ENERGY ARGUMENTS UNDER A "WAVY LID" - A NEW APPROACH TO CAPSIZING AND OTHER HIGHLY NONLINEAR PROBLEMS

R.C.T.RAINEY

WS Atkins Engineering Sciences, Woodcote Grove, Epsom, Surrey, UK

Introduction

Highly nonlinear problems such as capsize and deck wetting are of great importance to naval architects, who require not so much a full numerical simulation of the phenomena (physical model tests satisfy that need), as a good simplified approach which can give them insight into the controlling parameters. The example of capsizing is particularly timely, since recent developments in the theory of nonlinear dynamics and chaos suggest that ships should have a well-defined regular waveheight beyond which they will capsize in transient conditions (Rainey & Thompson, 1990). This gives a rational measure of the stability of both physical and numerical models, setting the stage for work on the required "good simplifications".

Previous Work

Attempts have been made (most recently in de Kat & Paulling 1989) to take the results from a linear 3-D diffraction analysis (i.e. added masses, diffraction forces, etc.) and use them in a time-history simulation. The difficulty is that our problems also require highly non-linear features in the simulation (e.g. unless its roll-stiffness characteristic is ultimately very non-linear, a ship cannot capsize at all!), and the two cannot be combined without glaring inconsistencies. For example, it is not at all clear what relevance the linear-theory heave added mass of a catamaran has to its hydrodynamic loads at a roll angle of 90 degrees.

A more interesting approach is derived from "strip theory", as reviewed for example in O'Dea & Walden (1985). The vertical hydrodynamic load F(x,t) at time t and position x along the length of a slender ship is taken as:

$$F(x,t) = D/Dt\{M(x,t)Dz/Dt\} + N(x,t)Dz/Dt + \rho gA(x,t)z$$

where $D/Dt = \partial/\partial t - V\partial/\partial x$

and V is ship forward speed, z is vertical motion of ship section relative to the water surface, and M,N & A are two-dimensional added mass, damping, and cross-sectional area respectively, which are all assumed to be functions of z as well as x.

This is a heuristic generalisation of Lighthill's rigorous result for a slender fully-immersed flexible body (Lighthill, 1960), using hydrodynamic parameters which incorporate the effects of the free surface. It appeals to intuitive ideas about "rates of change of momentum" - the problem is that the fluid "momentum" around a moving body is in fact not well-defined (Lamb, 1932 art 119), because the reaction forces at the far boundaries cannot be ignored. As far as that type of argument goes, for example, one might conclude that the lateral velocity of a neutrally-bouyant circular cylinder ought to double if it moved broadside out of the water (since its effective mass had halved), implying a doubling of the kinetic energy (half the mass and double the velocity), which is obviously unsatisfactory.

<u>Energy Arguments Under a "Wavy Lid"</u>

An equally fundamental objection to the above strip-theory approach is Ursell's remarkable demonstration (Ursell, 1968) that in head seas the waves seen at any section of a ship will be distorted by sections previously encountered by the wave, no matter how slender the ship is. This means that even in small waves no formula of the above type can be justified by appeal to diffraction theory, and suggests that greater consistency may be achieved with an altogether different approximation. One such is the "wavy lid" approximation introduced in Rainey (1989), in which the motion of the water surface is assumed to be unaffected by the presence of the ship, but otherwise all potential-flow effects are retained. In particular, the water pressure <u>is</u> affected by the ship, unlike the superficially-similar but much cruder "Froude-Krilov" approach.

The attraction of this approximation is that it removes all the degrees of freedom associated with the free surface, so that the hydrodynamic loads on the ship are "memoryless" and determined solely by its instantaneous position and velocity. In these circumstances the ship has a fully-nonlinear equation of motion, which can be determined from the fluid kinetic energy by the classical Lagrange argument (Lamb, 1932 art. 136), from which the Lagrange coordinates can be eliminated to give (Rainey, 1989, eqn. 6.8):

$$d\underline{N}/dt = \underline{Q} - d\underline{I}/dt + \{0,(\underline{m}+\underline{j})_{\wedge}\underline{\nu}\} + \Delta e/\Delta\underline{X}$$

Here $\underline{N} = (\underline{m},\underline{h})$ is the linear and angular "momentum" (including the "impulse" from the added mass), $\underline{I} = (\underline{j},\underline{k})$ is a similar "wave impulse", $\underline{\nu}$ is the ship's linear velocity, $\underline{Q}$ is any additional non-hydrodynamic force and moment, and $\Delta e/\Delta\underline{X}$ is the position-derivative of the fluid kinetic energy e.

