On the Kelvin wave–source potential

by

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Abstract.

The potential of the Kelvin wave source in the free surface consists of a double integral and a single integral, as is well known. The single integral expresses the wavelike behaviour and is difficult to evaluate numerically near the source. An important advance was reported by Baar at the Second Workshop. The single integral involves the function

$$F(x,\rho,\alpha) = \int_{-\infty}^{\infty} \exp\{-\tfrac{1}{2}\rho \cosh(2u-i\alpha)\}\cos(x \cosh u)du;$$

Baar found that for large values of $x^2/4\rho$ this was closely approximated by the asymptotic expansion (due to Bessho 1964)

$$F(x,\rho,\alpha) \sim -\pi I_0(\tfrac{1}{2}\rho)Y_0(x) - 2\pi \sum_1^{\infty} I_m(\tfrac{1}{2}\rho)Y_{2m}(x)\cos m\alpha, \quad (S)$$

(where I_m, Y_{2m} are the usual Bessel functions,) except near the free surface $\alpha = \pm\frac{1}{2}\pi$. Baar also found that the numerical evaluation of the series was much more economical (by a factor of about 100) than the numerical evaluation of the integral.

Bessho's derivation of the expansion (S) was purely formal. A rigorous derivation has now been obtained which shows that the expansion is indeed asymptotic, except near $\alpha = \pm\frac{1}{2}\pi$ when an additional integral term is needed.