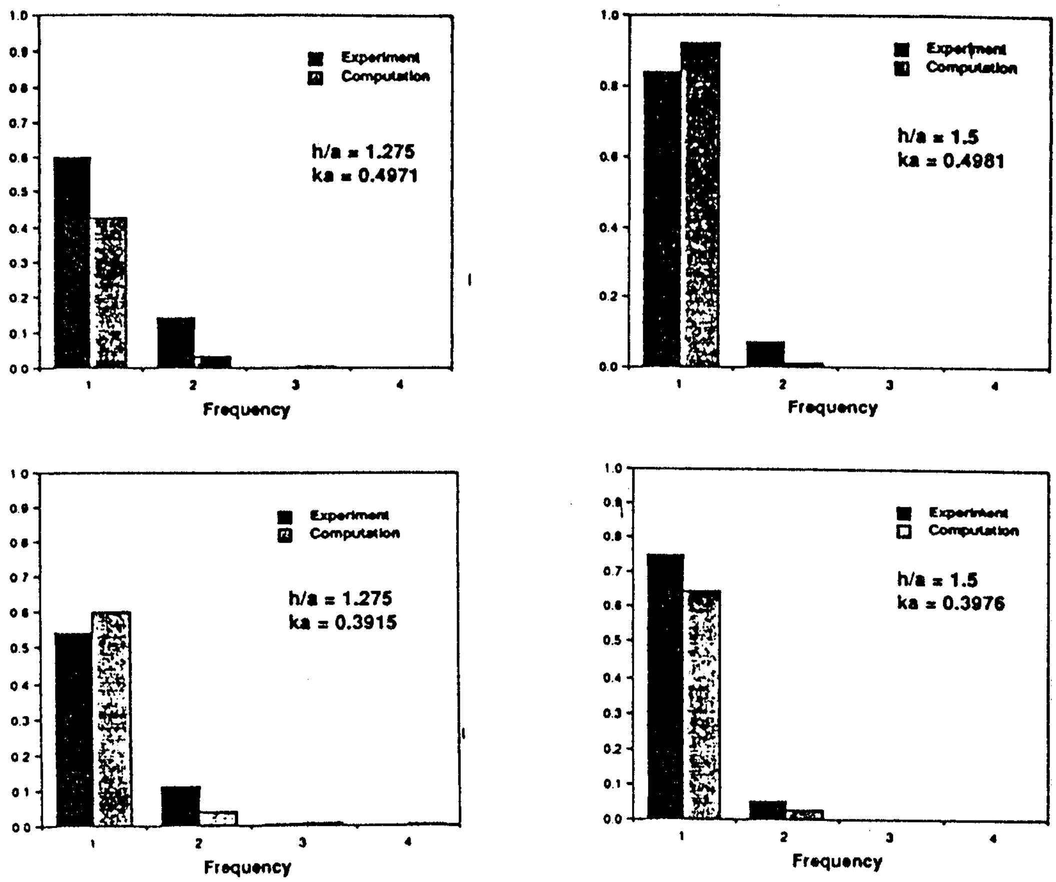

Fig 5. A_T^2/A_I^2 of Fourier Components of Transmitted Wave for Circular Cylinder

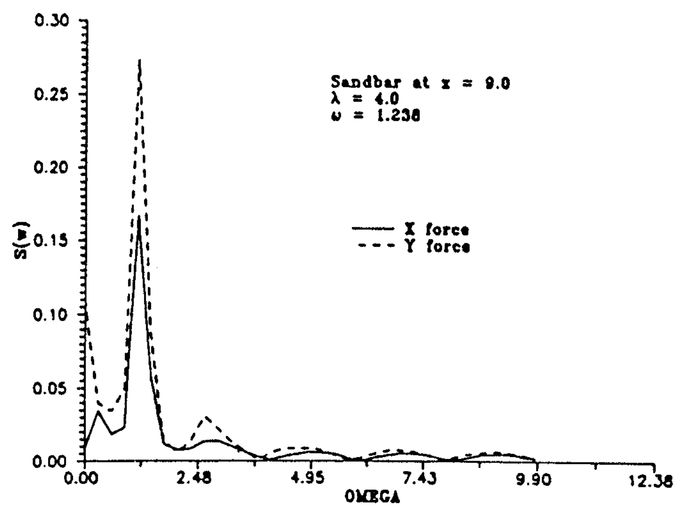

Fig 7. Force Spectrum for Submerged Semi Circular Mound

DISCUSSION

Grue: In experiments, I have observed that well-defined transients travel back and forth in a wave flume for a long time. Also, I have not observed reflected waves from a submerged circular cylinder. How do these observations fit with your numerical experiments?

Yeung & Vaidhyanathan: Our simulations are not carried out long enough to allow the transients to be reflected from the right boundary. Furthermore, since the wavemaker is started smoothly (i.e. with zero velocity), such transients are not pronounced. The reflections observed upstream of the cylinder, at least for deep submergence, are mainly spurious and are due to the coarseness of the grid; refinement of the grid tends to eliminate them.

Vada: Are the wavelengths of the small ripples of the same size as the mesh? If so, are they due to numerical inaccuracies?

Yeung & Vaidhyanathan: The small ripples are due to nonlinear effects, which can be observed when a long wave passes over a shallowly submerged body. However, extremely short waves can eventually be generated by the nonlinear process, and these cannot be resolved by the grid. The presence of such short waves is also apparent in Ref. 3. Our numerical scheme is stable.

Peregrine: A comment: the short waves also appear in our boundary integral computations for cases where a moving cylinder generates waves; they appear to be a real nonlinear effect rather than purely numerical noise.

Cao: Why do you use a finite-difference scheme instead of a boundary-element method?

Yeung & Vaidhyanathan: We chose to pursue the finite-difference method because it has the capability of including viscous effects. The motivation was also explained in Ref. 3.

Thomas: Detailed experiments (performed at the University of Bristol in 1983) for regular waves incident upon a submerged horizontal cylinder, in which both pressures and forces were measured, gave the following results:

(i) For linear waves, a model using a boundary-element method gives excellent agreement between theory and experiment.

(ii) For nonlinear waves, vortex shedding is noticeable and cannot be ignored.

Yeung & Vaidhyanathan: Thank you. It appears that there is a regime in which even small-amplitude waves can excite nonlinearities. It was, in fact, observed by Hin Chiu (Ref. 4) that for a sufficiently small gap, the transmission coefficient is far from being unity (and thus deviates from the results of linear theory), and this is the situation where the present calculations can be helpful.