HYDRODYNAMIC FORCES ON A SUBMERGED SPHERE MOVING IN A CIRCULAR PATH

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1. INTRODUCTION

An ocean-going ship normally follows a straight line course. Cases, however, do arise when a ship has to follow a curved path, as for example during manoeuvring or navigation in restricted waters. This leads to some difficulties in obtaining the mathematical solution of the flow around a ship, even for the well known linearized potential theory. In particular, the problem associated with a ship moving in a curved line is time-dependent. The solution at each stage depends on the entire history before that. One special case is that of a ship moving in a circular path at constant angular velocity. This leads to a drastic simplification of the mathematical solution, in that the problem becomes steady after a sufficient length of time. Apart from mathematical simplification, this case is a useful basis for estimating steady turning radius of the ship and gives some insights into the general solution of a ship moving in a curved path. Havelock (1950) analysed this problem by considering a submerged sphere. He used a dipole without free surface effects to represent the sphere, in an initial value formulation in the time domain, and then took the time limit to infinity. He derived an equation for calculating the wave resistance and radial force. The problem was also considered by Sretenskii (1957) in a slightly different manner. While their investigations were limited to a certain degree of approximation, the present work tries to obtain the exact solution of the linearized potential problem of a submerged sphere moving in a circular path. A distribution of sources over the body surface is expanded into a series of Legendre functions. The governing equations are satisfied by use of the appropriate Green function and by choosing the coefficients in the series of Legendre functions.

2. THE GOVERNING EQUATIONS

To describe the problem, various coordinate systems are shown in figure 1. O_0-$x_0y_0z_0$ is fixed in the space with the origin on the undisturbed free surface and z pointing upwards; O-xyz rotates about point O_0 at the same angular velocity Ω as the sphere. The polar coordinate system $(\bar{\omega},\beta,z_0)$ is defined so that β=0 indicates the ray O_0O. It is apparent that the problem becomes steady in the polar coordinate system as the time tends to infinity. Based on the linearized potential theory, we have the following governing equations for the velocity potential

$$\nabla^2\phi = 0 \tag{1}$$

in the whole fluid domain;

$$\frac{\partial^2\phi}{\partial\beta^2} + \frac{g}{\Omega^2}\frac{\partial\phi}{\partial z} = 0 \tag{2}$$

on the free surface z=0, where g is the gravitational acceleration;

$$\nabla\phi = 0 \qquad z\to-\infty; \tag{3}$$

$$\frac{\partial\phi}{\partial n} = \Omega(x+d)n_y - \Omega y n_x \tag{4}$$

on the body surface S_0, where d is the distance between O_0 and O, n is the inward normal of the body, and n_x and n_y are its components in the x and y directions respectively.

To obtain the solution of the above equations, we write the potential using a source distribution $\sigma(\xi,\eta,\zeta)$ over the body surface

$$\phi = \int_{S_0} G(x,y,z,\xi,\eta,\zeta)\sigma(\xi,\eta,\zeta)\, dS \tag{5}$$

where G is the Green function satisfying equations (1), (2), (3) and the appropriate radiation condition. It can be derived as (Havelock 1950)

$$G = \frac{1}{R} - \frac{1}{R_1} + 4\sum_{s=1}^{\infty} pv\int_0^\infty \frac{ke^{k(z_0+\zeta_0)}}{k-s^2\Omega^2/g} J_s(k\bar\omega)J_s(k\bar\omega_0)\cos s(\beta-\beta_0)dk$$
$$- \frac{4\pi\Omega^2}{g}\sum_{s=1}^{\infty} s^2 J_s(s^2\Omega^2\bar\omega/g)J_s(s^2\Omega^2\bar\omega_0/g)\exp[s^2\Omega^2(z_0+\zeta_0)/g]\sin s(\beta-\beta_0) \tag{6}$$

where pv indicates the principal integration, $(\bar\omega_0,\beta_0,\zeta_0)$ is the position of the source, $J_n(x)$ is the Bessel function, and R and R_1 are distances from the field point $(\bar\omega,\beta,\zeta)$ to the source and its mirror image about the free surface respectively.

