A MODEL OF THE SHOCK PRESSURES FROM BREAKING WAVES

by M.J.Cooker & D.H.Peregrine
Department of Mathematics, University of Bristol, Bristol, BS8 1TW, U.K.

ABSTRACT

This work is concerned with the forces produced by a breaking wave when it slams into a solid surface. Numerous experimenters have measured very high pressures (tonnes force per square metre) of very short duration (less than a millisecond). Breaking waves striking vertical walls have been studied by Bagnold (1939), Mitsuyasu (1962), and Kirkgöz (1982), amongst others. The vertical wall is generally fronted by a steep beach, so that the incident waves are made to break at the wall. This is also true of coastal structures. Consequently, there is a wide variety of wave shapes possible at the moment of impact. However, experimenters from Bagnold (1939) onwards agree that the highest pressures occur when the breaking wave has a vertical face at the instant it strikes the wall.

The fluid motion near the wall, during the short period of impact, is extremely complex. The wave traps air bubbles and the impact of the free surface with the wall may consist of many collisions at different part of the wall. Some experimental pressure data record such multiple collisions.

Despite these complexities we can model the sudden horizontal deceleration of the fluid at the wall, as being due to the action of large pressures p enduring only over a small time interval Δt. The impulsive pressure of reaction by the wall on the fluid induces an impulsive pressure field $P(x,y)$ (which is the time integral of pressure) in the liquid. See Lamb (1932, §11) and Batchelor (1973, §6.10).

$P(x,y)$ is harmonic and $\nabla P/\rho$ is the jump in fluid velocity due to the wave impact. P vanishes at a free surface, and $\partial P/\partial n = \rho U_n$ is prescribed at a boundary where the fluid velocity component normal to the boundary changes from U_n before impact, to zero after impact.

Exact unsteady water wave computations presented in another abstract (see that by Peregrine and Cooker) show that certain shallow water waves can break at a vertical wall in such a way that, just before impact, the fluid domain has the form of a semi-infinite strip - see figure 1. Nagai (1960) reports experimental wave profiles with a similar geometry. As a simple example let the wave impact zone occupy the upper fraction μ of the wall, and let the impact speed $U_n = U_0$ a constant. The other boundary conditions are shown in figure 1, and Fourier analysis gives a solution for the impulsive pressure

$$P(x,y) = \sum_{n=0}^{\infty} \frac{2\rho U_0}{h\lambda_n^2} (-1)^n (1-\cos \lambda_n \mu h) \cos \lambda_n (y+h) \exp(-\lambda_n x) \qquad (1)$$

where $\lambda_n = (n+\frac{1}{2})\pi/h$, and h is the depth.

The pressure impulse decays exponentially with distance x from the wall, so the impulse near the wall is insensitive to the shape of the wave tail. The result of a large variation in shape can be shown explicitly for a triangular wave whose free surface is a 45° diagonal extending from a point on the wall to the bed. The impulsive pressures in that case are about one half that for the semi-infinite wave of figure 1.

Returning to the wave in figure 1 the pressure impulse at the wall is, from (1),

$$P(0,y) = \sum_{n=0}^{\infty} \frac{2\rho U_0}{h\lambda_n^2} (-1)^n(1-\cos \lambda_n \mu h) \cos \lambda_n (y+h) \qquad (2)$$

and in figure 2 this is drawn for several values of μ. The peak shock pressures at the wall can be estimated from (2) by dividing it by a suitable constant time interval, Δt, corresponding to "the pressure rise time" of experimental measurements, and is less than a millisecond for waves of height $0(\frac{1}{2}h) = 0(10cm)$. So each curve in figure 2 is also a (scaled) illustration of the peak pressure on the wall.

A suitable choice of Δt can be made by estimating the time it takes a sound wave to travel from the wall a distance L into the fluid, where L is the "momentum length" defined by

$$L\mu h U_0 = I_w \qquad (3)$$

where

$$I_w = \int_{-h}^{0} P(0,y)dy \qquad (4)$$

is the impulsive force on the wall, and equals the momentum lost by the fluid. So $\Delta t = L/c$ where c is speed of sound in the liquid.

