MOTIONS OF 2D BODIES AT SMALL FORWARD SPEED

Chunsheng Hu
Wimpey Offshore,
London, U.K.

R. Eatock Taylor
University of Oxford,
U.K.

In this work we study the linearized unsteady motions of two dimensional cylinders at forward speed. A major difficulty in solving this problem is the presence of the steady potential in the free surface condition for the unsteady potentials. For bodies submerged below the free surface, a simple treatment is to ignore the steady potential in the free surface condition. Here, a method is developed to solve the unsteady velocity potentials while the disturbance of the steady potential is included in all boundary conditions. The results obtained suggest that the steady potential has a significant influence. Neglecting its disturbance in the free surface condition would underestimate the wave loadings and dynamic responses of the body.

The method used here is the same as we suggested in the 4th Workshop (1), but results presented there did not include the disturbance of the steady potential in the free surface condition. The idea of the method is to expand the unsteady velocity potential into a zero-speed solution plus a forward speed correction term, which is linearly proportional to the forward speed. Any other terms in the expansion are assumed to be negligible under the small forward speed condition. The boundary value problem is thus solved in terms of a zero-speed problem and a forward speed correction problem. The free surface condition for the latter contains terms involving the steady potential as well as the zero-speed unsteady solution. Green's second identity is applied to form boundary integral expressions, and the pulsating source potential is used for both problems.

1. Statement of the Problem

A moving Cartesian coordinate system is used to describe the boundary value problem of a body moving at forward speed U in waves. The x axis is defined pointing in the direction of forward speed and the z axis is defined pointing upwards with z=0 on the mean free surface. Let the velocity potential be denoted in a complex form $\Phi = \mathrm{Re}[\phi e^{i\omega t}]$ with ω the encounter frequency. Up to the first order of the Froude number, the boundary conditions for the diffraction potential (ϕ_7) and the radiation potentials (ϕ_j, j=2,3,4) may be written as follows:

$$-k\phi + 2i\tau\vec{\nabla}(\bar{\phi}-x).\vec{\nabla}\phi + \phi_z - i\tau\phi\bar{\phi}_{zz} = 0, \quad \text{on the free surface } z=0, \tag{1}$$

$$\frac{\partial\phi_7}{\partial n} = -\frac{\partial\phi_0}{\partial n}, \quad \text{on the mean body surface } S_B, \tag{2}$$

$$\frac{\partial\phi_j}{\partial n} = n_j + \frac{U}{i\omega}m_j, \quad \text{on } S_B, \quad (j=2,3,4) \tag{3}$$

$$\frac{\partial \phi_j}{\partial x} + ik(1+2\tau)\phi_j \to 0 \qquad \text{as} \qquad x \to +\infty, \tag{4}$$

$$\frac{\partial \phi_j}{\partial x} - ik(1-2\tau)\phi_j \to 0 \qquad \text{as} \qquad x \to -\infty, \tag{5}$$

where $\omega = \omega_0 - Uk_0\cos\beta = \omega_0(1-\tau_0\cos\beta)$, with ω_0, k_0 the frequency and wavenumber of an incident wave at an angle of β (β=0 or π only in 2D); $k = \omega^2/g$; and $\tau = U\omega/g = \tau_0(1-\tau_0\cos\beta)$, with $\tau_0 = U\omega_0/g$ and $\omega_0^2 = gk_0$. The steady potential $\bar{\phi}$ is taken as the "double-body" flow. The ϕ in the free surface condition represents each of the three radiation potentials or the sum of the diffraction potential and the incident potential. The radiation conditions may be deduced from the asymptotic expression of a Green function given by Haskind (2).

2. The Pertubation Formulation

As outlined in Reference 1, the problem is solved by expanding it into a zero-speed problem and a forward speed correction problem. Each is solved in turn, on the basis of the hypothesis that the velocity potential ϕ_j can be expanded into a series of the form:

$$\phi_j = \phi_{j0} + \tau_0\phi_{j1} + \ldots .$$

In particular, the free surface condition is expanded into:

$$-k_0\phi_{j0} + \frac{\partial \phi_{j0}}{\partial z} = 0, \qquad \text{on } z=0, \tag{6}$$

$$-k_0\phi_{j1} + \frac{\partial \phi_{j1}}{\partial z} = 2i\frac{\partial \phi_{j0}}{\partial x} - 2k_0\cos\beta\phi_{j0} + 2iQ_j, \qquad \text{on } z=0, \tag{7}$$

where

$$Q_j = \begin{cases} -\vec{\nabla}\bar{\phi}.\vec{\nabla}\phi_{j0} - \frac{1}{2}(\bar{\phi}_{xx}+\bar{\phi}_{yy})\phi_{j0}, & j=2,3,4, \\ -\vec{\nabla}\bar{\phi}.\vec{\nabla}(\phi_{j0}+\phi_{00}) - \frac{1}{2}(\bar{\phi}_{xx}+\bar{\phi}_{yy})(\phi_{j0}+\phi_{00}), & j=7. \end{cases} \tag{8}$$

