FAMILIES OF TRIAL FUNCTIONS FOR THE THREE-DIMENSIONAL VARIATIONAL METHOD ON WAVE-BODY INTERACTION PROBLEMS

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Introduction

In the analysis of the wave-body interaction problems focus is usually put on computing some global quantities, as added mass, radiation damping, linear and nonlinear exciting forces, etc. These global quantities are supposed to be weakly dependent on the detailed description of the flow field. Indeed, it has been recently shown, see Aranha & Pesce (1989), that they can be computed by means of a *variational method*, in which relatively crude approximations taken for the velocity potential provide much better results for these quantities. Numerical results, obtained for the two-dimensional linear problem, indicate that the method works very well, recovering the values of these quantities with the use of only three or four potential flow singularities, placed at convenient points *within* the body contour.

Mathematical Background

It is convenient to split the velocity potential into two parts: an *evanescent* and an *ondulatory* one. Such a partition is made by constructing a cylindrical surface of radius a surrounding the body and by introducing the operators

$$L_n(\psi) = \int_S \psi(a,\theta,z)\, \frac{1}{2\sqrt{\pi a}} \left[\left(1 + (-1)^n \right) \cos n\theta + \left(1 - (-1)^n \right) \sin n\theta \right] e^{kz} dS \qquad (1)$$

where S is the cylinder suface. If now $\left\{ q_n(x,y,z);\ n=0,1,2,\ldots \right\}$

is an arbitrary set of functions with the properties $L_n(q_m)=\delta_{mn}$, then, given any $\psi(x,y,z)$ defined within S, one can write

$$\psi(x,y,z) = \psi_e(x,y,z) + \sum_{n=0}^{\infty} L_n(\psi)\ q_n(x,y,z) \qquad (2)$$

where $\psi_e(x,y,z)$ is *evanescent*, i.e.

$$\int_{-\infty}^{o} \psi_e(a,\theta,z)\ e^{kz} dz = 0 \qquad (3)$$

Once the *evanescent potential* $\phi_e(x,y,z)$ is determined one can construct the *scattering matrix* S and solve the infinite-dimensional linear system

$$[\ S_{nm}\ ]\ \langle L_m(\phi)\rangle = \langle\ V_n\rangle \qquad (4)$$

The generalized *evanescent potential* ϕ_e is solution of the weak equation

$$G(\phi_e;\psi_e) = V\ (\psi_e) \qquad (5)$$

valid for all ψ_e in the *evanescent space*. The discretization problem reduces, then, to equation (5), solved in a finite-dimensional sub-space spanned by a set of convenient (*evanescent*) trial functions.

It can be shown (see Aranha & Pesce (1989)) that equation (5) represents the stationarity condition for the Lagrangian associated to the present problem and that the above mentioned global quantities are stationary values of well defined functionals constructed over $G(\cdot;\cdot)$ and $V(\cdot)$.

Families of evanescent trial functions

Let $h(x,y,z;\alpha_i)$ be a potential flow *singularity* in an unbounded fluid region, characterized by a set of parameters α_i, referring to its location. This *singularity* can be either a pole or a x,y,z-dipole, or else it can represent a line distribution of elementary singularities: a source line, a dipole line or even a vortex ring. Each one of these *singularities* has a very simple analytical expression. The parameters α_i must be chosen such as to locate the *singularities* within the body.

Consider now the function

$$H_e(x,y,z;\alpha_i) = \left(k + \frac{\partial}{\partial z} \right) \left(h(x,y,z;\alpha_i) - h(x,y,-z;\alpha_i) \right) \quad (6)$$

It is an easy task to verify that $H_e(x,y,z;\alpha_i)$ satisfies both Laplace's equation and the free surface boundary condition. Further more $H_e(x,y,z;\alpha_i)$ is an evanescent function since

$$\int_{-\infty}^{0} H_e(x,y,z;\alpha_i)\, e^{kz}\, dz = 0 \quad (7)$$

Within the class of functions given by (6) the bi-linear form $G(\cdot;\cdot)$ reduces to a regular integral over the body surface, simplifying the solution of equation (5). By properly choosing the family of *trial functions* and the parameters α_i one can imitate the gross features of the fluid flow around the body. The global quantities can then be computed by means of the variational method with an error of order δ^2 (they are stationary values) if the fluid flow is only roughly described with an error of order δ.

