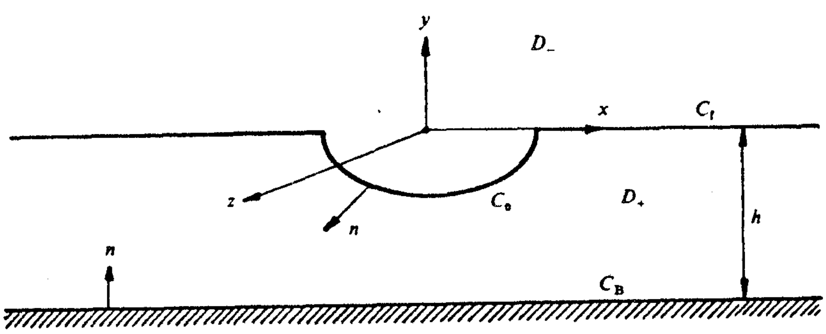
Some Questions in Optimal Design of Floating Bodies

T. S. Angell and R. E. Kleinman
Center for the Mathematics of Waves
Department of Mathematical Sciences
University of Delaware
Newark, Delaware 19716

We consider the problem of a floating body, partially submerged in an infinite, ideal, inviscid, irrotational fluid, subjected to periodic displacement or rotation. Our interest is not primarily in determining the velocity of the fluid induced by the oscillatory ship motion but rather in determining hull forms which optimize functionals of the velocity. In particular our goal is to find a constructive method for finding a shape in a constrained family of admissible surfaces which optimizes added mass or damping.

In this paper we describe such a method and indicate the mathematical questions that remain to be answered in order to prove existence of optimal shapes. However these mathematical results are not needed in order to implement the method numerically.

In order to set the problem recall the mathematical formulation of the floating body problem: Orient a Cartesian coordinate system with origin on the projection of the wetted portion of the hull, Γ, on the free surface (see figure). We seek a velocity potential ϕ which satisfies Laplace's equation $\nabla^2\phi = 0$ in the fluid domain, a radiation condition at infinity, the linearized free surface condition $\frac{\partial\phi}{\partial y} - K\phi = 0$ on $y = 0$, the boundary condition $\frac{\partial\phi}{\partial n} = G$ on Γ and, in the case of finite depth, $\frac{\partial\phi}{\partial n} = 0$ on the sea floor.



The problem may be cast in two dimensions (in which case Γ is a curve) or in three dimensions (in which case Γ is a surface), and either finite or infinite depth. The radiation condition takes the form $\rho^{\frac{n-2}{2}}\left(\frac{\partial\phi}{\partial\rho} - iku\phi\right) = o(1)$ as $\rho \to \infty$ where for $n = 2$, $\rho = x$, whereas for $n = 3$, $\rho = \sqrt{x^2+z^2}$. For infinite depth $k_0 = k$ while for finite depth $(y = -h)$, k_0 is the root with largest real part of the equation $k_0 \sinh k_0 h = K \cosh k_0 h$.

The problem we consider is that of finding Γ in some suitable class which optimizes some functional of the solution of this problem. We choose to concentrate attention on the added mass and define the cost functional

$$L(\phi_{\Gamma,G}) := \mathrm{Re} \int_\Gamma \phi_{\Gamma,G}\, G \, d\Gamma$$

where $\phi_{\Gamma,G}$ is the solution of the floating body problem with normal derivative G on Γ. Our goal is to find a surface Γ which optimizes this functional without solving a succession of direct floating body problems for different surfaces. The underlying mathematical question is whether we can choose a family of admissible surfaces, which is large enough to be of physical interest, and which contains an optimizer of the cost functional, that is, a surface Γ_0 such that

$$L(\phi_{\Gamma_0,G}) \leq L(\phi_{\Gamma,G})$$

for all admissible Γ. In that case the optimization problem is said to be solvable.

Our approach is guided by our work on the similar problem for the fully submerged body. The two problems ar closely related but have significant differences. First of all, the class of surfaces for which either problem may be shown to be uniquely solvable is restricted geometrically. For the floating body the restrictions under which John [3] originally established uniqueness have been relaxed somewhat (Kleinman [4], Simon and Ursell [7]) while for the submerged body, entirely different restrictions were needed by Maz'ja [6] (see also Hulme [2]) to establish uniqueness for solutions of the boundary value problem. These restrictions must be incorporated into the class of admissible surfaces for the optimization problem. Another difference in the two problems arises from the integral equation formulation. For the submerged body the use of Green's theorem using John's Green function, which satisfies the free surface condition, leads to a uniquely solvable integral equation. For the floating body, the same procedure leads to an integral equation which exhibits the well known irregular frequency malady, frequencies at which the integral equation is not uniquely solvable. This uniqueness problem is intrinsic to the integral equation formulation and is not to be confused with the uniqueness questions for the boundary value problem. The integral equation formulation plays a key role in establishing the existence of a solution of the optimization problem. The idea is to show that the functional varies continuously in some appropriate sense with changes in the boundary Γ. Then by restricting Γ to lie in a compact set the functional will be continuous on a compact set and hence will assume its minimum (and maximum) on the set. In the submerged case the uniquely solvable integral equation was used to establish this desired continuity. For the floating body, a uniquely solvable integral equation is needed and is available e.g. Kleinman [4], Wienert [9], however the equation is more complicated than that used in the submerged case in order to eliminate the occurrence of irregular frequencies. The analysis required to establish continuity of the functional is yet to be completed.

