WAVE SCATTERING BY A VERTICALLY AXISYMMETRIC BODY IN A CHANNEL

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Introduction

When evaluating the results of model tests in wave tanks it is important to account for the interference effects resulting from the presence of the tank walls. Spring and Monkmeyer (1975) have calculated the wave forces on a vertical, circular cylinder extending throughout the depth and centrally placed between channel walls. More recently, Yeung and Sphaier (1989) have given an eigenfunction method for a truncated cylinder that can, in principle, account fully for the images of the body in the walls. However, in their calculations, evanescent modes are neglected when accounting for interactions between the body and its images. In the present work, an approximate procedure is given in which hydrodynamic characteristics of a vertically axisymmetric body when in a narrow tank are deduced from the characteristics in open water by a relatively simple calculation.

The method is based upon the plane-wave approximation proposed by Simon (1982) who was interested in the performance of arrays of axisymmetric wave-energy devices. The basic idea is to replace the waves scattered or radiated by one body with an approximating plane wave in the vicinity of another body. This is essentially a wide-spacing approximation and is easily visualised by considering the case of outwardly propagating circular waves. At sufficiently large distances the curvature of the wave crests is negligible on the scale of the wavelength and the waves may be considered as locally plane. The method was developed further by McIver and Evans (1984) and McIver (1984) who included a non-plane "correction" term.

A body placed in a channel is equivalent to an infinite row of bodies and the scattering and radiation problems may be analysed by a simple adaptation of the plane-wave method. Preliminary work on the application of the plane-wave method in this context was carried out by Simon (1981).

Formulation

The usual assumptions of the linearised theory of water waves are made. For simplicity of presentation, the scattering of waves by a fixed body centrally placed in a channel of uniform depth h and constant width b is considered. The extension to the radiation problem is trivial and an off-centre body may be treated by an obvious, but algebraically more complex, development of the same ideas. Cartesian coordinates (x, y, z) are chosen with origin O at the body axis and in the mean free surface, Ox is directed parallel to the channel walls and Oz vertically upwards. It is assumed that the motion is time harmonic with radian frequency ω. As the depth of the channel is constant, the complete motion will consist of evanescent modes, that decay exponentially away from the body, and propagating modes. Therefore, in the region "not too close" to the body evanescent modes may be neglected and the velocity potential written as

$$\Phi(x, y, z, t) = \mathrm{Re}\,\{\phi(x, y) \cosh k(z+h)\, e^{-i\omega t}\} \tag{1}$$

where k is the positive real root of $\omega^2 = gk \tanh kh$ and the complex valued function $\phi(x, y)$ satisfies the Helmholtz equation.

A plane wave

$$\phi_I = e^{ikx} = \sum_{n=-\infty}^{\infty} i^n J_n(kr)\, e^{in\theta}, \tag{2}$$

where $x = r\cos\theta$ and $y = r\sin\theta$, is incident upon the body. When the body is in open water the scattered wave field is given by

$$\phi_S = \sum_{n=-\infty}^{\infty} A_n i^n H_n(kr)\, e^{in\theta}, \tag{3}$$

say, where the A_n are assumed known. With the body placed in the channel each image in the walls will scatter the incident wave in this way giving further non-plane waves incident on the original body and a subsequent modification to the scattered wave field. For a centrally placed body, the total wave field scattered by each image is identical and is written as

$$\phi_m = \sum_{n=-\infty}^{\infty} B_n i^n H_n(kr_m)\, e^{in\theta_m} \tag{4}$$

where (r_m, θ_m) are polar coordinates with origin at $(x, y) = (0, mb)$, the axis of image m.

In order to relate the unknown coefficients B_n to the known A_n the plane-wave approximation is used to estimate the additional incident waves due to the image bodies. This requires the assumption that the body is widely separated from its images, that is $k(b - 2a) \gg 1$, where a is a typical body radius. To a first approximation, in the vicinity of the body the waves scattered by the images in $y > 0$ may be represented by a plane wave incident from large positive y. The images in $y < 0$ contribute a plane wave of the same amplitude but propagating in the opposite direction. An approximation to the total wave field incident upon the body is therefore

$$\phi_I' \sim e^{ikx} + Ce^{-iky} + Ce^{iky} = e^{ikr\cos\theta} + C\left(e^{ikr\cos(\theta - \frac{1}{2}\pi)} + e^{ikr\cos(\theta + \frac{1}{2}\pi)}\right) \tag{5}$$

and the body may be thought of as being in open water but with additional incident plane waves due to the images. Equation (3) describes the scattering of a plane wave incident from $\theta = 0$ when the body is in open water, combining the three plane waves in equation (5) gives the total scattered wave field

$$\phi_S' = \sum_{n=-\infty}^{\infty} A_n i^n H_n(kr)\, e^{in\theta}\{1 + C((-i)^n + i^n)\}, \tag{6}$$

where evanescent modes have been neglected. Comparison of equation (4), with $m = 0$, and equation (6) gives

$$B_n = A_n\{1 + C((-i)^n + i^n)\}. \tag{7}$$

It remains to determine C, the amplitude of the plane wave representing the waves scattered from the images in $y > 0$ (or $y < 0$). By an addition theorem for Bessel functions,

$$i^n H_n(kr_m)e^{in\theta_m} = \sum_{p=-\infty}^{\infty} i^p J_p(kr)\, H_{n+p}(mkb)\, e^{-ip\theta} \sim S_m(-i)^n e^{-iky}, \tag{8}$$

using the large argument expansion of the Hankel function, where

$$S_m = \left[\frac{2}{\pi mkb}\right]^{\frac{1}{2}} e^{i(mkb - \frac{1}{4}\pi)}. \tag{9}$$

