Hydrodynamic Impact Analysis of a Cylinder: Modelling the Jet

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Introduction

In a previous paper (Cointe and Armand, 1987), the hydrodynamic impact problem was addressed for a circular cylinder. The method of matched asymptotic expansions was used. The small parameter was the ratio of the penetration depth to the wetted width. An outer solution was found on a length scale equal to the wetted width of the cylinder. An inner solution with a jet was found in the vicinity of the spray root, i.e. the intersection of the outer free surface with the impacting body. Matching of the two solutions yielded the thickness of the jet. This solution is very general and can be extended to a wide range of geometries, including the wedge. The method of matched asymptotic expansions allows the result of Wagner (1932) concerning a wedge with small deadrise angle to be recovered.

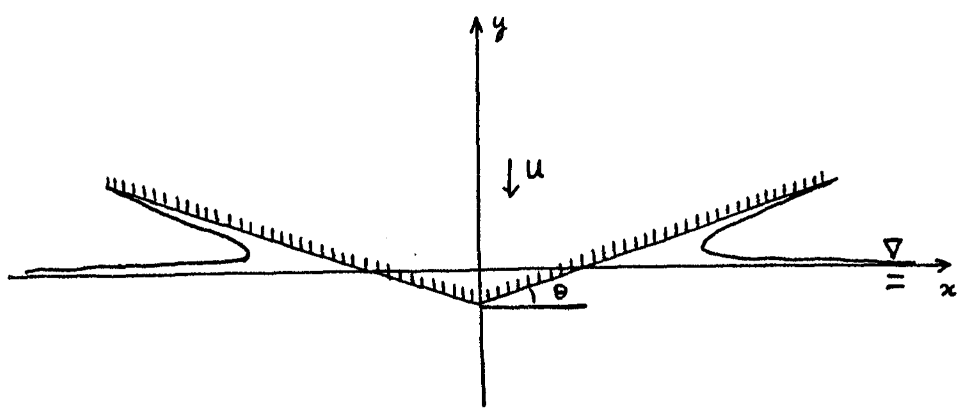
In this case, and in the absence of gravity, the resulting flow is self-similar. A consequence is that the length of a free surface arc should remain constant. In the inner domain, the jet has a constant thickness and an infinite length. The length conservation property is therefore violated by the solution resulting from the matching of the outer and inner solutions.

In order to overcome this difficulty, it is suggested that a third solution has to be introduced, *the jet solution*. This jet solution is matched to the inner solution which is itself matched to the outer solution.

The method allows the size of the respective domains and the corresponding equations to be determined. It should be very useful as a guide for future numerical computations. The procedure is outlined is this paper in the case of the wedge. When the flow can be regarded as self-similar, an analytical solution is found. The extension to a flow that is not self-similar is discussed.

Equations of the problem

We assume that the fluid is incompressible and that the flow is irrotational. The wedge is supposed to be rigid and is moving downward at the constant velocity U. The x-axis coincides with the undisturbed free surface. The y-axis is the axis of symmetry of the wedge.



If the deadrise angle θ is small, the wetted width is of order Ut/θ. This quantity is taken as length scale. If the velocity scale is U, the corresponding non-dimensional equations for the velocity potential ϕ (which is harmonic in the fluid domain) and the free surface position are:

$$\frac{g\,t}{U} y + t\,\phi_t + \phi - x\,\phi_x - y\,\phi_y + \frac{1}{2}\theta\,\nabla\phi \,.\, \nabla\phi = 0$$

$$t\,S_t + \theta\,S_x\,\phi_x + \theta\,S_y\,\phi_y - x\,S_x - y\,S_y = 0 \qquad \text{for} \qquad S\,(x,y,t) = 0$$

$$\phi_x \sin\theta - \phi_y \cos\theta = \cos\theta \qquad \text{for} \qquad 0 < x < \lambda,\ (y+\theta) = x\tan\theta$$

$$\phi_x = 0 \qquad \text{for} \qquad x = 0,\ y < -\theta$$

Note that due account has been taken of the fact that the length scale is time dependant.

Outer Solution

We now assume that the deadrise angle, θ, is small. In the outer domain, the following expansions are then assumed:

$$S\,(x,y,t) \approx -\,y + \theta\,\eta\,(x,t) + o\,(\theta)\,, \quad \phi \approx \Phi + o\,(1)$$

At the leading order in θ, the equations of the problem in the outer domain become:

$$\theta\,\frac{g\,t}{U}\eta + t\,\Phi_t + \Phi - x\,\Phi_x = 0$$

$$t\,\eta_t - \Phi_y - x\,\eta_x + \eta = 0 \qquad \text{for} \qquad x > \lambda,\ y = 0$$

$$\Phi_y = -\,1 \qquad \text{for} \qquad 0 < x < \lambda,\ y = 0$$

$$\Phi_x = 0 \qquad \text{for} \qquad x = 0,\ y < 0$$

If $t \ll U/g\theta$, a self-similar outer solution, here a solution independant of time in non-dimensional variables, can be found. The potential Φ corresponds to the unbounded flow around a flat plate of half width λ. The free surface elevation and the wetted width are obtained by solving the kinematic free surface boundary condition (see Cointe and Armand, 1987). This gives:

$$\lambda = \frac{\pi}{2}\,, \quad \eta = \{-1 - \frac{x}{\lambda}\,(\arccos\frac{\lambda}{x} - \lambda)\}$$

