Two-dimensional Numerical Modelling of Large Motions of Floating Bodies in Steep Waves

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The paper presents a continuation of the earlier work [1,2] on the development of a relatively simple algorithm for the numerical modelling of large motions of floating or submerged bodies in steep waves. The problem is considered in two dimensions and the fluid flow is assumed to be irrotational and acyclic. A boundary element method, based on an application of Green's second identity in a finite control domain, is used to solve for the unknown values of the velocity potential (ϕ) or its normal derivative ($\partial\phi/\partial n$) at the domain boundaries. The evolution of the free surface in time (t) is followed by means of an Eulerian method in which motions of the collocation points are restricted to vertical displacements. Earlier results [1,2] have shown that with the application of Airy or Stoke's 2nd order wave potentials on the upstream control boundary, and by adopting a form of Orlanski's radiation condition at the downstream boundary, propagation of steep waves in the control domain can be modelled effectively. Good results have also been obtained for the interaction of steep waves with vertical walls, [1].

In the present work the method is extended by introducing a floating rigid body in the control domain. The impermeability boundary condition is imposed on the wetted surface of the body. The generalized hydrodynamic forces exerted by the fluid on the surface of the body are derived by direct integration of pressure (p) determined from Bernoulli's equation:

$$p = -\rho[\frac{\partial}{\partial t}\phi + (\frac{\partial \phi}{\partial \bar{x}})^2 + gz] \tag{1}$$

The motions of the body are obtained by integration in time of Newton's equations of motion:

$$M_i \frac{d^2 q_i}{dt^2} = F_i \tag{2}$$

for $i = 1,2,3$, with M_i denoting the mass of the body for $i = 1,2$, and its moment of inertia about C.G. for $i = 3$. F_i represent generalized forces. At collocation points which are fixed on the body surface, $\partial\phi/\partial t$ in (1) is determined from:

$$\frac{\partial \phi}{\partial t} = \frac{d\phi}{dt} - \vec{v} \cdot \frac{\partial \phi}{\partial \bar{x}} \tag{3}$$

where $d\phi/dt$ denotes the rate of change in time of the potential at the collocation point moving with velocity $\vec{v}$.

In the application of the boundary element scheme, the motion of the body with respect to the free surface may result in large variations of the size of the discretization grid in the vicinity of the intersection of the free surface with the body surface. This in turn is found to introduce destabilizing force effects, through the computation of $d\phi/dt$ in (3). The difficulty is rectified by maintaining a uniform grid size with the use of an appropriate regridding on the free surface and wetted surface of the body, at each time step. The required information on the new grid is obtained by spatial interpolation. Difficulties related to the $d\phi/dt$ term also occur in the application of fourth order predictor-corrector A-B-M scheme for the integration of equations of motion (2), which is used successfully in the computation of the trajectories of free surface collocation points. These are eliminated if $d\phi/dt$ values are determined by means of a central difference formula at the corrector level of the A-B-M scheme.

In order to evaluate the numerical model a series of experiments was carried out in which a rigid cylindrical body of a rounded-off rectangular cross-section (fig. 1) was subjected to wave excitations in a channel. The width of the channel corresponded to the length of the body, and displacements of the body were restricted to heave and roll modes by an appropriate mounting device. Oncoming wave profiles, in and without the presence of the body, were measured at several stations. The measurements of the body responses included displacements in heave and roll modes, and swaying force. The algorithm described above is being used to replicate the experimental results. The computations and comparisons between experimental and computed values have not yet been completed. Here, the results obtained for two wave lengths, $B/\lambda = 0.19$ and 0.21 are shown, corresponding to the wave steepness of $H/\lambda = 0.04$. In both cases, the body motion is close to heave resonance and higher wave steepness could not be achieved experimentally owing to flooding. The matching between experimental and computed wave heights is shown in figs. 2a and 3a. The synchronization between the experimental and computed time histories was achieved by referencing the time histories to the extrapolated undisturbed wave elevations, shown in figs. 2b and 3b, at the location of body's C.G. In general, taking into account the resonant behaviour of the body, good agreement is observed between the measured and computed values in the region of comparison indicated by t_1 and t_2 in the figures. In both cases, experimental and computed values of roll were less than 4^o and are not shown here. In fig. 4, a sample of a numerical simulation in which large roll amplitudes was achieved is displayed.

