Low Frequency Motions of Semisubmersibles

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Introduction

A need to explain and ultimately predict the low frequency motions of vessels in irregular seas has been the motivation behind many recent developements in second order potential theory.

Whilst exact solutions for the slowly varying potential forces and moments are now available, experimental investigations [1] indicate a lack of knowledge of the damping of such motions. Indeed for semisubmersible like structures — which do not always fall clearly into the diffraction regime — it is as of yet uncertain that the second order potential forces are the only significant excitation forces [2].

Here we are particularly interested in the vertical roll and pitch low frequency responses. The presented approach utilizes the principle of the separation of the hydrodynamic forces into: a) potential flow forces and b) vortex flow forces, [3]. The significance of components a) and b) are investigated as excitation forces and as damping forces in bi-chromatic seas. Some experimental results are also presented.

Description of Motion

The rigid body motion is described in terms of the displacement of the centre of gravity $\underline{\xi} = (\xi_1, \xi_2, \xi_3)$ and the roll, pitch and yaw motions about the centre of gravity $\underline{\alpha} = (\xi_4, \xi_5, \xi_6)$. The solution for $\underline{\xi}$ and $\underline{\alpha}$ is sought in the frequency domain in terms of the first order motions at the incident frequencies ω_a, ω_b, and the second order motion at the difference frequency $\omega_- = \omega_a - \omega_b$.

Potential Flow Forces and Moments

The slowly varying second order potential flow forces are evaluated using the approach adopted by Matsui [4]. The linear potentials and motions being evaluated using the 3-D source distribution technique.

Vortex Flow Forces and Moments

Traditionally the viscous force on a cylinder of diameter D is expressed in terms of the empirical viscous drag coefficient C_d:

$$\underline{F} = \frac{1}{2}\rho D C_d \underline{U}|\underline{U}|$$

where $\underline{U}$ is the relative velocity normal to the cylinder. The above expression has proven to be reliable in cases of steady and sinusoidal flows [5]. Cases where $\underline{U}$ consists of a high frequency and a low frequency oscilation have been investigated by Koterayama and Nakamura [6]. They show that the above expression may still be used, although a modification to the C_d value is required. In the present application $\underline{U}$ is of the form:

$$\underline{U} = Re\{\underline{u}_a e^{-i\omega_a t} + \underline{u}_b e^{-i\omega_b t} + \underline{u}_- e^{-i\omega_- t}\}$$

At present little is known about drag coefficients in such flows. We will use the well documented steady stream values, incorporating modifications to C_d at a later stage.

The viscous forces—being dependant on the motions — must be found iteratively: initially a guess for the motions is used in evaluating the viscous forces and moments on the whole structure in the time domain. This is then frequency analised and the relevent frequency components are extracted and incorporated into the equation of motion to obtain the next approximation to the motions. This is repeated until convergence on the motions is achieved.

Equations of Motion

From Newtons laws of motion we have the second order equations of motion:

$$m\ddot{\underline{\xi}}_2 = \underline{F}_2$$

$$I\ddot{\underline{\alpha}}_2 = \underline{G}_2 - I(\dot{\underline{\omega}}_2 - \ddot{\underline{\alpha}}_2) - \dot{\underline{\alpha}}_1 \times I\dot{\underline{\alpha}}_1 - \underline{\alpha}_1 \times I\ddot{\underline{\alpha}}_1$$

where m is the mass, I is the inertia tensor, $\underline{\omega}$ is angular velocity and a numerical suffix denotes order of perturbation. $\underline{F}_2$ and $\underline{G}_2$ are the hydrodynamic forces on the vessel—they are decomposed into added mass, potential damping, restoring, potential exitation and viscous excitation terms. The equations of motion are reduced to a 6×6 matrix equation for the unknown $\underline{\xi}_2$ and $\underline{\alpha}_2$.

Experimental Tests

A number of experimental tests have been performed on an Aker H-3 model semisubmersible in bi-chromatic waves. The scale model was free to move in all six degrees of freedom being restrained only by low stiffnes elastic ties.

Conclusions

It was found that the slowly varying pitch potential flow moment was significant throughout the relevant range of differencce frequences and dominated over the pitch vortex flow moment . In roll, however the potential flow moment was approximately double the vortex flow moment and both increased in a roughly linear manner with the difference frequency. Quantitive agreement between the numerical results and experimental results were poor — this is thought to be due to experimental errors. However, some interesting trends were identifiable. The pitch response was found to be quadratic in the wave amplitude away from resonance and linear at resonance. Whereas the roll response was quadratic at and above resonance and cubic below resonance.

References

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[2] Aage c., Lundgren H., Jensen O., Per V., "Scale Effects in Model Testing of Offshore Structures", O.M.A.E.-960, Tokyo 1986.

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DISCUSSION

Standing: I was interested to see that your paper concluded with a series of questions. This is fairly typical of this kind of study; we have had similar experiences ourselves:

(i) Moorings, even if simple elastic ones, can make a surprisingly large contribution to the damping.

(ii) There are many experimental problems, particularly those of spurious long waves in model basins.

(iii) Some of the experimental results suggest that the response may depend on the amplitude cubed. This suggests that viscous drag may be important for the forcing — as found in other studies.

Pizer: No, I have not considered the damping contributed by the moorings. It does look as if a more intricate experimental set-up will have to be devised in future if these small effects are to be investigated.

Rainey: Taking Dr. Standing's characteristically modest point that he ended up with more questions than answers with his approach, I wonder if the key to these problems is perhaps provided by Chen & Molin, in their interesting paper on TLP's (these Proceedings). It is to use more than one approach simultaneously (as indeed also advocated by Prof. Newman, in his comment on my paper), with the following evident advantages:

(i) it increases physical insight, because the different approaches are making different approximations, allowing particular physical effects to be isolated; and

(ii) it reduces the risk of being misled by programming errors, because, in certain circumstances, the different approaches should agree and can thus be cross-checked.

Following this line on the semisubmersible problem, I believe that a good additional approach to the present one would be that described in [1], which should work well as the relevant wavelengths are quite long. An encouraging sign is that this relatively simple approach appears to explain the 'semisub tilt' phenomena nicely; see my 1986 paper cited in [1].

Pizer: First, it should be clarified that the difficulties referred to by Dr. Standing were concerned with the experimental aspects of a second-order study, rather than with our common theoretical approach.

Yes, a time-domain approach such as your's, which permits large motions, may well provide useful physical insight into nonlinear phenomena which is not attainable via second-order theory. However, one should be cautious in using Morison's equation. Eatock Taylor & Hung [2]have shown that, at low frequencies, the total horizontal drift force on a group of N cylinders is proportional to N^2. Thus, *even for long wavelengths*, Morison's equation is inappropriate since it ignores interference effects.

Sclavounos: It would be interesting to attempt the solution of the second-order equations of motion with a different ordering of the rotation angles, and then to observe if the resulting displacements are identical. In particular, there appears to be a good reason to start with the yaw rotation and then to continue with either pitch or roll, since yaw does not introduce any hydrostatic effects.

Pizer: I would expect that a different ordering would lead to different values of 'roll', 'pitch' and 'yaw'. However, these different values should represent the same motions (to second order).

Yes, choosing yaw first does result in a welcome simplification of the hydrostatics.

References

[1] R.C.T. Rainey, 'A new equation for calculating wave loads on offshore structures', *J. Fluid Mech.* 204 (1989) 295–324.

[2] R. Eatock Taylor & S.M. Hung, 'Wave drift enhancement effects in multi-column structures', *Appl. Ocean Res.* 7 (1985) 128–137.