Inner Solution

This solution is singular near the spray root ($x = \lambda$, $y = \theta(\lambda-1)$), i.e. the intersection of the (outer) free surface and the wedge. This stems from the fact that the asymptotic process is incorrect near this point. An inner solution can be found by introducing the inner variables:

$$\theta^2 x^* = x - \lambda\ ,\ \ \theta^2 y^* = y - \theta(\lambda-1)\ ,\ \ \theta\, \phi^* = \phi\ ,\ t^* = t\ ,$$

the free surface being given in the inner domain by:

$$S^*\ (x^*,y^*,t) = 0$$

At the leading order in θ (still assuming $t << U/g\theta$), the inner domain equations are:

$$-\lambda\, \phi^*_x + \frac{1}{2} \nabla\phi^* \,.\, \nabla\phi^* = 0$$

$$S^*_x\, \phi^*_x + S^*_y\, \phi^*_y - \lambda\, S^*_x = 0 \qquad \text{for} \qquad S^*\ (x^*,y^*,t) = 0$$

$$\phi^*_y = 0 \qquad \text{for} \qquad y^* = 0$$

A solution to this problem can be found with a jet of constant thickness δ^* in which the velocity is 2λ, i.e. twice the horizontal velocity of the spray root (see Cointe and Armand, 1987). Matching with the outer solution yields the thickness of the jet,

$$\delta^* = \frac{\pi}{8\,\lambda} = \frac{1}{4}$$

This result agrees with that of Wagner (1932).

Jet Solution

If the outer and inner solutions are matched, we obtain a jet going all the way to infinity. As stated above, this violates the arc length conservation property that the self-similar solution should satisfy. Since the shortening of the outer free surface is of order 1 and the thickness of the inner jet is of order θ^2, we expect to find a jet along the wedge boundary on a length scale equal to 1 and a thickness scale equal to θ^2. We, therefore, define the new variables:

$$s = \cos\theta\ x + \sin\theta\ (y+\theta)\ ,\ \ \theta^2\, n = -\sin\theta\ x + \cos\theta\ (y+\theta)\ ,\ \ \varphi = \theta\,\phi - \theta^3\, n$$

the free surface being given in these new variables by $n = h\ (s,t)$.

At the leading order in θ (still assuming $t << U/g\theta$), Laplace's equation and the body boundary condition yield $\varphi = \varphi\ (s)$. The free surface boundary condition now yields:

$$t\,\varphi_t + \varphi - s\,\varphi_s + \frac{1}{2}\,\varphi_s^2 = f\ (t)$$

$$t\, h_t + h_s\, \varphi_s - s\, h_s + h = 0$$

Matching with the inner solution implies that

$$\varphi_s(\lambda) = 2\lambda \text{ and } h(\lambda) = -\delta^*.$$

This finally gives the solution

$$\varphi_s = 2\lambda \text{ and } h = -\delta^*(2 - s/\lambda).$$

According to this model, the velocity in the jet is constant while the thickness is linearly decreasing with the distance from the spray root. As a consequence, the length of the jet is equal to λ. Note that this is just equal, at leading order in θ, to the shortening of the outer free surface. The arc length conservation property is therefore satisfied by the present composite solution.

A rather interesting consequence is that the intersection angle between the jet and the wedge is simply given by :

$$\tan\beta = \theta^2 \frac{\delta^*}{\lambda} = \frac{\theta^2}{2\pi}$$

In particular, it goes to zero with the deadrise angle, in agreement with numerical results obtained by Dobrovol'skaya (1969) but in disagreement with the 9.5° limit obtained analytically by Johnstone and Mackie (1974). It should be noted, however, that the jet model might not be relevant to a local analysis near the tip.

Conclusion

The impact problem has been discussed in the case of a wedge with small deadrise angle. The jet appearing in the vicinity of the spray root has been described by equations similar to those of Saint-Venant (shallow water equations) in dimensional variables. Boundary conditions for these equations are provided by matching the inner solution, valid in the vicinity of the spray root, and the outer solution. The problem has been solved in the case of a self-similar flow. The outer equations could however be solved numerically in the general case (given initial conditions, non-constant impacting velocity, large times, etc...). As long as the inner solution remains the same, the jet equations could then be solved using as boundary conditions the jet thickness and the jet velocity resulting from the matching between the outer and inner solutions. In particular, this should allow the development of the jet to be studied.

References

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