The impulsive force is plotted in figure 3, as a function of μ and the impulse due to a triangular wave (of height h) is shown for comparison.

Experimentally measured peak shock pressures are too widely scattered to make convincing comparisons, but where pressure and force impulses have been recorded, expressions (2) and (4)give good agreement. The qualitative variation of pressure in figure 2 accords with measured impulsive pressures. See figure 5 in Partenscky and Tounsi (1989), and figure 5 of Denny (1950). In particular note that significant impulsive pressures act all the way down to the bed.

Given the uncertainty of measuring peak pressures more elaborate theories than this are hardly worth the extra effort. Further work is in progress to evaluate the effect of changing the geometry of the wall and wave near the impact zone in 2D and 3D. Boundary integral schemes have also been used by the authors to compute wave motion up to the moment of impact (see Peregrine and Cooker abstract) and further studies aim to relate the two theories.

ACKNOWLEDGEMENT

Support for M Cooker from SERC is acknowledged.

REFERENCES

BAGNOLD, R A (1938-39) Interim report on wave-pressure research. *J Inst Civil Engineers*, **12**, 201-226 + 4 plates.

BATCHELOR, G K (1973) An introduction to fluid dynamics. *Camb Univ Press.*

DENNY, D F (1950) Further experiments on wave pressures. *J Inst Civil Engineers*, **35**, 330-345.

KIRKGÖZ, M S (1982) Shock pressure of breaking waves on vertical walls. *Proc ASCE.* WW1, **108**, 81-95.

LAMB, H (1932) Hydrodynamics. *Camb Univ Press.* 6th edition.

MITSUYASU, H (1962) Experimental study on wave force against a wall. Report N°47. Transport Tech Res Inst, Unyu-gijutsu Kenkyujo Mejiro, Tokyo, Japan. 39pp.

NAGAI, S (1960) Shock pressures exerted by breaking waves on breakwaters. *Proc ASCE.* WW2 **86**, 1-38.

PARTENSCKY, H W and TOUNSI, K(1989) Theoretical analysis of shock pressures caused by waves breaking at vertical structures. Int Assoc for Hydraulic Res XXIII Congress, Ottawa, Canada. Aug 1989. 6pp.

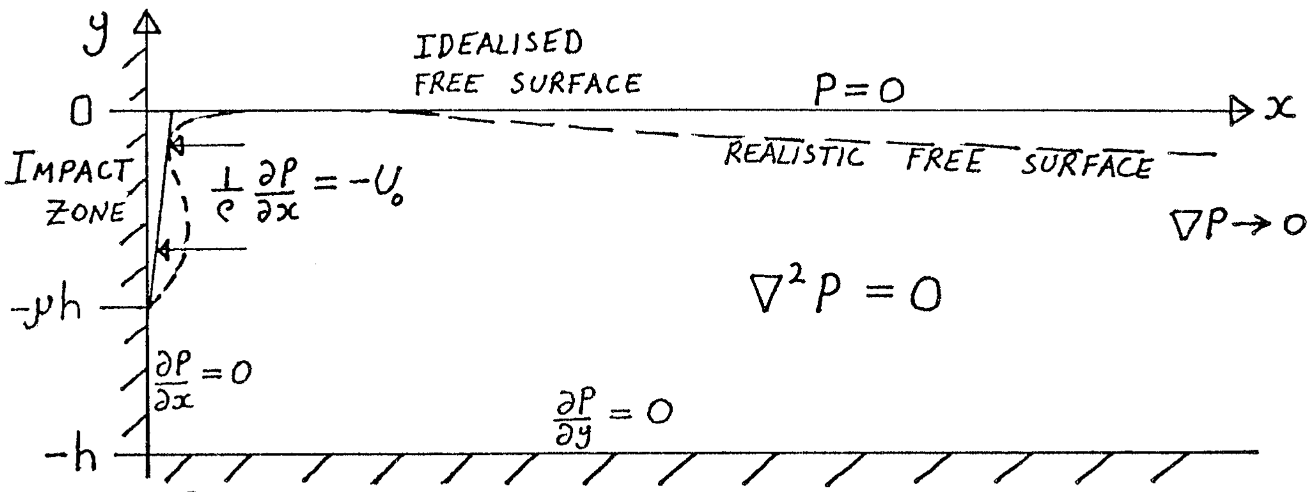

FIGURE 1.

