CONVERGENCE OF THE NEUMANN-KELVIN PROBLEM

Carl A. Scragg
John C. Talcott

In recent papers Doctors and Beck have discussed the limiting behavior of the Neumann-Kelvin problem for increasing numbers of panels and integration steps. The results of their calculations presented in [1] demonstrate that their Neumann-Kelvin solutions converge smoothly, although slowly, for the case of a submerged body even when remarkably few integration steps were used. Their results for a surface ship (the Wigley hull) given in [2] are somewhat less encouraging, exhibiting some surprising oscillations in the convergence as the number of panels was increased, but still limiting to a reasonable value.

In a similar study, we have examined the results obtained with our own Neumann-Kelvin solver as the number of panels was systematically increased. We found that the Neumann-Kelvin solution was quite well behaved for the Wigley hull, with the wave resistance coefficient converging smoothly to a finite value which is in good agreement with experimental data. However, for two hull forms which are not wall-sided, the limiting behavior was found to be quite different, with calculated wave resistance coefficients apparently increasing without bound as the panel density was increased.

Our formulation of the problem is quite similar to that given by Doctors and Beck, but our numerical approach differs, primarily in the method of evaluating the Havelock source. The velocity potential is given by:

$$\Phi = -Ux + \sum_{i=1}^{N} \sigma_i \phi_i \quad .$$

The potential ϕ_i due to a unit source strength distributed uniformly over the ith panel is given by:

$$\phi_i = \int_{s_i} ds\, G \quad - \quad \frac{U^2}{g} \int_{c_i} dl\, n_x \tau_y\, G \quad .$$

The first term on the right hand side is to be integrated over the panel surface, and the second term is integrated along the waterline if the panel intersects the free surface. The zero normal velocity boundary condition is imposed at the centroid of each panel in order to set up a system of equations which can be solved for the unknown source strengths σ_i :

$$\sum_{i=1}^{N} \sigma_i \nabla(\phi_i) \cdot \hat{n}_j = U (n_x)_j , \qquad \text{for } j = 1, N .$$

We use a form of the Green function given by Wehausen [3], but the variables of integration have been transformed from (k,θ) into the rectangular wave number coordinates (k_x, k_y):

$$G(x,y,z;\ x',y',z') = -\frac{1}{r} + \frac{1}{r^*}$$

$$+ \frac{4}{\pi} k_o \int_0^\infty dk_y \int_0^\infty dk_x \frac{e^{k(z+z')}}{k_x^{\,2} - k_o k} \cos[\, k_x(x-x')\,] \ \cos[\, k_y(y-y')\,]$$

$$+ k_o \int_0^\infty dk_y \frac{\beta(k_y)}{k_x'} e^{k'(z+z')} \sin[\, k_x'(x-x')\,] \ \cos[\, k_y(y-y')\,]$$

We have found that numerical integration of the Green function over (k,θ) requires extremely small integration step sizes to accurately evaluate the energy contained in the higher wave numbers which are important near the free surface. This difficulty may explain why Doctors and Beck found that their results converged more smoothly for the submerged body than for the surface ship. The use of (k_x, k_y) as the variables of integration has the distinct advantage that the integration step size is dependent only upon the length of the ship and its forward speed. Since the energy at high wave numbers will depend exponentially upon the depth of the source point and the field point, we can easily adjust the domain of integration for each source panel on the hull to obtain a specified level of accuracy.

Our approach to examining the convergence of the Neumann-Kelvin problem for the Wigley hull was very similar to that used by Doctors and Beck. The panel aspect ratio (panel length divided by panel height) was held at a value of 2.0 for consistency. As we plotted the singularity distribution calculated for each successively finer panelization of the hull, we noted that the most significant differences occured on the panels nearest the free surface. This is due to the fact that our Neumann-Kelvin solution satisfies the hull boundary condition at the centroid of each panel, and for successively smaller panels we are imposing a zero normal flow condition at points nearer the hull/free-surface intersection where higher wave numbers become more important. Therefore, we concluded that the dominant parameter in the convergence study was the panel height. Plotting the wave resistance coefficient versus the panel height (equivalent to plotting versus the inverse square root of the total number of panels), we found that the results converged smoothly and rapidly to a limiting value which agrees well with experimental data [4]. We concluded that the Neumann-Kelvin problem is quite well behaved for this simple wall-sided hull form, and the primary difficulty in obtaining an accurate solution for large numbers of panels lies in resolving the shorter wavelengths on the shallowest control points.

