THE NEGATIVE WAVE RESISTANCE PARADOX IN DAWSON'S METHOD

Hoyte C. Raven
Maritime Research Institute Netherlands (MARIN), P.O. Box 28, 6700 AA Wageningen

In a previous study [1] a comparison was made of wave resistance predictions obtained by Dawson's method and a Neumann-Kelvin approach, both implemented by a Rankine source distribution on the undisturbed free surface, and the neglected nonlinear terms in the free surface condition (FSC) were evaluated. It was found that the predictions by both methods differ by only a few percent for ships with a block coefficient up to about 0.70. More evidence of this has been collected since by applying both methods to several practical ship forms. There thus seems to be no preference for either of these FSC's with a practical discretization.

However, an important exception is the class of full-formed ships at low speeds. For several of these, wave resistance curves like those shown in Fig. 1 were found. At higher speeds both predictions are similar again. But for decreasing speed (still in a practical range) quite substantial differences appear; the slow-ship FSC predicts a rapid decrease of the resistance even to negative values, while the Kelvin FSC still yields a large positive resistance, far exceeding the experimental value.

It is striking that the very similar wave patterns shown in Fig. 2 correspond to such drastically different resistances. Dawson's condition typically results in a small forward shift of the bow wave, yielding a large difference in resistance due to the strong curvature of the bow waterlines for full ships. Almost all differences in longitudinal forces on the hull are found at the forebody.

The fact that a negative wave resistance is sometimes predicted is known to most people that work with Dawson's method, and is generally attributed to numerical inaccuracies in the imposition of the hull boundary condition or in the pressure integration over the hull. But in the cases considered successive refinement of the hull paneling (up to 1415 panels on one half of the hull) made the resistance converge to a negative value for Dawson's FSC, so discretization errors can be rejected as cause of the negative resistance here.

ENERGY BALANCE

A negative value of the wave resistance in the presence of waves radiated by the ship is, of course, paradoxical. Since in harmonic deep-water gravity waves energy travels with half the phase speed, the flow in a control volume moving with the ship looses energy through its aft plane. A steady wave pattern can only exist if energy is supplied at the same rate by the propulsion of the ship; but a negative R_W means that the ship instead extracts energy from the flow. Apparently a qualitatively correct wave pattern need not have the correct energy budget.

By considering the energy fluxes through the boundaries of a control volume moving with the ship in a fixed frame of reference, we can express the wave resistance in terms of an integral over the aft plane S (not necessarily at infinity) plus a line integral along its intersection with the free surface:

$$R_W(1) = \tfrac{1}{2}\,\rho \iint_S (-\phi_x^2 + \phi_y^2 + \phi_z^2)\,dS + \tfrac{1}{2}\,\rho g \int \eta^2\,dz \tag{1}$$

The derivation supposes a zero energy flux through the free surface, which is true for zero normal velocity and zero pressure at the free surface, i.e. an exact satisfaction of both the dynamic and the kinematic FSC.

Let us now redefine the control volume by taking not the exact free surface but the undisturbed free surface y = 0 as its upper boundary, in accordance with the imposition of a linearized FSC. We then find:

$$R_W(2) = \tfrac{1}{2}\,\rho \iint_{S_0} (-\phi_x^2 + \phi_y^2 + \phi_z^2)\,dS - \rho \int\int_{FS_0} \phi_x\,\phi_y\,dS \tag{2}$$

$R_W(2)$ is now the pressure force acting on the hull under the undisturbed free surface, which differs from $R_W(1)$ by an integral along the waterline:

$$R_W(1) = R_W(2) - ½ \rho g \int_{WL} \eta^2 n_x \, d\ell \tag{3}$$

Compared with (1) the aft plane integral is unchanged but extends now only to y = 0; the line integral has been substituted by an integral over the undisturbed free surface FSO. In this expression, no assumption on the FSC imposed has been made yet.

For simple harmonic waves $R_W(1)$ is positive definite and independent of the position of the aft plane, due to the fact that there is a constant horizontal energy transport. In expression (2) however, the horizontal energy flux is not constant: the potential energy is absent as the control volume extends to y = 0 only. Still (2) gives a resistance independent of x, because the difference of the fluxes of energy through two vertical planes is removed (or supplied) through FSO.

If for a more general wave form we use the Kelvin condition to evaluate the integral over FSO, we find by partial integration again the expression (1) plus an integral along the waterline akin to (3). So if the Kelvin condition is imposed and due account is taken of the upper limits of integration, the expressions (1) and (2) give the same result up to higher order terms. In other words, the resistance from pressure integration over the hull in our linear problem, which satisfies (2,3), is equal to the resistance deduced from the wave energy radiated by the ship (corresponding to (1)), hence it is positive if a system of harmonic waves is present at the aft plane. Again the energy flux through FSO just takes into account the variations of the potential energy.