We may then expand the Green function and $\sigma(\xi,\eta,\zeta)$ into series of Legendre functions in the spherical coordinate system

$$x = r\sin\theta\cos\phi \tag{7a}$$
$$y = r\sin\theta\sin\phi \tag{7b}$$
$$z = r\cos\theta - h \tag{7c}$$

where h is the distance between the centre of the sphere and the free surface. Details of the solution procedure are similar to those given by

Wu & Eatock Taylor (1988) in an analysis of the submerged sphere advancing in a straight line in waves. After the potential has been found the hydrodynamic pressure can be obtained from

$$p = -\rho(-\Omega\frac{\partial\phi}{\partial\beta} - \frac{1}{2}\nabla\phi\nabla\phi) , \qquad (8)$$

where ρ is the density of the fluid; whereas the forces F_i can be obtained by integrating the pressure over the body surface. Figure 2 gives the hydrodynamic forces on the sphere submerged at h=2a and moving in circles with d=2a, 5a, 8a and ∞ respectively, as a function of Froude number. The forces are nondimensionalized as $f_i = F_i/\rho g(4/3)\pi a^3(a/h)^3$ in a similar manner to Havelock. The results corresponding to $d=\infty$ are obtained from Wu & Eatock Taylor (1988). As d increases, we find that the result tends to that of straight line motion, where the radial force becomes zero. From the figure it can be seen that the forces oscillate with Fn; but the oscillation decreases as the radius of the circle increases. It seems remarkable that for each radius of the path taken by the sphere, the mean of the tangential forces lies close to the case of straight line motion.

3. Conclusions

The hydrodynamic problem of a submerged sphere moving in a circular path is solved based on the linearized velocity potential theory. It is found that the radius of the circle has a significant effect on the hydrodynamical forces on the sphere. The present method can be further used to extend Havelock's approximation for the submerged spheroid (1950), by replacing equation (7) by spheroidal coordinates (Wu and Eatock Taylor 1987,1989). Existing numerical methods could be used with the Green function in equation (6) to analyse the problem of an arbitrary surface ship, if one could assume that the linearized potential theory remains valid in such a case.

References

Havelock, T. H. (1950) "The forces on a submerged spheroid moving in a circular path" Proc. R. Soc. Lond. A201, 297-205

Sretenskii, L.N. (1957) "Computation of tangential forces of wave resistance of a sphere moving on a circular path" (in Russian) Akad. Nauk SSSR, 3-17

Wu, G.X. & Eatock Taylor, R (1987) "The exciting force of a submerged spheroid in regular waves" J. Fluid Mech. 181, 441-426

Wu, G.X & Eatock Taylor, R (1988) "Radiation and diffraction of water waves by a submerged sphere at forward speed" Proc. R. Soc. Lond. A417, 433-461

Wu, G.X & Eatock Taylor, R (1989) "On the radiation and diffraction of surface waves by submerged spheroids" To appear in J. Ship Res.