1. Sen, D., Pawlowski, J.S. Two-dimensional Flow with the Free Surface. 2nd International Workshop on Water Waves and Floating Bodies, Bristol, April 1987.

2. Sen, D., Pawlowski, J.S. Simulation of Unsteady Propagation of Steep Waves by Boundary Element Method, 7th Int. Conf. OMAE, Feb. 1988.

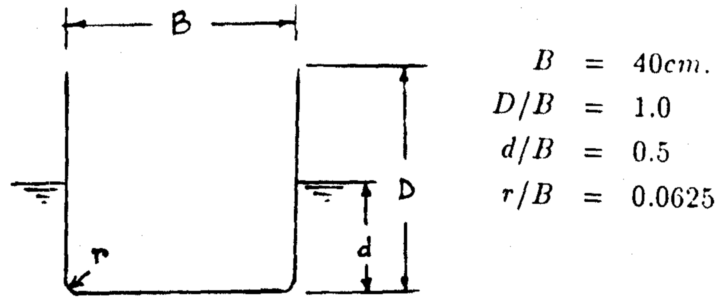

Fig. 1 The body geometry

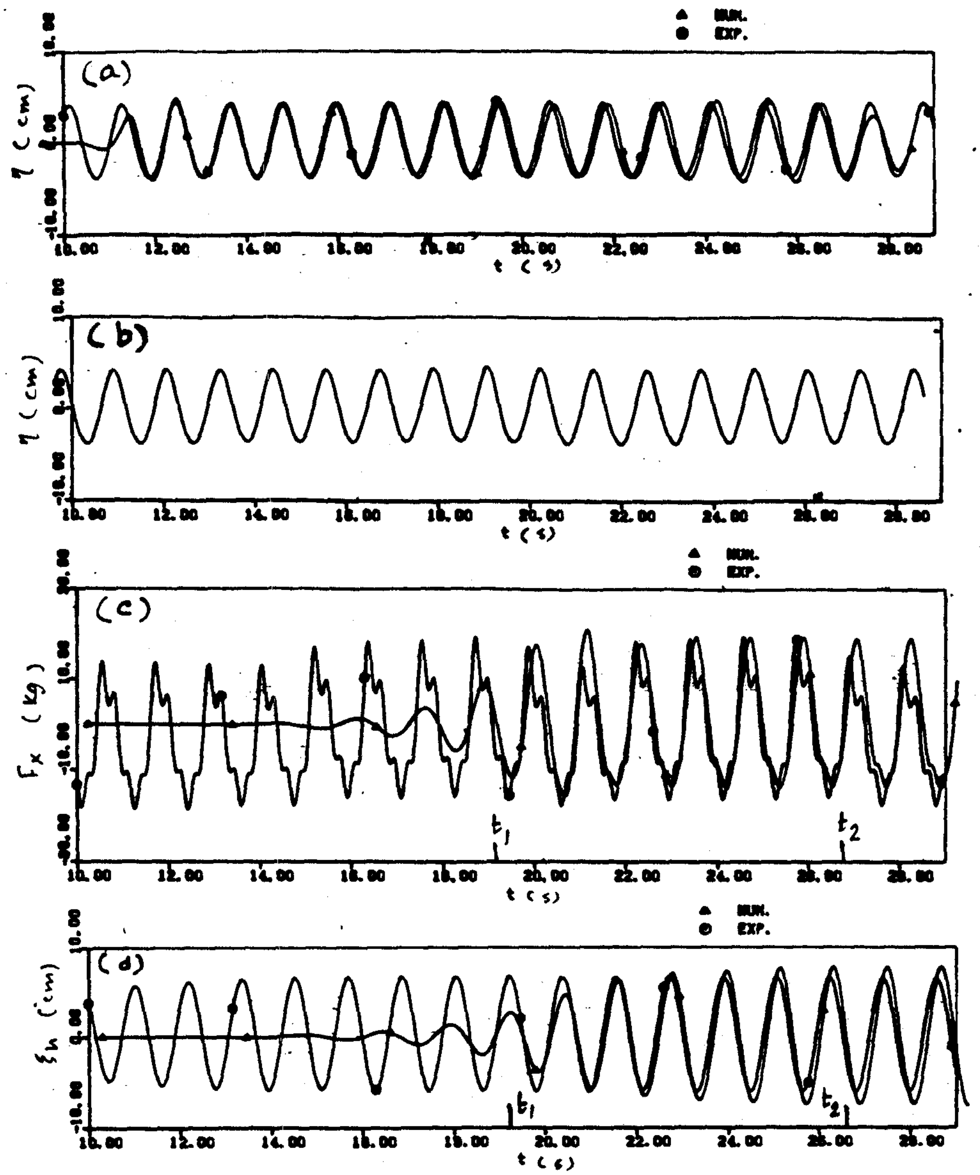

Fig. 2 Comparison of experimental and computed results (C.G. located at 3.25λ from the excitation boundary); $B/\lambda = 0.19$, $H/\lambda = 0.04$.