However, for free surface conditions other than Kelvin's this equality may be lost. If a realistic wave system is present behind the ship (1) will again give a positive resistance; but $R_W(2)$ may be different as part of the energy flux may have been supplied through the free surface. In the extreme case the pressure integration over the hull may give a negative resistance: the ship 'rides' on waves generated by the free surface condition.

In order to obtain a more realistic wave resistance prediction two approaches now suggest themselves: either to use the far-field resistance expression (1), or to modify the free surface condition so as to eliminate the excess energy flux through the free surface.

FAR-FIELD RESISTANCE CALCULATION

An important difficulty is present in the evaluation of the far field momentum or energy flux according to (1). In the usual Rankine source implementation, the energy or momentum flux through a plane S_0 located at or behind the aft edge of the free surface panel distribution is always zero! Since no singularities are present outside a control volume surrounding the double hull and the free surface panel distribution, no momentum is left in the flow behind it due to d'Alemberts paradox. The wave resistance is an internal force in this whole collection of source panels, and is compensated by an opposite force exerted by the hull on the free surface sources. Thus the first term of (1) is dominated by the effect of truncation of the free surface and will only give a useful result if the free surface paneling is continued a large distance beyond S_0. The same is likely to be true for wave pattern analysis methods, which after all are based on (1).

ENERGY FLUX THROUGH THE FREE SURFACE

Another remedy would be to eliminate the excess energy flux through the free surface, $dE/dt = \iint_{FS} (\rho U \phi_x \cdot \Delta V_n - U \cdot \Delta p \cdot \eta_x) dS$, where ΔV_n is the remaining normal velocity and Δp the pressure at the calculated free surface. To eliminate the energy flux at every point of the free surface we have to satisfy the exact nonlinear FSC's; the best to be achieved in a linear method is a reduction of the energy flux to higher order in the perturbation parameter, Fn. Thus to leading order (Fn^4) the far-field and pressure resistance coefficients are equal if both ΔV_n and Δp are $O(Fn^6)$.

Consider the best estimate of the p=0 surface ($y = \eta^*$) in the notation of [1]). The energy flux through this surface is determined by the violation of the kinematic FSC only, ΔV_n. This is equal to minus the sum of the terms 3 to 7 evaluated in [1], i.e. nonlinear terms of $O(Fn^6)$ plus the linear transfer terms $\eta\Phi_{yy} + \eta_r \phi'_{yy}$ which are of $O(Fn^2, Fn^4)$ and are neglected in Dawson's FSC. Hence the desired order of the energy flux is only achieved if these transfer terms are included.

Now this is exactly what has been derived in a somewhat different way by Eggers [2]. His FSC, derived from the required invariance to leading order of the far-field resistance, differs from Dawson's FSC just by these transfer terms and is, therefore, consistent to $O(Fn^4)$. Surprisingly however, calculations with this FSC for our test case predict an even more strongly negative resistance as a result of a still larger forward shift of the bow wave. Moreover, for higher Fn the results become quite unrealistic, with local flow reversal at the free surface. The precise cause of this could not be determined yet; but the singularity in the FSC at the point where the double body flow velocity $|\nabla\Phi| = 1/\sqrt{3}$, [2], may play a role. Further study is needed to clarify this behaviour.

CONCLUSIONS

This study has shown that the negative wave resistances found for full hull forms at low speed are not a result of numerical inaccuracy but of the formulation of the free surface boundary condition. The use of a linearized free surface condition imposed on y = 0 changes the energy balance in such a way that a negative resistance is not ruled out. The slow ship FSC may locally supply energy to the waves which has no counterpart in a wave resistance acting on the hull. This energy flux through the free surface is directly linked to the order up to which the free surface condition is satisfied. Eggers's formulation of the FSC in principle reduces the energy flux to higher order, but did not give more realistic results for the present case. A method solving the exact dynamic and kinematic FSC is perhaps necessary to exclude the occurrence of negative resistance.

