DISSIPATION AND DIFFRACTION OF AN INCOMING WAVE DUE TO A SUBMERGED, HORIZONTAL, PERMEABLE, CIRCULAR CYLINDER

by

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We are studying the dissipation and diffraction of an incoming sinusoidal wave due to a submerged, horizontal circular cylinder which is either (i) a porous cylinder, or (ii) a thin cylindrical porous shell. It is assumed that Darcy's Law is valid in the porous media. The problem is 2D, the equations are linearized, and the fluid is incompressible, homogeneous, and of infinite depth.

It will be shown that, in both cases, the wave reflection is zero, in spite of dissipation of wave energy. This result is an extension of the classical result of Dean (1948) and Ursell (1950) that there is no reflection from an impermeable, horizontal circular cylinder. To show this, we use the technique of integral equations as in Grue and Palm (1984). In problem (i), the Laplacian equation is valid in both the fluid layer and the porous layer.

In both these regions we apply Green's theorem with a Green function satisfying the boundary conditions in the fluid layer. Using the boundary conditions at the porous cylinder we obtain a coupled set of integral equations which is easily reduced to one integral equation. In case (ii), we use the same procedure for the fluid layers within and outside the cylindrical shell. In this case we are not able to reduce the two coupled integral equations.

The equations are solved by a standard collocation method, and the results show that the wave reflection is zero for both the circular cylinders. We have also proven this result analytically. In case (i), the velocity potential is developed in a Fourier series at the cylinder surface.

It is shown that half of the Fourier components are zero, from which it readily follows that the coefficient of reflection is zero. A similar procedure is also used to prove no reflection in case (ii).

It is also shown analytically that the horizontal and vertical first order forces are equal, apart from a difference of $\pi/2$ in phases, exactly as for an impenetrable circular cylinder.

The forces on the cylinder have been computed numerically. For the porous cylinder, the effect of porosity on the water waves is relatively small in the Darcy region. For the porous shell, however, we obtain a considerable damping of the incoming wave in the Darcy region, assuming that the porous layer is thin and the permeability small. In this case, Darcy's Law is given in the form of $\Delta p = (\mu/\alpha)\upsilon$ where Δp is the pressure jump over the porous shell, and υ the normal velocity. The amount of damping is dependent on the value of μ/α, the acoustic impedance. The damping and forces have been computed for various values of this parameter. We found that for maximal damping the transmitted energy is about 60% of the incoming energy.

We thus have (at least on paper) a device which absorbs a significant part of the incoming energy and is not leading to any reflection of the incoming harmonic wave.

Both the first order forces and the mean second order vertical force are smaller than the corresponding forces for an impenetrable, circular cylinder. The mean second order horizontal force is very small – much smaller than should be expected from the change in flux of wave momentum due to the cylinder. The reason for this seems to be that the change in wave momentum flux is partly balanced by the change in momentum flux of the fluid penetrating the cylindrical shell.

References

DEAN, W.R. 1948. On the reflection of surface waves by a submerged circular cylinder. Proc. Camb. Phil. Soc. 77.

URSELL, F. 1950. Surface waves on deep water in the presence of a submerged circular cylinder. Proc. Camb. Phil. Soc. 79.

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DISCUSSION

Tuck: A generalisation of your work would be to allow the porosity to be non-constant. Of course, it cannot be too general, since, after all, we can construct an impermeable cylinder of *arbitrary* shape by a suitable step-function porosity in a massively porous circular cylinder. But surely if the porosity was any function of radius alone (i.e. of the polar coordinate r), there would still be zero reflection. I wonder just how much variation we can allow and still obtain zero reflection.

Palm: I believe that you are thinking of a variation in *permeability* rather than in porosity: a change in porosity without a change in permeability does not alter the form of the equations. I agree also that a purely radial variation in permeability probably leads to zero reflection. This may be difficult to show, since the governing equation inside the cylinder is no longer Laplace's equation. I believe that it is relatively simple to extend the result of zero wave reflection to the case where the permeability is a step-function of r.