Following the procedures outlined in Reference 1, the zero-speed potential is written as:

$$\phi_{j0} = \frac{1}{\alpha} \int_{S_B} \left(G\frac{\partial\phi_{j0}}{\partial n} - \phi_{j0}\frac{\partial G}{\partial n}\right) dS. \tag{9}$$

The foward speed correction term for head seas ($\beta=\pi$) is written as:

$$\phi_{j1} = \frac{1}{\alpha} \int_{S_B} \left(G\frac{\partial\phi_{j1}}{\partial n} - \phi_{j1}\frac{\partial G}{\partial n}\right) dS + \frac{1}{\alpha} J, \tag{10}$$

where (for 2D)

$$J = 2i\left[\int_{S_F} G\left(\frac{\partial\phi_{j0}}{\partial x} - ik_0\phi_{j0} + Q_j\right) dS - (G\phi_{j0})_{+\infty}\right]. \tag{11}$$

The expression for the following sea ($\beta=0$) is similar, but different in some signs. G used above i the pulsating source potential. Multipole expansions are used to compute the zero-speed potential in the J-integral. Coupled numerical methods are used. Details may be found in Reference 3.

Computations including or not including the Q_j terms in equation (8) have been made and compared. The results presented in Fig. 1 to Fig. 8 are for a submerged circular cylinder with its axis one and half radii below the free surface. The results show that neglecting the Q_j terms underestimates variation of added mass, damping, exciting forces and responses. Computations for a floating semi-circle and a floating rectangle have also been carried out.

References

(1) Hu, C.S. and Eatock Taylor, R. A small forward speed perturbation method for wave-body problems. 4th Int. Workshop on Water Waves and Floating Bodies, Oystese, Norway, pp.97-100, 1989.

(2) Haskind, M.D. On wave motion of a heavy fluid, Prikl. Mat. Mekh. Vol.18, pp.15-26, 1954.

(3) Hu, C.S. Wave Forces on Oscillating Bodies At Small Forward Speed. PhD Thesis, University of London, 1989.