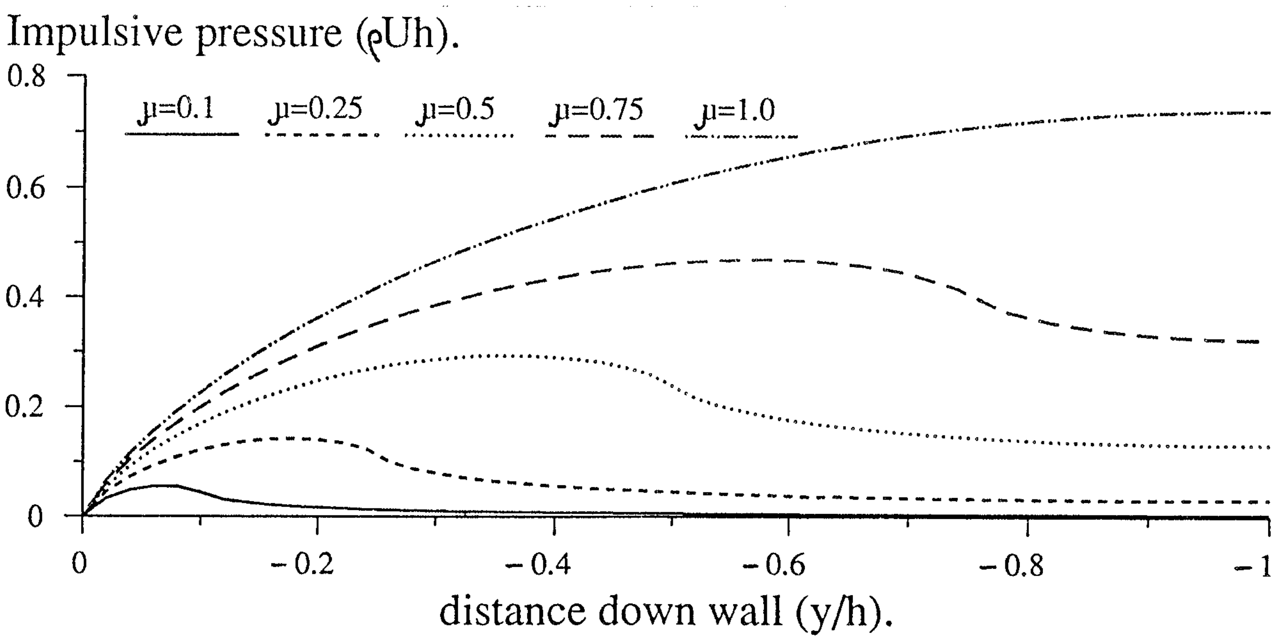

Figure 2: Impulsive pressures at the wall due to wave in fig. 1. Waterline is at y=0 and bed is at y/h= -1.