However, the constructive optimization method relies not on the integral equation formulation but on the availability of a complete family of solutions by which we mean a family of functions $\{u_j\}_{j=0}^{\infty}$ such that $\nabla^2 u_j = 0$ exterior to every surface in the admissible class, and u_j satisfies the free surface condition, and the radiation condition, the boundary condition on the sea floor in the finite depth case. Moreover the normal derivatives of u_j are to be complete and linearly independent on every surface Γ in the admissible class, that is, if $(u, u_j)_{L^2(\Gamma)} = 0$ for every j, then $u = 0$, where $(\cdot, \cdot)_{L^2(\Gamma)}$ denotes the L^2 inner product on Γ. The existence of such a family in the submerged case has been shown, Angell and Kleinman [1], while for the floating body case such a family is available in the form of Ursell's multiple potentials [8] whose completeness properties were established by Martin [5].

With the availability of such a family a sequence of penalized finite dimensional optimization problems may be defined as follows: Find $c_j^{(N)}$ and Γ to minimize

$$L_\nu^{(N)}[c_j^{(N)},\Gamma] := \operatorname{Re} \int_\Gamma \sum_{j=0}^{N} c_j^{(N)}\, u_j(P)\, G(P)\, d\Gamma_P + \nu \int_\Gamma \left| \sum_{j=0}^{N} c_j^{(N)} \frac{\partial u_j(P)}{\partial n} - G(P) \right|^2 d\Gamma_P\,.$$

In order to solve this nonlinear optimization problem the set of admissible surfaces Γ must be carefully defined and it will prove convenient to transform the integrals to integrate over a reference surface introducing the unknown Γ in the Jacobian of the transformation.

A precise characterization of the admissible surfaces and the way in which solutions of the finite dimensional optimization problems converge to solutions of the original optimization problem will be presented.

References

1. T. S. Angell and R. E. Kleinman, A constructive method for shape optimization: a problem in hydromechanics, to appear.
2. A. Hulme, Some applications of Maz'ja's uniqueness theorem to a class of linear water wave problems, *Math. Proc. Comb. Phil. Soc.* **95** (1984), 165–174.
3. F. John, On the motion of floating bodies II, *Comm. Proc. Appl. Math.* **3** (1950), 45–101.
4. R. E. Kleinman, On the mathematical theory of the motion of floating bodies – an update, David W. Taylor Naval Ship Research and Development Center Report DTNSRDC–82/074, Bethesda, MD.
5. P. Martin, On the null–field equations for water–wave scattering problems, *IMA Journ. Appl. Math.* **33** (1984), 55–69.
6. V. G. Maz'ja, Solvability of the problem on the oscillations of a fluid containing a submerged body, *Journ. Soviet Math.* **10** (1978), 86–89.
7. N. J. Simon and F. Ursell, Uniqueness in linearized two–dimensional water–wave problems, *J. Fluid Mech.* **148** (1984), 137–154.
8. F. Ursell, On the heaving motion of a circular cylinder on the surface of a fluid, *Q. Jour. Mech. Appl. Math.* **2** (1949), 218–231.
9. L. Wienert, An existence proof for a boundary–value problem with non–smooth boundary from the theory of water waves, *IMA Journ. Appl. Math.* **40** (1988), 95–112.

DISCUSSION

Pesce: I would like to add three comments on your interesting lecture. First, regarding the 'cost functional' to be chosen, it seems to me that exciting forces or even first-order motions should also be considered in a general problem.

Second, regarding irregular frequencies, I should mention that the problem can be formulated in a weak sense; it can be proved that there are no irregular frequencies with this formulation.

Finally, regarding the choice of trial functions, can the requirement of completeness be somewhat relaxed, provided the functions are chosen to imitate properly the hydrodynamic problem? Examples are the elementary trial functions (poles, dipoles, lines of dipoles and, particularly, vortex rings) mentioned in my lecture [Pesce & Aranha, these Proceedings]. Could not completeness be sacrificed in order to achieve more rapid convergence?

Kleinman: Certainly, other cost functionals could be considered. Added mass was merely an example. Using the weak formulation rather than the integral equation is an interesting idea. One would have to investigate whether this would simplify the proof of continuity of the functional with respect to the surface. As far as other choices of the expansion functions, I agree that there are many possibilities. However, I do not think that it is necessary to sacrifice completeness. Terms of the sort you suggest could be included in the family but completeness is necessary in order to prove convergence of solutions of the finite-dimensional optimization problem. In practice, of course, only a finite number of members of the family will ever be employed.

Martin: You are working at one fixed value of K. Can you say anything about your optimal surface Γ_0 as a function of K?

Kleinman: What I spoke about was indeed a cost functional for one fixed value of K. However, nothing would change (except the complication of the finite-dimensional optimization problem) by taking the cost functional to be a sum of terms of the type discussed for different values of K. Alternatively, the cost functional could be taken to be an integral over K, but in that case continuity with respect to the surface would have to be reexamined.