- **(a)** Comparison of measured and computed wave elevations; the wave was measured without the presence of the body and the computed wave elevation was taken at 0.5λ from the excitation boundary, with the body present.
- **(b)** Measured wave elevation plotted at the body C.G. location; time histories in figs. (c) and (d) are synchronized with respect to this wave elevation.
- **(c)** Sway force
- **(d)** Heave motion

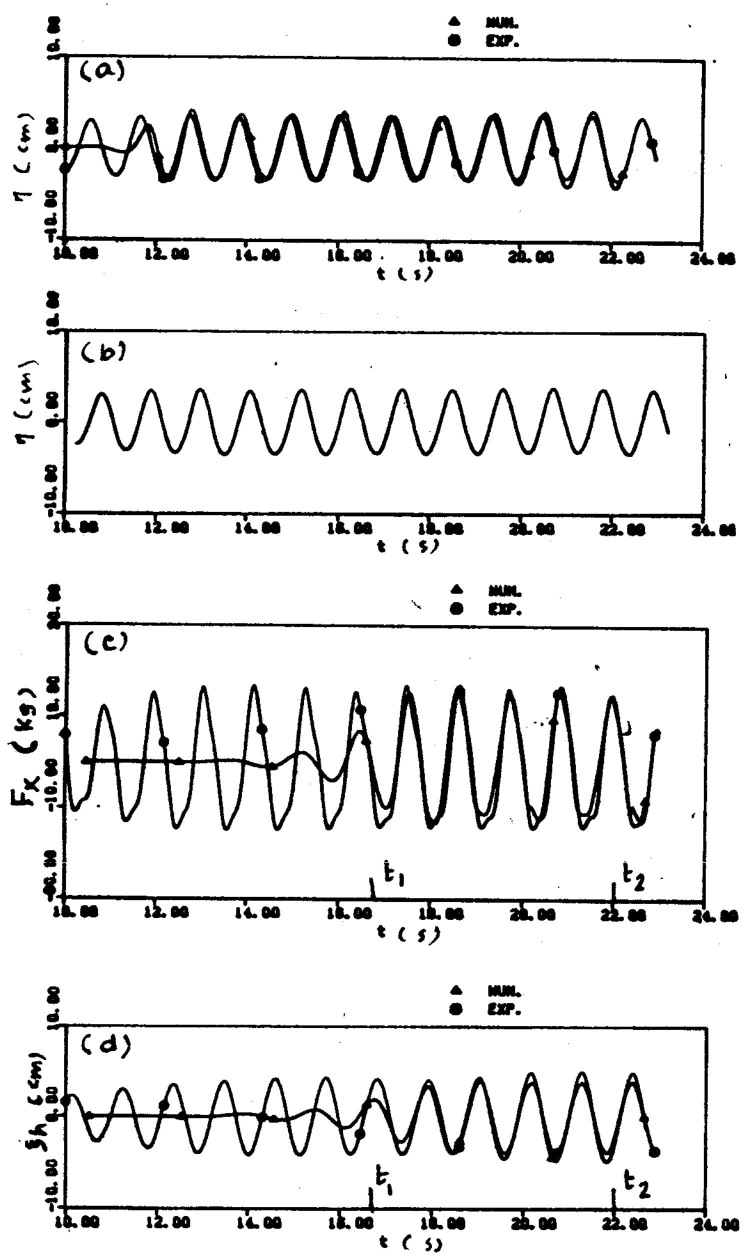

Fig. 3 Comparison for $B/\lambda = 0.21, H/\lambda = 0.04$ (for the computed results, C.G is located at 2.5λ from the excitation boundary); (a)-(d) are same as in fig. 2.