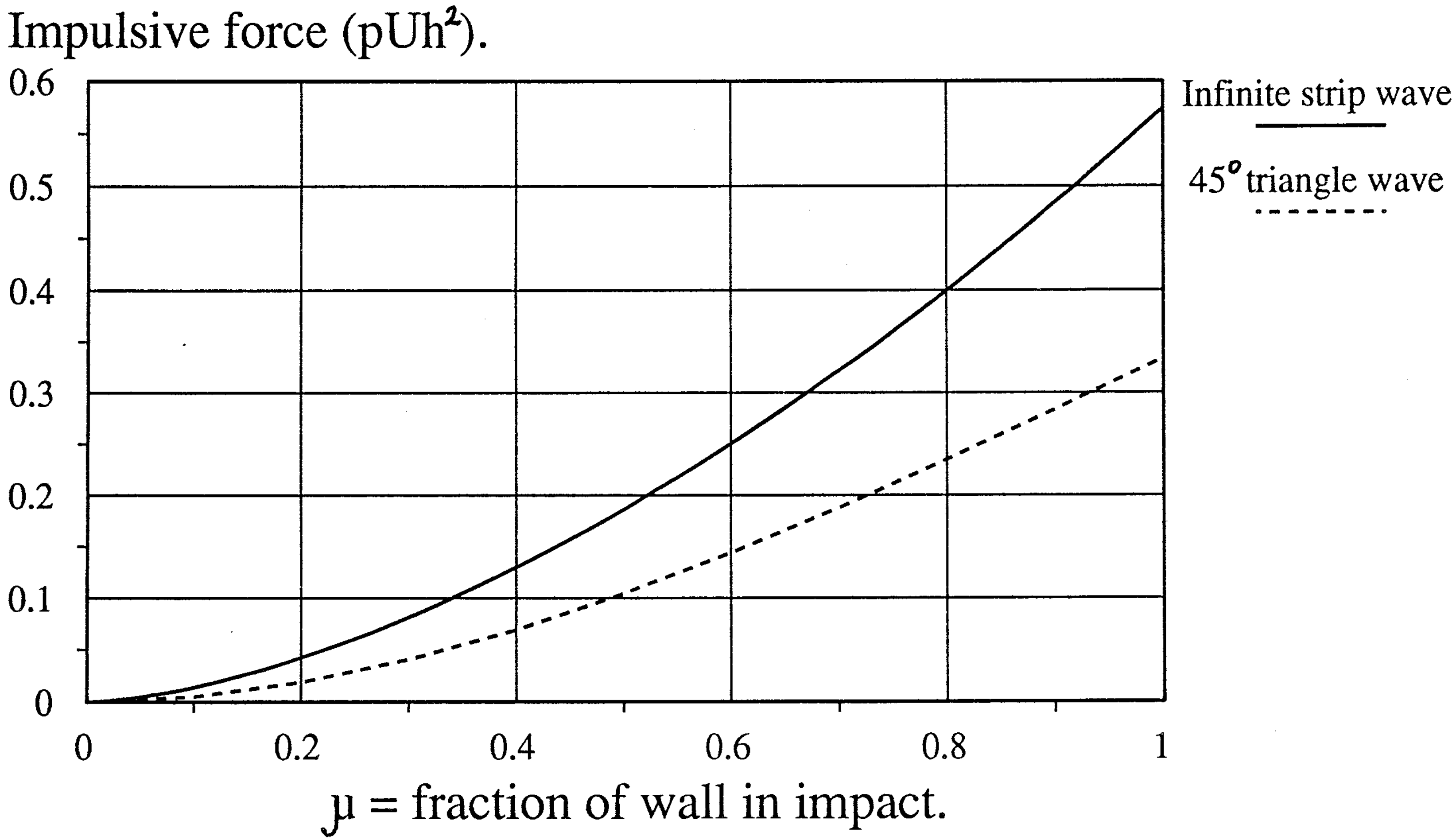

Figure 3: Impulsive force on vertical wall due to wave in fig.1 and triangular wave of height h.

DISCUSSION

Schultz: Rather than using $p = 0$ for the entrapped bubble, it might seem appropriate to set $p = C$, with C determined from the intersection point. Would this alter any of your results qualitatively?

Cooker: If we have an air bubble below the impact zone with boundary condition $p = C = 0$, I would expect the peak pressure distribution on the wall to be concentrated within the impact zone, and to be very small (at the wall) *below* the bubble. If $p = C > 0$, this is like Bagnold's air cushion model, in which the wave face is halted by air compression. Finding an appropriate value of C to use in the boundary condition is equivalent to putting the air cushion inside my impact zone. So, for *appropriate* values of C in the boundary condition, I would not expect *qualitatively* different results to those presented.

Anderson: You showed a slide with four different published results. Why is the one due to Nagai (1960) such that the peak pressure does not tend to zero at the bed, whereas all the others have non-zero pressures all the way down to the bed?

Cooker: I think that this can be explained theoretically. If I had included an air bubble below the impact zone, the pressure impulse (and hence the peak pressure) would be concentrated in the impact zone. Below the bubble, the theoretical calculation will give a very small pressure impulse. So, I think Nagai's experiments had wave impact with an air bubble (uncompressed) beneath the overturning surface.