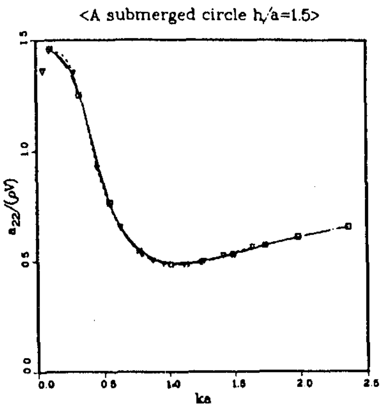


Fig. 1 Sway added mass

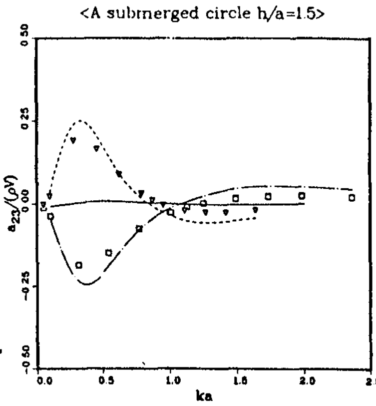


Fig. 2 Sway-heave added mass

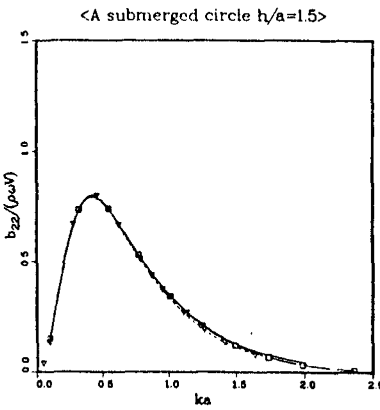


Fig. 3 Sway radiation damping

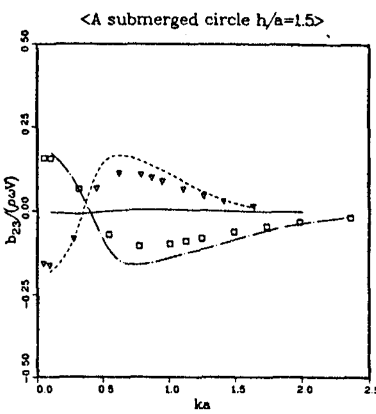


Fig. 4 Sway-heave radiation damping

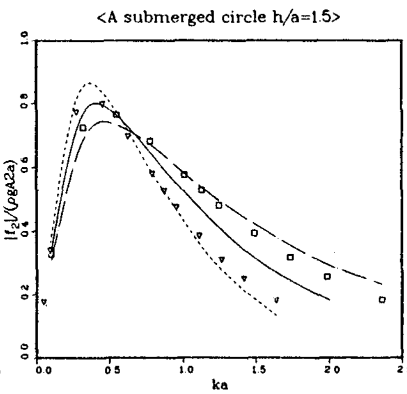


Fig. 5 Sway exciting force

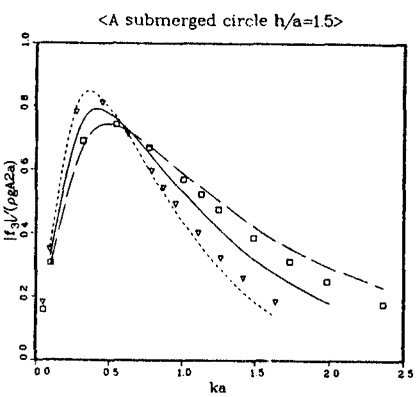


Fig. 6 Heave exciting force

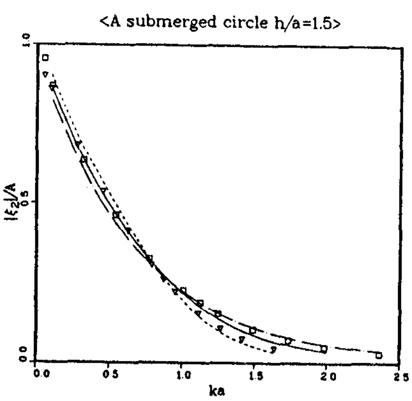


Fig. 7 Sway response

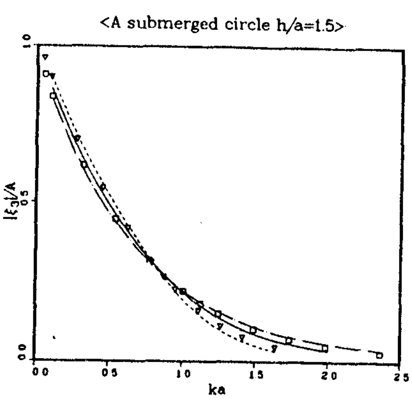
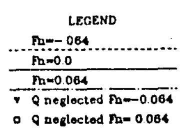


Fig. 8 Heave response



DISCUSSION

Kleinman: Is there any rigorous justification for the particular form of the perturbation used? For example, has it been established that the potential has a convergent Taylor expansion in powers of τ_0 or that the expansion is asymptotically correct, i.e.

$$\lim_{\tau_0 \to 0} \frac{1}{\tau_0} \{\phi_j - \phi_{j0} - \tau_0 \phi_{j1}\} = 0 \quad ?$$

Hu: The rigour of the perturbation form used here was discussed in Reference (1), and full details can be found in Reference (3). It has been shown that the perturbation may not be appropriate in the far field (i.e. for radiation conditions), but it is acceptable in the near field. In particular, the expression (10) is valid in a finite fluid domain around the body; see Reference (3) for a rigorous justification.

Sclavounos: Have you observed the sensitivity of the linear and second-order forces upon the Q terms in the free-surface condition? I have observed that for the diffraction problem around a vertical cylinder, the double-body flow does not appear to enter the evaluation of the slow-drift damping.

Hu: Yes. As shown in the paper, the first-order exciting forces, cross-coupling added mass and damping coefficients are all sensitive to the Q terms for a shallowly submerged circular cylinder ($h/a \leq 1.5$). The responses are less sensitive. For floating bodies, we have specifically investigated this sensitivity, because, theoretically, it is not justifiable to neglect the steady potential disturbance in the free-surface condition for bluff bodies (e.g., the stagnation-point condition would not be satisfied).

For diffraction by a vertical cylinder, we have an asymptotic approximation for the mean drift force in the long wave limit [1]. As discussed in [1], the presence of the steady potential (Q terms) does not appear to influence the far-field behaviour of the diffraction potential, because the double-body flow decays faster than the diffraction potential. The mean drift force was shown to be insensitive to the Q terms. However, if the body is allowed to oscillate, the answer is presently unknown.

Reference

[1] R. Eatock Taylor, C.S. Hu & F.G. Nielsen, 'Mean drift forces on a slowly advancing vertical cylinder in long waves', *Appl. Ocean Res.*, to appear.