The great advantage of deriving fluid loads in this way from the fluid kinetic energy is that the complexities of reaction forces at the far boundaries are avoided, and <u>a relatively crude approximation to the flow can be used</u>, compared with the standard approach of deriving fluid loads by integrating surface pressures. An example (taken from Rainey 1989) is that a simple estimate of the added mass of a slender cylinder, ignoring end-effects, is sufficient to give the correct expression for the "Munk moment" via the $\underline{m}_{\wedge}\underline{\nu}$ term above - obtaining this result by surface pressure integration would require a detailed picture of the end-flows.

A Suitable Fluid Kinetic Energy Expression for a Slender Ship

Rainey (1989) is concerned with the problem of a lattice-type offshore structure, for which surface intersections are relatively unimportant, so that the fluid kinetic energy may be expressed by a strip-theory formula that is exact in the limiting case of slender structural members (and incidentally allows a unification with diffraction theory, to second order). That argument fails in the present case of a slender ship, however, because the surface intersection extends its whole length, and means that on a two-dimensional strip-theory view the fluid velocity will only fall inversely with the radial distance (owing to the volumetric changes produced by variations in A above). This would lead to infinite sectional added masses M(x,t) above, and thus infinite kinetic energy - so clearly an important feature of the problem, keeping the kinetic energy finite, is that the far flow field is three-dimensional, because the ship has a finite length.

It appears necessary, therefore, to model the flow field in two stages. First, we can distribute three-dimensional monopole sources of strength per unit length s(x,t) along the centreline of the ship's waterplane, in such a way as to give the correct volumetric changes, i.e. set

$$s(x,t) = DA/Dt$$

where D/Dt has the meaning above, except that it would appear to be advantageous to make V the forward velocity relative to the mean incident water velocity over A. We also see that this velocity field does not satisfy the rigid-lid boundary condition precisely, unless the water surface is flat. The error involved clearly reduces with waveheight, but of course we are not here concerned with unification with diffraction theory, but merely with an approximation to the fluid flow good enough to give a faithful reflection of its kinetic energy.

The second stage is to add similarly distributed dipoles, quadrupoles, octupoles etc., to give a closer approximation to the local flow around the hull. In all cases we must choose multipoles with the symmetry necessary to give zero flow normal to a flat free surface, as we had with the monopole. This time, however, the kinetic energy falls off much more rapidly with radial distance, so that the flow can be approximated in the normal two-dimensional strip-theory manner. For example, the flow produced by a semicircular-sectioned hull moving sideways in still water will be described by a two-dimensional dipole with its axis parallel to the water surface.

To find the kinetic energy of the flow, we follow the argument in Rainey (1989) sect. 5, and transform all terms involving the incident velocity potential into simple integrals over the immersed volume or waterplane. This leaves the kinetic energy of the monopoles, dipoles etc. above, acting on their own. We can now take advantage of the orthogonal property of cylindrical harmonics (Lamb, 1932, ch. 4) to see that the kinetic energy can to a good approximation be further divided into that from the monopoles acting alone, and that from the rest acting alone. The latter can be expressed in the normal two-dimensional strip-theory manner in terms of integrals along the ship's length of sectional added masses (for sway, roll, or heave as appropriate) of a special sourceless kind, multiplied by incident-wave and local ship velocity. The former, on the other hand, cannot be treated in strip-theory fashion as we saw above, but must be left as an integral over the ship's wetted surface S (i.e. Rainey 1989 eqn. 5.4). This requires numerical integration around the edge of a series of sections spaced along the length of the ship - it may be sufficient to treat the sections as semicircular (i.e. to ignore the kinetic energy of the fluid between S and such a semicircular surface), so that advantage may be taken of axial symmetry.

In any event, the total kinetic energy will have the quadratic form given in Rainey (1989) eqn. 5.11, thereby defining $\underline{I}$ and $\underline{N}$ for the purposes of the above equation of motion.

Concluding Remarks

The above is intended as an outline sketch of the new approach only - the important point is that by means of a single approximation (the "wavy lid"), the problem is reduced to the relatively mundane one of calculating fluid kinetic energy, for which good approximate strip-theory treatments are fairly obvious, and need less details of the flow than a conventional surface pressure integration.

We can however note a couple of the more interesting non-linear aspects to the new approach. For example, it will produce "squat" and "trim" effects depending on the ship's forward speed, because the kinetic energy will vary as a function of pitch and heave (cf Lamb, 1932 art. 137). In addition, it will produce a "slam" effect during large changes of waterplane area (caused e.g. by bow emergence in large waves, or superstructure immersion during large rolls) as a consequence of the DA/Dt term above (cf Rainey 1989 eqn. 7.4 et seq.).