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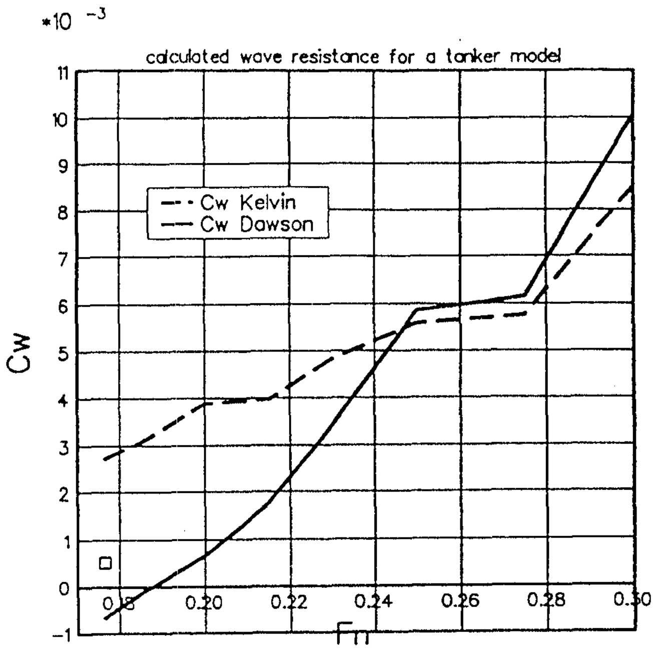

□ Experimental C_w

Fig. 1 Wave resistance coefficients calculated by various methods

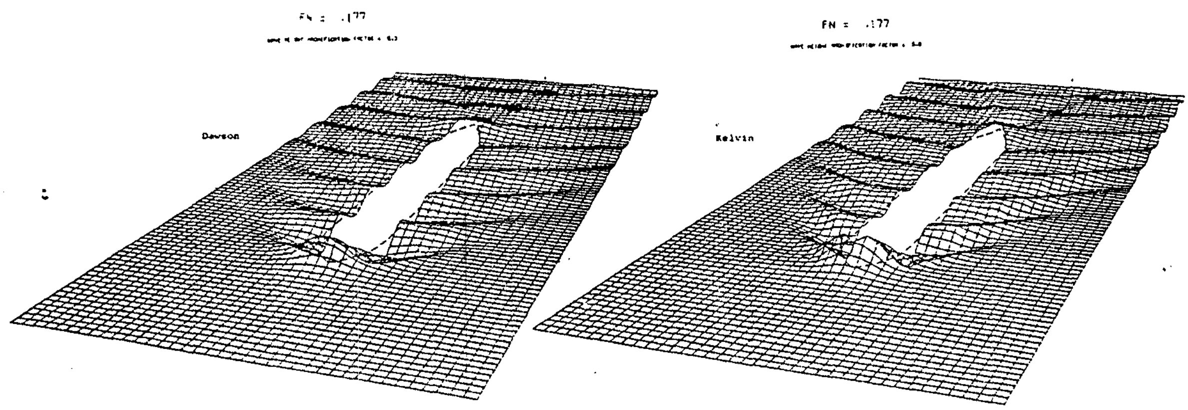

Fig. 2a Calculated wave patterns, with Dawson's (left) and Kelvin FSC (right), at Fn = 0.177 (service speed)

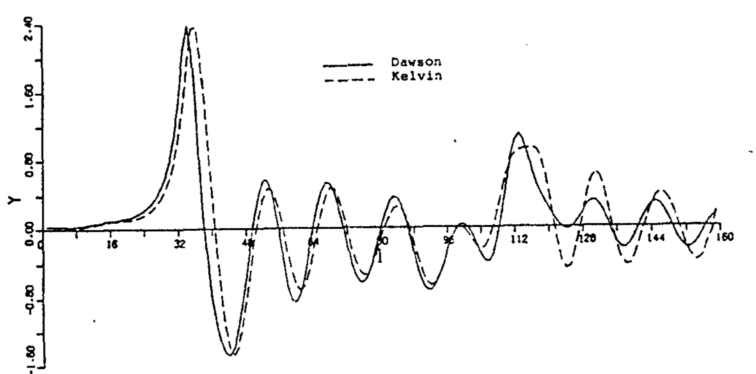

Fig. 2b Wave profile along the hull and centreline

DISCUSSION

Wu: Could I just clarify the implication of your third paragraph: both wave patterns correspond to positive resistances, but negative resistance only arises from the equation used to calculate it?

Raven: This is a matter of terminology. For me, the most basic 'resistance' is the pressure resistance, since it represents the natural way in which a force is being exerted on the hull. The far-field momentum or energy flux may have better properties, but that is another question.

Grue: What is the problem with choosing any control surface surrounding the body for evaluating the energy flux and the momentum flux?

Raven: As explained in the paper, there are no problems in the exact case, but two basic problems for a linearized method:

(1) Energy can be supplied through the free surface, except for special FSC-formulations; thus, the resistance may depend on the choice of control surface. In the case considered here, a control surface fitting tightly around the hull would give a negative resistance, whereas one at a larger distance would give a positive resistance.

(2) If the control volume encloses the entire free-surface panel distribution, the main part of the momentum and energy flux is zero due to d'Alembert's paradox.