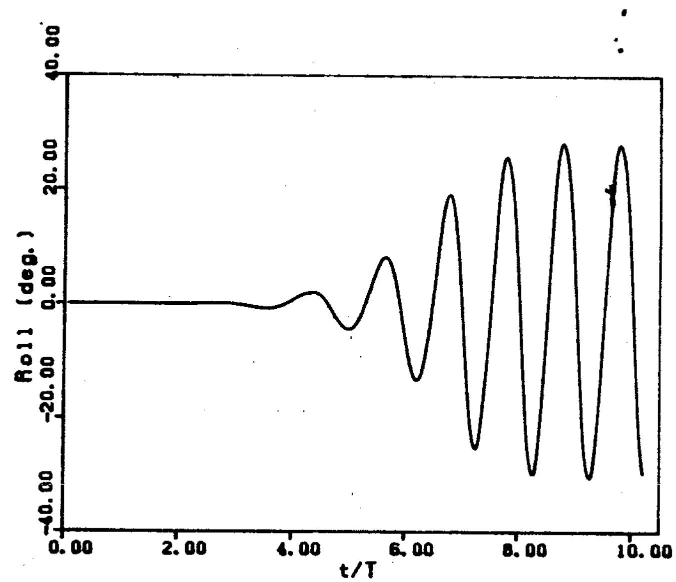

Fig. 4 A numerical simulation of roll motion of the rectangular body for $B/d = 1.0$; $H/\lambda = 0.05$ and $B/\lambda = 1/6$

Papanikolaou On the basis of your numerical model you could simulate the free oscillations of a 2D body in response to a steep wave. In this case there exist nonlinear hydrodynamic couplings between roll, sway, and heave and mean displacements of the body in the modes with restoring forces [*i.e.* in roll (steady heel) and heave (steady sinking or lift]. Did you discover these effects in your simulations? In particular I do not see a clear indication of the coupling between sway and roll. I would recommend checking the results in the steady-state case against well established values from frequency-domain calculations. Ref: A. Papanikolaou, ***Schffstehnik*** 1984.

Sen & Pawlowski: In our algorithm, total hydrodynamic forces are computed directly by the integration of pressure on the wetted surface of the body. Since the only restrictions imposed on the flow are that the velocity field is described by a single-valued velocity potential and wave elevation, the nonlinear hydrodynamic effects are implicitly represented by the algorithm, including possible hydrodynamic couplings between the modes of body motion.

The results shown here have been prepared to demonstrate the validity of the algorithm with respect to a specially designed series of model tests with a heaving, or heaving and rolling, model excited by waves, over a wide range of wavelength and wave steepness including very steep waves. In addition to the motions, time histories of the sway force were compared. The computations were found to reproduce, faithfully, experimental results including the roll mode after viscous damping was added semi-empirically.

In the results of Reference 1, free propagation of steep waves is demonstrated. Also, in the case of the interaction of steep waves and a vertical wall, a comparison of computed parameters with data available in the literature was carried out, showing very good agreement.

The algorithm has not yet been used for a systematic comparison with other motion and force data available in the literature or for a study of nonlinear phenomena of special interest, but the possibility of carrying out such studies provided the motivation for the development of the algorithm.

Grue: Can you explain in more detail the radiation condition which is applied in the numerical wave-tank. Will this radiation condition be effective for *any* transient nonlinear wave?

Sen & Pawlowski: The radiation condition at the downstream boundary assures that the velocity potential of the outgoing wave is represented by a wave-form translating out of the control domain. The celerity is equal to that of an Airy wave, the potential of which is applied at the upstream control boundary to induce a wave propagating in the control domain. The scheme is sensitive to the correct matching of the two celerities.

Transient excitations were present in the computed simulations, since in wave propagation and body motion simulations initial conditions assured that the fluid or the fluid and body were initially at rest. The magnitude of the transients and their high-frequency content were kept at a minimum due to the application of the modulation function which sets the excitation potential and its time derivative initially at zero. In none of the computed cases were reflection effects due to transients noticed.

However, this does not mean that the radiation condition is expected to produce correct results for any transient wave, and the sensitivity to the chosen celerity suggests that it would not, at least in its present form.