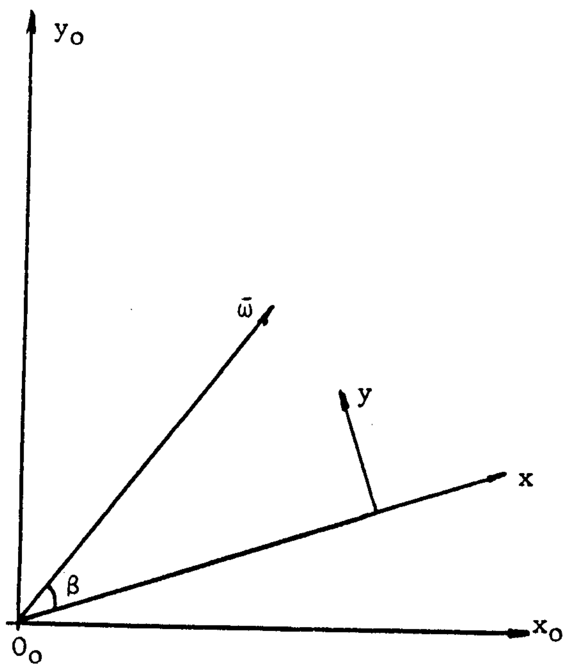

Figure 1

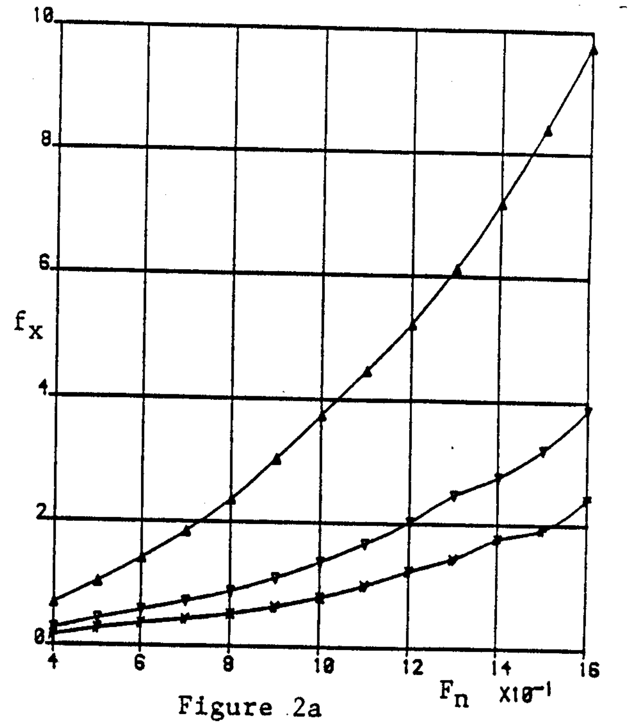

Figure 2a

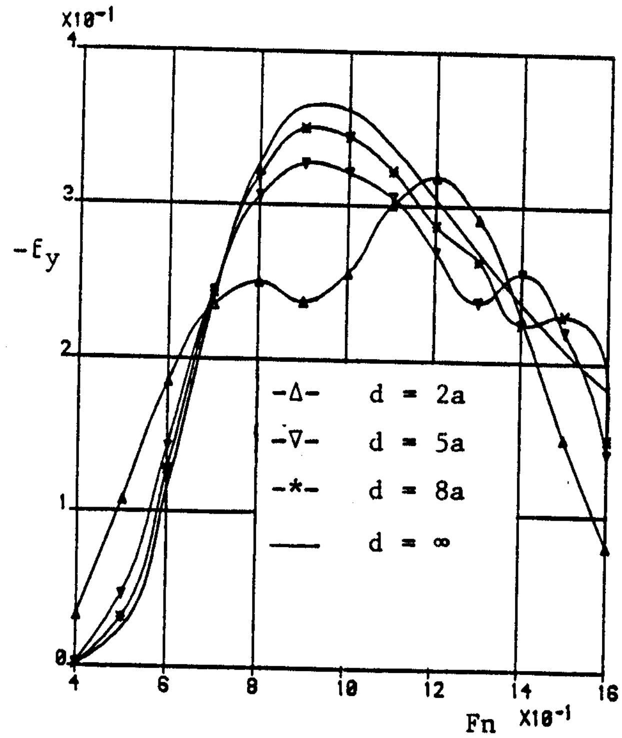

Figure 2b

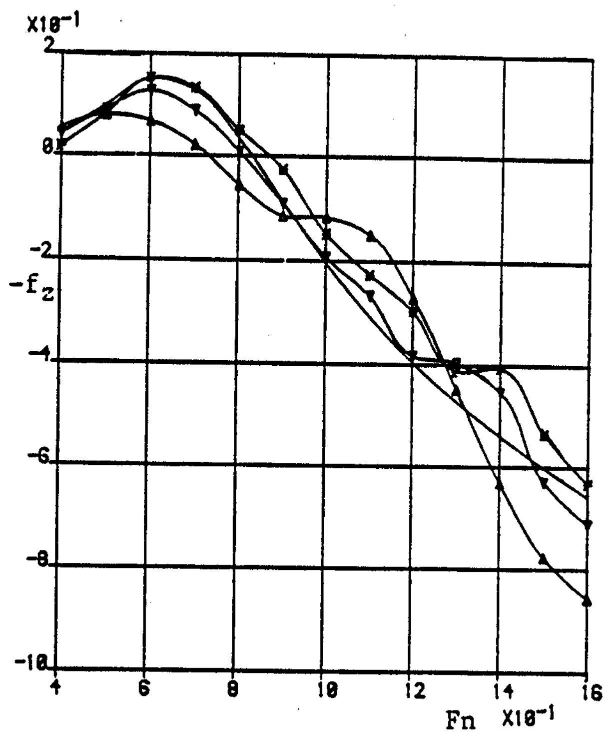

Figure 2c