ACCURACY OF FREE SURFACE CONDITIONS FOR THE WAVE RESISTANCE PROBLEM

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1. INTRODUCTION

The wave resistance of a ship moving steadily in still water is generally calculated from the potential flow about the hull subject to a linearized free surface boundary condition (FSC). This may be either the Kelvin condition or a slow-ship linearized condition such as that adopted by Dawson. Not much is known about the adequacy of various FSC's in practical applications. In the past Workshops on Ship Wave Resistance Computations, differences between FSC's were obscured by numerical errors. More specifically, a fair comparison between the FSC of Dawson and the Kelvin condition was impossible, since the former was implemented using Rankine sources and the latter using Kelvin sources, the two methods thus containing numerical errors of an entirely different nature. The first objective of the present study is therefore, to make such a comparison using exactly the same numerical method, such that differences are directly attributable to the FSC's. This has been achieved by applying our program DAWSON [1], without any change for the (corrected) Dawson condition, and with the double body velocities replaced by a uniform flow for the Neumann-Kelvin problem.

Secondly, it is desirable to collect information on the magnitude of the neglected nonlinear terms as these indicate the adequacy of the linearization and drive the iteration process necessary to solve the exact problem. In addition this may provide some insight in the correctness of certain basic assumptions made in the linearization of the FSC.

For lack of sufficiently detailed and accurate experimental data or solutions of the exact nonlinear problem, in the present study the magnitude of nonlinear terms has been derived from a posteriori estimates, using the flow field calculated with a linearized method. Although of course not altogether reliable this method is easily applied and gives information heretofore unavailable as far as known to this author.

2. RESISTANCE AND WAVE PROFILES

The test cases in this study were: the Wigley hull at Fn = 0.40; the Series 60, Cb = 0.60 hull at Fn = 0.22 to 0.38, and the so-called "strut-like hull" [2], a full form with vertical sides, an elliptic bow and a block coefficient 0.86, at Fn = 0.25 and 0.18.

For the Wigley hull, the Kelvin condition predicts a wave resistance coefficient only 1% higher than the Dawson condition. This is not surprising since the double body flow is almost uniform for this thin ship. Even so, at the 1979 workshop [3] all Neumann-Kelvin predictions for this case, however scattered, were substantially higher than the Rankine source and slow-ship predictions. More relevant is the comparison for the Series 60 ship. Figure 1 shows the wave resistance coefficient, compared with some experimental data. It is evident that the differences between the methods are totally negligible again, except perhaps above Fn = 0.32. Also the predicted wave profiles are in close agreement; the general tendency is, that the Dawson FSC results in somewhat more pointed bow and stern waves. There is little difference in wave amplitude. Both methods underestimate the bow wave height; otherwise the agreement with experimental data is fairly good.

For the strut-like hull, The Neumann-Kelvin solution predicts a 12% lower Cv. This is not much difference in view of the considerable nonuniformity of the double body flow for this full hull form. The Kelvin condition leads to significantly higher waves behind the stern in this particular case.

3. NONLINEAR TERMS

To assess the validity of the linearized FSC's, I have computed both the linear terms included in the FSC and the neglected nonlinear terms. Now the prime difficulty here is that the exact FSC should be applied at the actual free surface, not at the undisturbed free surface y = 0. This was partially overcome by using Taylor expansions for the transfer of the boundary condition to y = 0, just as is done in the derivation of the FSC itself. However, the convergence of such expansions for the short wave disturbance at low Fn is disputable.

The nonlinear terms can be formulated consistently, including only the contributions of leading order in the perturbation parameter, or inconsistently so as to approximate the exact FSC as closely as possible. For the strut-like hull at Fn = 0.25 this gives important differences, indicating a poor convergence of the perturbation expansions. For the other cases the consistent and inconsistent forms give similar results.

For the slow-ship FSC with the basic assumptions according to Eggers (see [1] for a further discussion) the consistent terms of $O(Fn^6)$ are complicated expressions containing third derivatives of the double-body potential. These are hard to calculate and susceptible to numerical oscillations. Therefore, a simplified form was derived by dropping the transfer terms connected with the double body potential. This is not just a matter of convenience; it can be argued that the extension of the double-body flow field above the undisturbed water plane, based on its assumed symmetry, only reduces the accuracy of the approximation, in particular for hulls with strongly flared sections at the water line.