Cao: Did you also calculate the wave resistance using Lagally's theorem? If so, how do the results compare with those obtained by pressure integration over the hull?

Raven: Yes, besides the pressure integration, two expressions based on Lagally's law are always evaluated. In general, the three results are almost equal, and, in this case, they were all negative. Indeed, one of the Lagally expressions is just the integral over FS_O in (2); this agrees with my statement that the other term, the integral over S_0, must be zero.

Bertram: A comment: if only we could succeed in obtaining the nonlinear solution for blunt bodies, the whole discussion would become obsolete! Unfortunately, at present nobody has yet succeeded for full bodies.

Raven: Perhaps I should mention that Ni [1] succeeded in calculating the nonlinear solution for the HSVA tanker model, but convergence of the iteration failed for this case as soon as certain numerical details were changed, so I do not consider this to be an entirely satisfactory result.

Tulin: Can a longitudinal wave cut be used to estimate the wave resistance?

Raven: In principle, yes; in practice, there are some drawbacks:

(i) You need a cut length of several times the ship length, at a sufficient transverse distance from the hull. This would require a much larger number of free-surface panels and a large increase in computation time.

(ii) The numerical damping inherent in the discretized method is likely to cause a decrease of the wave-pattern resistance with distance from the hull.

(iii) The effects of truncation of the free surface will be larger than on the pressure integration over the hull.

Similar experiences are reported in [2] for a transverse cut.

Tulin: It has been pointed out several times that Dawson's method (in fact, all so-called 'slow-ship' theories) is *not* asymptotic to zero Froude number. I think that the same is true for the Kelvin-Neumann problem. Perhaps this mathematical failure is part of the reason for the observed discrepancies?

Raven: This may be a correct way of interpreting some of the results. Of course, energy must be conserved whatever the FSC; and the explanation in terms of energy flux through FS is not affected. But, if the slow-ship condition is not asymptotically correct for $Fn \to 0$, expansions in Fn may be meaningless. In that case, there is no reason *a priori* why the energy flux will be smaller with a FSC consistent in Fn. Although without further study I cannot affirm that this is the cause of the unexpected results with Eggers's FSC, I appreciate your suggestion very much.

Eggers: Let me recall that, in my understanding, the comparison presented here is between a (corrected) Dawson approach and its modification by exchanging the free surface condition for the form given by Kelvin. This means that you investigate a Rankine source approach to the Kelvin-Neumann problem formulated by

Brard [3], hence without the line source distribution advocated by Brard for compensating singular effects due to Kelvin sources.

I trust that it is well known in our community that from a formal point of view Dawson's FSC has two deficiencies:

(i) No attempt is made to admit terms from a vertical Taylor expansion in the transfer of the boundary condition to the undisturbed free surface.

(ii) The influence of the double-body flow curvature is omitted when converting the FSC to streamline-orientated coordinates.

On the other hand, Dawson does not discard vital terms connected with first derivatives of the disturbance potential, as would be required under orthodox application of an order change rule.

In defining his approximation for the wave resistance, Dawson inconsistently disregards the line integral of η^2 along the waterline, representing the action of pressure or (with inverse sign) the momentum flux in addition.

If we agree that wave resistance tends to zero with a high power of Fn, we must accordingly satisfy the FSC up to that power at least. In a comparison with an approach using genuine Kelvin sources, I would expect that Dawson's approach cannot exclude energy inflow from upstream, as he could hardly simulate evanescent wave free modes; but this should equally hold for your modified approach. For the present comparison, I wonder if for a blunt bulbous bow form the amount of transverse flow components for the influx integral, which represent positive energy flux from upstream, may be stronger with Dawson's FSC?

Raven: Thank you. Perhaps I may point out that in my program the erroneous transformation to the streamline grid has been corrected and the double-body flow curvature is incorporated; see [4] for some remarks on this. However, the deficiency (i) is still there. It is these transfer terms that should be added so as to reduce the energy flux through the free surface to a higher-order quantity. The results of calculations with the FSC so obtained are, however, most puzzling.

Your remark on the waterline integral over η^2 (see (3)) is perfectly correct. In the present case, this integral would reduce (but not eliminate) the negative resistance for Dawson's FSC, but is negligible for your FSC-formulation. I follow Dawson in disregarding it, since according to my experience it sometimes spoils the wave-resistance prediction. But I agree that this is not quite a scientific argument! Perhaps the predicted sinkage of the ship should be incorporated in the wave elevation at the waterline, as in [5]?

Concerning your last point, in the present Rankine-source comparison, there is no significant difference in the transverse velocities near the upstream edge of the free-surface domain between Kelvin's and Dawson's FSC; in both cases, their magnitudes are small, less than 0.3% of the ship speed, so the energy influx must be negligible.

References

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