(KOCHIN-TYPE) SECOND-ORDER WAVE EXCITING FORCES

by
Chang-Ho Lee
Department of Ocean Engineering
MIT, USA

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At the 4th workshop, Lee(1989) presented a paper on expressions for the sum-frequency second order forces derived in the form of Fourier integrals of Kochin functions. In those expressions two types of force components are considered following the definition of Sclavounos(1988). One component (IB component) is due to the interaction between the body disturbance (B) and incident wave (I) and the other component (BB) is due to the interaction of two body disturbances. In this study, we concentrate on the computation of the 'IB' component which involves one Fourier integration pair. For the wall-sided bodies, this Fourier integration is performed using analytical techniques for large values of the wavenumber k. For small to moderate k numerical quadrature is used. The efficient evaluation of this part of the force is also important for the computation of the 'BB' component since the 'IB' appears as the integrand of the Fourier integral for the 'BB' component.

SECOND ORDER FORCES

For the sake of convenience, we further divide the 'IB' component into two parts (A- and B-type). The derivation of the A-type force X_{iA}^{+} is given in Lee(1989) with the result

$$X_{iA}^{+} = \frac{-i\rho\omega_1(\omega_1+\omega_2)^2}{2\pi g} A_1 e^{i\delta_1} \int_0^{2\pi} d\theta \int_0^{\infty} dk \frac{k}{k-N} \frac{\nu_1+\nu_2-k\cos(\theta-\beta_1)}{l-\nu_2} \cdot H_i(k,\theta)\cdot S_2(l,\alpha) \qquad (1)$$

where

ω_i : frequency of linear wave source potential

ν, N : linear and sum frequency wave numbers

σ_i : strength of linear wave source potential

A_i, β_i, δ_i : amplitude, heading angle, and phase of incident wave

$$l = \sqrt{k^2+\nu_1^2-2k\nu_1\cos(\theta-\beta_1)}$$

and

$$\alpha = \tan^{-1}\frac{k\sin\theta-\nu_1\sin\beta_1}{k\cos\theta-\nu_1\cos\beta_1}$$

The contour of integration in (1) must pass above the poles at $k=N$ and $k=\nu_2$ to satisfy the appropriate radiation condition at infinity.

The Kochin functions $H_i(k,\theta)$ and $S_i(l,\alpha)$ in (1) are given as

$$H_i(k,\theta) = \iint_{S_b} (n_i(\vec{\xi}) - \psi_i(\vec{\xi})\frac{\partial}{\partial n})e^{k\varsigma - ik\xi\cos\theta - ik\eta\sin\theta} \tag{2}$$

and

$$S_i(l,\alpha) = \iint_{S_b} \sigma_i(\vec{\xi})e^{l\varsigma + il\xi\cos\alpha + il\eta\sin\alpha} \tag{3}$$

The practical usefulness of the expression (1) hinges on the proper approximation of the Kochin functions for large k such that a simple expression for Fourier integral may be available in that domain.

Especially, for the wall-sided bodies, the body integration in (2) and (3) can be divided into a integration in ς and a integration along the body contour in horizontal planes. Since the integrands of (2) and (3) decay exponentially, the ς integration can be expanded in terms of inverse powers of k and l, with the result

$$\begin{aligned} H_i(k,\theta) = \int_{Cw} e^{-ik\xi\cos\theta - ik\eta\sin\theta}[&i\psi_i(n_1\cos\theta + n_2\sin\theta) + n_i/k \\ &-iN\psi_i(n_1\cos\theta + n_2\sin\theta)/k + O(1/k^2)] \end{aligned} \tag{4}$$

and

$$S_i(l,\alpha) = \int_{C_w} e^{il\xi\cos\alpha + il\eta\sin\alpha}[\sigma_i/l + O(1/l^2)] \tag{5}$$

Upon substituting (4) and (5) into (1) followed by the successive applications of the binomial expansion to l, $1/(l-\nu_2)$ and $1/(k-N)$ and by the exchange of Fourier and waterline integrations, we obtain of X^+_{iA} when $k \geq K$ in the form

$$\begin{aligned} F_K = &\frac{\rho\omega_1(\omega_1+\omega_2)^2}{2g}A_1 e^{i\delta_1}\int_{C_w}\sigma_2 e^{-i\nu_1(\xi_2\cos\beta_1 + \eta_2\sin\beta_1)}\int_{C_w}\psi_i e^{ikR\cos(\gamma-\theta)} \\ &\cdot(C_1\int_K^\infty \frac{J_o(kR)}{k}dk - C_2\frac{J_1(KR)}{KR}) \end{aligned} \tag{6}$$

where $C_1 = n_1\cos\beta + n_2\sin\beta$ and $C_2 = n_1\cos(2\gamma-\beta) + n_2\sin(2\gamma-\beta)$. In addition, R represents the distance between two points on the waterline; $R = (\xi_2-\xi_1)^2 + (\eta_2-\eta_1)^2$ and $\gamma = \tan^{-1}((\eta_2-\eta_1)/(\xi_2-\xi_1))$.

Computational results for the second order forces on offshore structures will be presented at the Workshop.

REFERENCES

Lee C.-H.(1989) Second-order wave forces on floating bodies. 4-th Int. Workshop on Water Waves and Floating Bodies, Øystetse, Norway.

Sclavounos P.D.(1988) Radiation and diffraction of second-order surface waves by floating bodies. J. Fluid Mech. Vol. 196, pp. 65-91

DISCUSSION

Chau: What is the numerical efficiency of the present method compared to 'conventional' methods?

Lee: In the present method, most of the computational effort is directed to the numerical integration over the small to medium range of Fourier wave number. In this region, I used the trapezoidal rule and Simpson's rule along the radial and circumferential directions, respectively. I think that more elaborate numerical quadrature could reduce the computational effort. In particular, the slow convergence due to poles may be avoided by integration in the complex plane.

Although direct comparison of the numerical efficiency between two methods is difficult at this point, I would say that the computing time for the 'IB' component of the present method using brute force numerical integration is comparable to that for the total force by conventional methods.