We now define the following decomposition:

$$\phi_y = \underbrace{\Phi_x\eta_x + \Phi_z\eta_z}_{(1)} + \underbrace{\phi'_x\eta_{rx} + \phi'_z\eta_{rz}}_{(2)} + \underbrace{\phi'_x\eta'_x + \phi'_z\eta'_z}_{(3)} + \underbrace{\left(\Phi_x\frac{\partial}{\partial x} + \Phi_z\frac{\partial}{\partial z}\right)\left(\bar{\eta} - \eta\right)}_{(4)} +$$

$$+ \underbrace{\left(\Phi_x\frac{\partial}{\partial x} + \Phi_z\frac{\partial}{\partial z}\right)\left(\eta^* - \bar{\eta}\right)}_{(5)} + \underbrace{\eta_r\left(\eta_{rx}\phi'_{xy} + \eta_{rz}\phi'_{yz}\right) - \tfrac{1}{2}\eta_r^2\phi'_{yyy}}_{(6)} +$$

$$- \underbrace{\eta'\phi'_{yy} - \eta_r\phi'_{yy}}_{(7)} + O\left(Fn^8\right)$$

where:

$$\eta_r = \tfrac{1}{2}Fn^2\left(1 - \Phi_x^2 - \Phi_z^2\right) \qquad \bar{\eta} = \eta - \tfrac{1}{2}Fn^2\left(\phi_x'^2 + \phi_y'^2 + \phi_z'^2\right)$$

$$\eta' = \eta - \eta_r = -Fn^2\left(\Phi_x\phi'_x + \Phi_z\phi'_z\right) \qquad \eta^* = \bar{\eta} - Fn^2\eta_r\left(\Phi_x\phi'_{xy} + \Phi_z\phi'_{yz}\right)$$

Φ = double body potential $\qquad$ ϕ' = perturbation potential

Here, terms 1 and 2 are normally included in Dawson's FSC. Terms 3 and 4 are nonlinear contributions to the FSC, while terms 5 and 6 result from the transfer to y = 0. Term 7 is neglected in Dawson's FSC but should be added for consistency.

Similarly, for the Kelvin FSC the perturbation parameter is the wave steepness; the consistent decomposition is:

$$\phi_y = \underbrace{\eta_x}_{(1)} + \underbrace{\phi'_x\eta_x + \phi'_z\eta_z}_{(3)} + \underbrace{\left(\bar{\eta}_x - \eta_x\right)}_{(4)} + \underbrace{\left(\eta^*_x - \bar{\eta}_x\right)}_{(5)} - \underbrace{\eta\phi'_{yy}}_{(6)} + O\left(\varepsilon^3\right)$$

with:

$$\eta = -Fn^2\phi'_x$$

$$\bar{\eta} = \eta - \tfrac{1}{2}Fn^2\left(\phi_x'^2 + \phi_y'^2 + \phi_z'^2\right)$$

$$\eta^* = \bar{\eta} - \eta\, Fn^2\phi'_{xy}$$

For the double body flow replaced by a uniform flow, the expressions for both FSC's become equal except η^* and term 5. As a result, in the consistent form these quantities are not completely comparable for both methods.

Evaluation of these decompositions for all test cases of course generated a massive amount of data, only some of which can be included here. Concerning the linear terms, we find that, compared with term 1, term 2 is only significant at bow and stern. But, particularly for higher Froude numbers, there is a considerable phase shift between η_r and η, and the derivative of η_r is often of the wrong sign; thus including term 2 reduces the accuracy. The linear, but neglected term 7 is generally larger than term 2, and is, for the Series 60 hull, about 8 to 10% of term 1 at the bow.

Of the nonlinear terms, the transfer terms 5 and 6 generally predominate. This means that any method solving the nonlinear problem must apply the FSC right at the true free surface, otherwise the most important nonlinear effects are missed or poorly represented by unreliable Taylor expansions; a fact not properly recognised in some of the methods proposed up to now.

The sum of the higher order terms (Figures 2 and 3), which indicates the accuracy of the linearized solution, turns out to be of the order of 100% of term 1 locally at the bow and stern wave crests in some cases, even though the wave resistance prediction may still be fairly accurate. In general Dawson's FSC leads to somewhat smaller nonlinear terms than the Kelvin condition. This difference becomes more pronounced for decreasing Froude number as expected, but is not quite decisive even at speeds rather low in the practical range. At higher Fn, the increasing phase difference between η_r and η is one cause of the loss of this advantage.

For relatively low Fn, the nonlinear terms become fairly small and suggest that the slow-ship condition in its present form is correct for Fn → 0. On the other hand, it has been argued [4] that the perturbation has a short-wave character, so differentiation reduces its order by Fn^2. In that case, term 5 for instance would only be the first of a series of terms of $O(Fn^2)$ (i.e. of the same order as term 1) and thus not representative for the total error. From the results available however, we find that term 5 decreases strongly relative to term 1 for decreasing Fn. We may have some confidence that the derivation without order reduction does lead to an FSC that is accurate in practice, if not asymptotically correct, for low Fn.