References

de Kat, J.O. & Paulling, J.R. 1989 The simulation of ship motions and capsizing in severe seas, Trans. Soc. Nav. Arch. & Marine Engrs., annual meeting 1989

Lamb, H. 1932 Hydrodynamics 6th edn. Cambridge: University Press

Lighthill, M.J. 1960 Note on the swimming of slender fish J. Fluid Mech. vol 9 pp 305-317

O'Dea, J.F. & Walden, D.A. 1985 The effect of bow shape and nonlinearities on the prediction of large amplitude motions and deck wetness, Proc. 15th Symp. Naval Hydrodynamics Washington DC: National Academy Press pp 163-176

Rainey, R.C.T. 1989 A new equation for calculating wave loads on offshore structures, J. Fluid Mech vol 204 pp 295-324

Rainey, R.C.T. & Thompson, J.M.T. 1990 The transient capsize diagram - a new method of quantifying stability in waves, J. Ship Research, in press

Ursell, F. 1968 On head seas travelling along a horizontal cylinder, J. Inst. Maths Applics vol 4 pp 414-427

DISCUSSION

Miloh: I would like to make a few comments:

(i) The Munk moment is not only valid for slender bodies but also for elongated (axisymmetric) bodies. In any case, it may be considered as a special case of the Kelvin-Lagrange moment equations.

(ii) Taylor's energy expression may also be deduced from Lagally's theorem.

(iii) Energy methods have been used by Lamb and by Milne-Thompson to compute the hydrodynamic loads acting on a sphere moving close to a rigid wall or a relief surface (the infinite-frequency limit), and in several of my papers on manoeuvring in shallow water and water-entry [1]–[4].

Rainey: Thank you for drawing my attention to your pioneering early papers on energy methods. I had hitherto only been aware of [5], which I noted briefly in (Rainey, 1989; cited as R below), as an example of an analysis retaining the free-surface degrees of freedom, and thus belonging to a different branch of the subject from my 'wavy-lid' approach.

I now find that [3] is closest to my work. In [3], equations are derived (32 & 33) for a body moving in the presence of fixed boundaries, which correspond precisely to those above, in the special case $\underline{j} = \underline{I} = 0$, so that only the Munk moment (which I accept as valid for arbitrary bodies; see pp. 304 & 312 of R) and $\Delta e/\Delta \underline{X}$ remain. The notation in [3] is much longer though: I believe the use of components in a body-fixed frame obscures the fundamental structure of the problem, and is only relevant if the resulting equations can be solved analytically, as in the classical literature. In particular, the computer program will be neater if $\underline{N}$ is used as the state vector. On the other hand, the notational and conceptual differences make cross-checks more valuable. For instance, R(6.8) can be deduced from (21) & (28) in [3], if R(5.11) is used. Thus, the lengthy algebra in [3] and in Appendix B of R confirm each other, increasing the credibility of both.

Turning to the classical example of the attraction of a sphere moving parallel to a wall (Lamb, §137), and also Lagally's theorem [6], I believe the former shows the superiority of energy arguments over the latter, because Kelvin & Tait could deduce the force using a potential (Lamb, §99, (3)) which is too crude to give this force by surface pressure integration, and therefore too crude to obtain using Lagally's theorem (which replicates pressure integration by virtue of (7) in [6]). This superiority is also well illustrated by your results, such as in your water-entry paper in these Proceedings.

Newman: You are in danger of overselling this method — there are problems which can only be tackled with panel codes, such as the 'microseism' second-order diffraction effect [7] and the pitch moments on submerged slender spheroids [8]. There is room for both — simple methods, like yours, and panel codes.

Rainey: Thank you for drawing your recent papers to my attention. They show very clearly the sort of wavelengths below which my 'wavy-lid' treatment breaks down.

Consider [7]. The analysis therein, applied to the vertical load on a TLP leg (vertical cylinder of radius a and draft $4a$), can be compared with the second-harmonic vertical load amplitude predicted by R(7.3) as

$$\rho g a A^2 (\pi/4) K a e^{-8Ka},$$

where K is the wavenumber and A is the amplitude. This is larger than the results in [7] (Fig. 2 and (28)) when $Ka \lesssim 0.2$, and the transition is sharp — the microseism effect dominates by a factor of 10 at $Ka \simeq 0.4$ and is dominated by a factor of 10 at $Ka \simeq 0.1$. For example, for the ISSC TLP (which has legs of the above proportions), $Ka = 0.2$ corresponds to waves of period 13 sec. We conclude that for this TLP, my second-harmonic calculation is almost irrelevant to second-harmonic-induced heave resonance effects, but that microseism effects are almost irrelevant to survival-wave calculations. This, of course, is only a rough guide: interactions between legs will magnify the microseism effect, and pontoon contributions (see below) will magnify second-harmonic loads. Also, for such small Ka, the TLP motions will produce second-order errors in the body boundary conditions which may dominate the second-order potential, an effect included by both panel codes and R.