We have observed a very different behavior for some highly flared hull shapes. We found that the calculated Cw changed quite radically when the number of panels was increased and did not seem to limit smoothly to a finite value. Concerned that the convergent behavior of the Neumann-Kelvin problem might be limited to wall-sided hull forms, we repeated our convergence study for a flared version of the Wigley hull. The results showed a lack of convergence, with the wave resistance coefficient increasing without bound as the panel density was increased.

In order to investigate the behavior further, we examined a simple wedge-shaped bow section with 45 degree flared sides. We used two different panelization schemes, each with a different panel aspect ratio. As the number of panels was increased, the results obtained with the two different aspect ratios came into agreement with each other, but they did not seem to converge to a finite Cw. We observed that as the panels became smaller, the singularity strengths on the row of panels nearest the waterline seemed to grow without bound. We speculated that this result was associated with the decreasing distance between the panel control point and the edge of the negative image panel. We examined the velocity induced at the control point due to each term in the Green function as the size of the panel was decreased. The positive Rankine term always produces a normal velocity at the centroid of the panel equal to 2π, and for submerged panels this term dominates all other terms. The normal velocities due to the wave terms associated with both the surface singularities and the waterline singularities are well behaved as the panel size is decreased. The negative Rankine image is the only term which leads to a rapidly increasing normal velocity as the control point approaches the waterline. For the particular case being examined, with 45 degree flare, the panel and its image meet at right angles and the normal velocity at the control point will become dominated by the tangential velocity component on the image panel. Since the tangential velocity will be proportional to the inverse square root of the distance from the image panel, we anticipated that the singularity strength would approach this inverse square root dependence upon the depth of the panel. To demonstrate this, we examined the convergence of the infinite Froude number solution for the same bow shape. The maximum calculated source strength on the panels does appear to limit to an inverse square root dependence upon panel size.

For the infinite Froude number case, the limiting behavior of the source strength is not a significant problem since the source strength integrated over the surface of the panel is still decreasing rapidly as the panel size is decreased. However, in our formulation of the Neumann-Kelvin problem, we assume that the source strength to be used in the waterline integral is equal to the source strength on the adjacent panel. Therefore, in the limiting case, we are distributing an infinite source strength right on the waterline, resulting in infinite wave resistance.

Note that this problem is never encountered for wall-sided hull forms. The image term's contribution to the normal velocity at the shallowest control point is identically zero since the panel and its image lie in a vertical plane. Accordingly, we examined the effect of rotating the top row of panels on our flared bow shape into a vertical plane. We selected a panelization in which the top row of panels were quite small, with panel height equal to approximately one percent of the hull draft. The resulting Neumann-Kelvin singularity distribution contained maximum source strengths which are an order of magnitude smaller that those calculated for the original flared panelization, indicating the sensitivity of the problem to the geometry at the hull/free-surface intersection.

We have demonstrated the limiting behavior of the Neumann-Kelvin problem as it is usually formulated. The solutions are well behaved in the limit of infinitesimal panel size only if the hull intersects the free surface at right angles (i.e. wall-sided hull forms). For highly flared hull shapes the calculated source strengths near the waterline are proportional to the inverse square root of the panel depth, and consequently the calculated wave resistance coefficients do not limit to finite values as the panel sizes are decreased to zero.

References

1. L. Doctors and R. Beck, 'Convergence Properties of the Neumann-Kelvin Problem for a Submerged Body,' J. Ship Res. Vol. 31, No. 4, pp. 227-234.
2. L. Doctors and R. Beck,'Numerical Aspects of the Neumann-Kelvin Problem,' J. Ship Res. Vol. 31, No. 1, pp. 1-13.
3. J. V. Wehausen, 'The Wave Resistance of Ships,' Advances in Applied Mechanics, 13, 1973.
4. Sangseon Ju, 'Study of Total and Viscous Resistance for the Wigley Hull Parabolic Ship Form,' Proc. 2nd DTNSRDC Workshop on Ship Wave-Resistance Computations, Bethesda, Md., Nov. 1983, pp 36-49.