4. CONCLUSIONS

Summarizing, it turns out that the differences in predictions obtained with the Kelvin FSC and the slow-ship FSC are far smaller than suggested by previous results, if at least the numerical errors are made comparable. The main contribution of Dawson seems to be the numerical treatment of the problem but not the particular form of the FSC.

Although asymptotically, or in the close vicinity of stagnation points, one of the FSC's may be better, in practical calculations the slow-ship FSC gives an at best marginally better approximation of the exact FSC. Both conditions become more adequate for decreasing Fn, and both have the same primary deficiency that they are applied at the undisturbed instead of the actual FS.

REFERENCES

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2. Second DTNSRDC Workshop on Ship Wave-Resistance Computations, Bethesda, Md., November 1983.
3. First DTNSRDC Workshop on Ship Wave-Resistance Computations, Bethesda, Md., November 1979.
4. Newman, J.N., "Linearized wave resistance theory", Int.Seminar on Wave Resistance, Japan 1976.

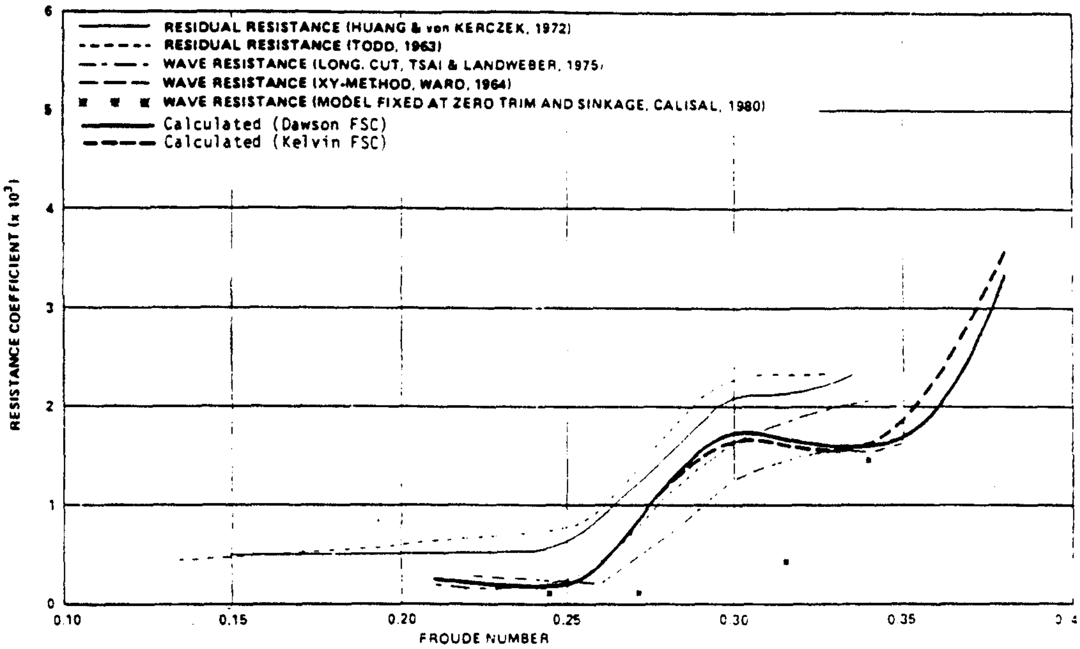

Fig. 1 Series 60, calculated wave resistance coefficients compared with experimental data (Taken from [3])

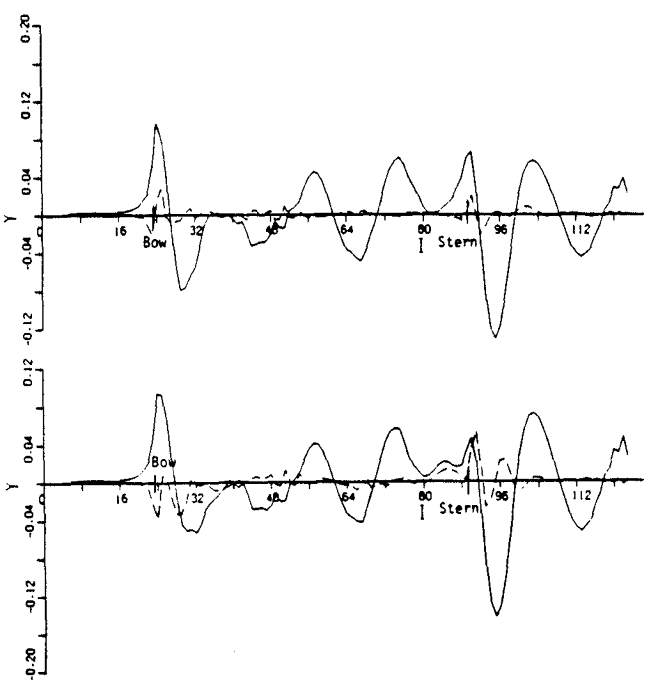

Fig. 2 Series 60, Fn = 0.22, with Davson FSC (above) and Kelvin FSC (below)