Thus, from equation (4), the total approximating plane wave due to the images in $y > 0$ is

$$\sum_{m=1}^{\infty}\phi_m = \sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty} B_n S_m (-i)^n e^{-iky} \tag{10}$$

so that the amplitude

$$C = \sum_{m=1}^{\infty} S_m \sum_{n=-\infty}^{\infty} B_n(-i)^n. \tag{11}$$

Combining equations (7) and (11) gives

$$C = \sum_{m=1}^{\infty} S_m \sum_{n=-\infty}^{\infty} A_n(-i)^n / \{1 - \sum_{m=1}^{\infty} S_m \sum_{n=-\infty}^{\infty} A_n(1+(-1)^n)\}. \tag{12}$$

The series in m in equation (12) is discussed by Yeung and Sphaier (1989) and may evaluated by expressing it as an integral.

Discussion

Substitution of equation (12) into equation (5) gives the total wave field incident on the body. If the wave forces due to an incident plane wave when the body is open water are known, then the forces for the body in a channel are found by combining results in a way that accounts for the amplitudes and directions of the plane waves given in equation (5). The additional contribution to the horizontal forces due to the presence of the channel walls will be zero (by symmetry) according to this approximation. Thus the vertical force on a body is more influenced by the channel walls than the horizontal forces, as noted by Yeung and Sphaier (1989). If the non-plane correction term introduced by McIver and Evans (1984) is included, by taking the next term in the expansion of the Hankel function in equation (8), then this does give a contribution to the horizontal force. Details of this are omitted here due to lack of space.

Spring and Monkmeyer (1975) have calculated accurately the horizontal force on a vertical cylinder extending throughout the depth. Comparison between those results and the plane wave method is given in figure 1; X is the ratio of the force on a cylinder in a channel to that when in open water. Comparison is made for three different values of b/λ, the ratio of channel width to wavelength. Strictly the plane-wave theory is valid only when b/λ is large, however good results are obtained even when this parameter is $O(1)$. The values of $2a/b$ on the figure correspond to $ka = 1$ for each of the values of b/λ. The approximation is less good when the cylinder diameter approaches the channel width, the limiting case corresponds to $2a/b = 1$.

Reflection and transmission coefficients may be calculated from the diffracted wave field. In figure 2, the present work is compared with results for the reflection coefficient R taken from

Dalrymple et al (1988) and calculated using an accurate method. The results given here use only plane waves to account for interactions between the body and its images; improved results should be obtained when the correction term is introduced.

References

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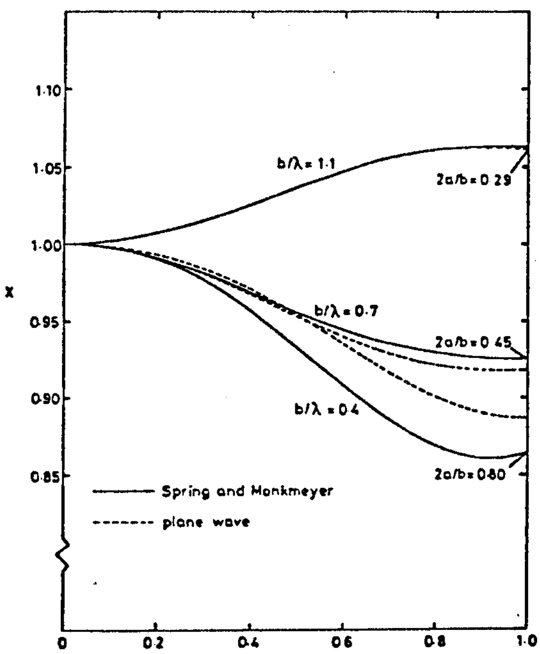

Figure 1

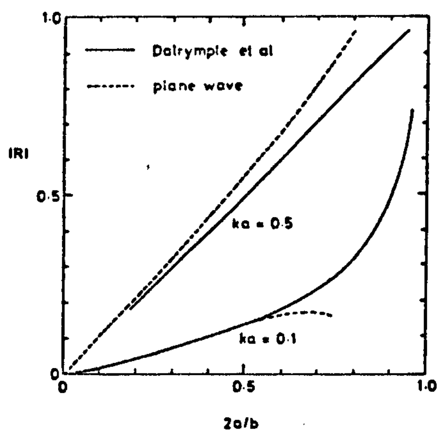

Figure 2

DISCUSSION

Yeung: I feel that the difficulty of using the plane-wave approximation is that one can never be absolutely confident of its effectiveness, unless an 'exact solution' is available as a check. My statement must not be construed as implying that the plane-wave approximation is not useful, but rather that simplicity can lead to inaccuracy, and proper caution must be exercised in its advocation.

McIver: This seems to be a general 'philosophical' point applicable to almost any method, not just to that under discussion here. Approximations are nearly always involved, but as long as one is fully aware of their nature then a reasoned judgement may be made concerning the applicability of a given method.