The analysis of a semi-submersible platform is under way in order to check the numerical accuracy provided by these families of *trial functions*.

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References

Aranha, J.A.P. & Pesce C.P. *A variational method for water wave radiation and diffraction problems*. J. Fluid Mech. (1989), vol. 204, pp. 135-157.

Pesce, C.P. & Aranha, J.A.P. Second-order forces in waves by means of a variational method. 6th *International Conference on Offshore Engineering*, Rio de Janeiro, Brazil, Aug. 1989.

DISCUSSION

Rainey: I completely agree with Dr. Pesce's point that velocity potentials which are too approximate to allow the forces on a floating body to be calculated by pressure integration over its surface *can*, paradoxically, give these forces accurately from a variational argument. It is splendid to have an ally on this point!

However, I believe that the paper by Aranha & Pesce (1989) is misleading when it claims (end of §2.2) to present a variational method with these advantages for calculating 'almost all global coefficients needed to analyse the interaction (to Nth order) between a floating structure and sea waves'.

The problem is that at, say, second order, their method calculates only the contribution to the second-order oscillatory force which comes directly from the second-order potential; it cannot calculate all the other contributions, coming from products of first-order terms. The difficulty is that, conventionally, these product terms require an accurate first-order potential — which is precisely what the variational method has neatly avoided calculating at the previous (first-order) stage.

For example, consider a deeply-immersed thin cylinder in oblique translational oscillations in still water (moving in a direction at 45° to its axis). At first order, a very crude approximation to the flow (2-D dipoles distributed along the axis, in strip-theory fashion) will clearly give a good estimate of the lateral added mass and thus of the first-order force. The second-order force, however, is just the 'Munk moment', felt in yaw, and rising to a peak twice per oscillation. It is traceable (see §3 of [1]) to the contribution from $\frac{1}{2}\rho|\nabla\phi|^2$ to the fluid pressure, and is felt not along the body of the cylinder, but at its ends. Thus, the first-order potential is required in considerable detail (2-D dipoles will clearly *not* do!) to evaluate it. This is in contrast to my own energy argument [1], which successfully derives the full second-order loading, *without* having to improve on the 2-D dipole model.

Pesce: Thank you. We would like to make the following points concerning our variational method.

(i) The method can be used to compute the effect of the Nth-order potential $\phi^{(N)}$ on the Nth-order force (such as the effect of the second-order potential on the slow-drift phenomenon). It can also be used to compute the horizontal second-order steady forces associated with $\phi^{(1)}$ (as they are related to the far-field wave amplitudes). The status of the vertical steady force is unclear, although we do not believe that the energy argument will fail in this case only.

(ii) In [1], diffraction is neglected and so are the steady forces. Dr. Rainey's intention, however, was to analyse a particular set of geometries, in the limit of small Ka; within this context, his 'wavy-lid' assumption seems appropriate. It seems that this assumption is needed so as to avoid radiation damping whence a pure Lagrangian formulation can be used. In our variational method, however, ϕ was separated into an undulatory part and an evanescent part. The Lagrangian approach is possible for the evanescent part. The undulatory part is determined by means of the 'scattering matrix', whose coefficients are determined variationally from the evanescent solution.

(iii) We agree that direct computation (pressure integration) of second-order forces requires very accurate knowledge of $\phi^{(1)}$ — for one reason, the second-order pressure depends on the gradient of $\phi^{(1)}$ which is, in general, much more difficult to compute by purely numerical methods than $\phi^{(1)}$ itself.

Reference

[1] R.C.T. Rainey, 'A new equation for calculating wave loads on offshore structures', *J. Fluid Mech.* **204** (1989) 295–324.