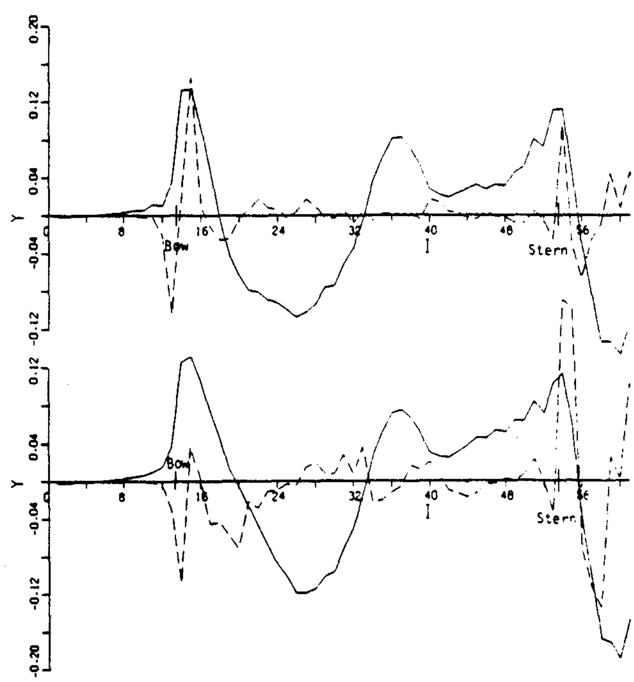

Fig. 3 Series 60, Fn = 0.35, with Davson FSC (above) and Kelvin FSC (below)

Sum of non-linear terms 3, 4, 5 and 6 (– – – – –), compared with term 1 (———————), on free surface panel strip along hull and centreline. Abscissa is panel number (uniform panel length); bow and stern location indicated.

DISCUSSION

Kleinman: Your results indicate that in many cases there is little difference between the two linearized free surface conditions. However your results also seem to support the conclusion that neither is a good approximation to the nonlinear condition. Would you comment on this?

Raven: As a matter of fact the magnitude of the nonlinear terms is larger than one would expect from the relative accuracy of the resistance and wave profile predictions. Apparently, some fortunate cancelling of errors occurs that makes the predictions still useful; on the other hand, estimating nonlinear terms from results of a linearized calculation perhaps gives a too pessimistic result. But the study does show that it is desirable to develop a method to satisfy the nonlinear FSC.

Nakos: In slow-ship linearization of the FSBC the second derivatives of the double-body flow are needed on the free-surface. It is true, however, that close to the stagnation points (bow-stem) these are singular (discontinuous). Can you comment on the error induced by the numerical treatment of such discountinuity and their share of contribution to the total error?

Raven: Thank you for raising this interesting point. What influence the local behariour near the stagnation points, if well resolved, would have on the resistance and wave pattern is not known to me. The singularity in η_{rx} that is present for angles of entrance other than zero or $\pi/2$ will perhaps analytically be eliminated by a zero φ_x' in term 2, such that the linear terms are not too much affected.

In practical methods this local behaviour is never resolved because of the finite panel dimensions both on the hull and on the free surface. In the present calculations up to 1300 free surface panels have been used, but still the collocation points near the stagnation points felt substantial nonzero double body velocities. Even a reduction of the width of the first free surface strip from 0.29B to 0.16B for the strut-like hull resulted in a very modest change of the results, e.g. 2% in the resistance, with Dawson's FSC.

So I do not believe that the linear terms and the predictions are much affected by the singularity in practice. The nonlinear terms 5 and 6 could perhaps be somewhat more sensitive.

Ursell: The use of Taylor expansions near a boundary is based on the assumption that the velocity potential can be continued into the space beyond the boundary. This seems reasonable when the boundary is smooth but cannot be strictly correct near corners and edges, also near stagnation points where the slope is not small. These form only a small part of the boundary, nevertheless Raven's work shows that they may have a noticeable effect on the outcome of his calculations.

Raven: Thank you very much for your comment. Perhaps I may add that in linearized methods, the Taylor expansions are not only used for analytic continuation of the potential beyond the boundary but also to express values at points inside the domain, at $y=0$, in values at the boundary $y=\eta$ where $\eta>0$. Although strictly the Taylor expansion must then converge for such points, at least away from corners at the boundary, it is the linearization that becomes invalid since higher order terms are of the same order of magnitude as the first term due to the character of the perturbation potential.