Consider [8]. It gives 'exact' panel-code calculations of the steady heave force and pitch moment on a submerged slender (10:1 aspect ratio) horizontal spheroid in head seas, and, for comparison, slender-body-theory calculations using the 'IB' approximation (IBA) introduced in [9]. On p. 240 of [9], it is claimed that the IBA, which assumes that no steady forces or moments are produced by body motions in calm water, is justified because such forces and moments vanish in comparison with steady wave-induced ones. I dispute that. A counter-example is a cylinder undergoing in-phase heave and surge motions — the 'Munk moment' felt in pitch will have a steady component which depends directly on the heave added mass (R, §3), and thus depends on slenderness in the same way as all other hydrodynamic forces. This moment is traceable to end-effects (R, §3), so I dispute the claim on p. 239 that steady forces and moments can be calculated without including end-effects (or taper). They can be seen, via $R(7.2)$, to produce a steady vertical force on a horizontal cylinder heaving in response to head seas, for example. Thus, the IBA involves additional approximations beyond slenderness alone, and so (contrary to the claim in [8]) it is not clear that it will replicate the results of my method, where no such additional assumptions are made. The IBA might be expected to give results which agree less well than mine with the panel-code results, especially in the 'free spheroid' cases in [8]. In the fixed-cylinder case, however, an argument similar to $R(7.6–7.8)$ (and given in my UK Dept. Energy Offshore Tech. Rept. OTH 89 311) gives the force per unit length as

$$\text{(A)} \qquad \rho c(\mathbf{a})_{\mathrm{T}} + \underline{\underline{m}}\,(\mathbf{a} + [\mathbf{1}\cdot\underline{\underline{v}}\mathbf{1}]\mathbf{v}) + [\underline{\underline{v}}\,\underline{\underline{m}}\mathbf{v}]_{\mathrm{T}} - \underline{\underline{m}}\,\underline{\underline{v}}(\mathbf{v})_{\mathrm{T}} \qquad (\mathbf{a} = \text{particle acceleration})$$

in the notation of R. In the special case of the steady vertical force on a horizontal cylinder in deep-water waves, with a principal added mass axis vertical, (A) agrees exactly with (54) and (55) of [9], indicating that the IBA is now correct. Also, the taper of the spheroid used in [8] now produces only a second harmonic and no steady force (at least vertically), so the fixed-spheroid IBA results should agree with my predictions.

So, let us compare these fixed-spheroid results with the panel code. As I would expect, the pitch moment decays in relation to the vertical force as Ka falls from 0.5 to 0.16, the lever-arm dropping from $0.035L$ to $0.005L$ (L = spheroid's length). But I am surprised that the two methods give vertical forces in a fixed ratio of ~ 0.7, for all Ka. This seems hard to reconcile with the physical mechanism postulated as causing the differences, which is the build-up of wave elevation along the body's length, for then one would expect a far better vertical force agreement once the pitch lever arm had dropped to only $0.005L$. I would be interested to see computations at different wave headings (where the wave build-up will be less) and of the second-harmonic heave force and pitch moment (which are predicted by (A) to be strong except for beam seas). Another avenue to explore is the enhancement of a cylinder's added mass caused by the proximity of my 'wavy lid'; according to $R(6.8)$, this ought to produce an additional steady vertical force through the term $\Delta e/\Delta\mathbf{X}$.

All this, of course, perhaps only illustrates the sagacity of Prof. Newman's final sentence!

References

[1] M. Bentwich & T. Miloh, 'On the exact solution for the two-sphere problem in axisymmetric potential flow', *J. Appl. Mech.* **45** (1978) 463-468.

[2] T. Miloh & A. Hauptman, 'Large amplitude motion of an elongated controlled body in shallow water', *J. Ship Res.* **24** (1980) 256–270.

[3] T. Miloh & L. Landweber, 'Generalization of the Kelvin-Kirchhoff equations for the motion of a body through a fluid', *Phys. Fluids* **24** (1981) 6–9.

[4] T. Miloh, 'Wave slam on a sphere penetrating a free surface', *J. Eng. Math.* **15** (1981) 221-240.

[5] T. Miloh, 'Hamilton's principle, Lagrange's method, and ship motion theory', *J. Ship Res.* **28** (1984) 229–237.

[6] L. Landweber & T. Miloh, 'The Lagally theorem for unsteady multipoles and deformable bodies', *J. Fluid Mech.* **96** (1980) 33–46.

[7] J.N. Newman, 'Second-harmonic wave diffraction at large depths', *J. Fluid Mech.* **213** (1990) 59–70.

[8] C.H. Lee & J.N. Newman, 'First- and second-order wave effects on a submerged spheroid', *J. Ship Res.*, submitted.

[9] C.M. Lee & J.N. Newman, 'The vertical mean force and moment of submerged bodies under waves', *J. Ship Res.* **15** (1971) 231–245.