A QUADRATIC SPLINE SCHEME FOR THE WAVE RESISTANCE PROBLEM

by

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The numerical solution of the linearized wave resistance problem by panel methods using simple Rankine singularities, has attracted considerable attention over the past decade. The promising work by Gadd (1976) and Dawson (1977) motivated several other studies.The evaluation of the various schemes ought to be based on their consistency, stability, order of numerical damping and dispersion, and on the numerical implementation of the radiation condition.In the present paper a scheme, of proper quality – according to these criteria, is developed and applied to various test cases.

The scheme

- The simple Rankine source $1/r$ is used as the elementary singularity in Green's 2nd identity in order to produce an integral formulation of the problem.
- The wetted body-surface and part of the free surface are discretized by plane quadrilateral panels.
- The unknown velocity potential is discretized by using a quadratic spline shape function over each panel. This shape function is constructed such that it intrinsically satisfies interelemental continuity of value and splopes at the midpoints of the panel sides.
- The end-conditions of the splines are applied on the boundary of the computational domain in such a way that they enforce the radiation condition.
- Collocation at the centroids of all the panels provides a system of linear simultaneous equations for the unknown spline coefficients.
- Because of the quadratic approximation, after this system is solved, the velocity and pressure fields are known without any additional numerical differentiations.

The single 3-D Havelock source.

The single Havelock source, located at $\vec{X}$, has been chosen as the most primitive application of the scheme. The integral equation is over the free surface only and, after nondimensionalization it reads:

$$\mathcal{L}\phi = b \ : \qquad 2\pi\phi(\vec{x}) + (Fr)^2 \iint_F \phi_{xx}(\vec{\xi})\,\frac{1}{\|\vec{x}-\vec{\xi}\|}\,ds \;=\; \frac{1}{\|\vec{x}-\vec{X}\|} \tag{1}$$

For sufficiently large submergence of the source the variation in the transverse direction is expected not to be as fast as in the logitudinal one. Thus a simpler version of the scheme is employed, which is much easier for analysis purposes.The shape function is quadratic spline in the logitudinal direction and constant in the transverse one, and the velocity potential in the j'th panel is approximated by:

$$\phi^{(j)}(\xi,\eta) = 4(a_{j-1}-2a_j+a_{j+1})\left(\frac{\xi}{h_\xi}\right)^2 + 4(a_{j-1}-a_{j+1})\left(\frac{\xi}{h_\xi}\right) + (a_{j-1}+6a_j+a_{j+1}) \tag{2}$$

Collocation at the centroid of the j'th panel results in the following algebraic equations:

$$\mathcal{L}_h \phi_h = b_h : \qquad 2\pi(a_{j-1} + 6a_j + a_{j+1}) + 8(Fr)^2 \sum_{i=1}^{N} \frac{a_{i-1} - 2a_i + a_{i+1}}{h_\xi^2} S_{ji} = R_j$$

$$\text{and :} \qquad \phi_j = a_{j-1} + 6a_j + a_{j+1} \tag{3}$$

Following Oskam (1982), the numerical analysis of the scheme can be carried out in the Fourier space. Taking advantage of the translational kernel we can Fourier-transform both the continuous and the discrete formulations (eqns 1 and 3). The spectrum of the discrete operator is found by using the semi-Discrete Fourier Transform (DFT).

$$\hat{\mathcal{L}} \cdot \hat{\phi} = \hat{b} \; ; \qquad \hat{\mathcal{L}} = 2\pi \left(1 - (Fr)^2 \frac{k_x^2}{\sqrt{k_x^2 + k_y^2}} \right) \tag{4}$$

$$\hat{\mathcal{L}}_h \cdot \hat{\phi}_h = \hat{b} \; ; \qquad \hat{\mathcal{L}}_h = 2\pi \left(1 - (Fr)^2 \frac{8}{h_\xi^2} \frac{1 - \cos k_\xi h_\xi}{3 + \cos k_\xi h_\xi} \frac{\hat{S}}{2\pi} \right) \tag{5}$$

$$\text{where :} \qquad \hat{S} = DFT \left\{ \iint_{-1/2}^{+1/2} \frac{du\, dv}{\sqrt{h_\xi^2 (j_1 - u)^2 + h_\eta^2 (j_2 - v)^2}} \right\} \tag{6}$$

Using the above and assuming that there is no approximation error in the RHS, we get the error equation in the Fourier space:

$$\hat{\mathcal{L}}_h \cdot (\hat{\phi}_h - \hat{\phi}) = (\hat{\mathcal{L}} - \hat{\mathcal{L}}_h) \cdot \hat{\phi} \tag{7}$$

The consistency criterion is met if $\hat{\mathcal{L}} - \hat{\mathcal{L}}_h \to 0$ as $h_\xi,\ h_\eta \to 0$. The numerical damping and dispersion are related to the distance between the root of $\hat{\mathcal{L}} = 0$ and $\hat{\mathcal{L}}_h = 0$. Finally the scheme is stable if there are no spurious roots of $\hat{\mathcal{L}}_h = 0$ in the principal interval $k_x \in (-\pi/h_\xi, \pi/h_\xi)$, $k_y \in (-\pi/h_\eta, \pi/h_\eta)$.

The proper radiation condition is implemented via the end-conditions of the splines. The strong hyperbolic-parabolic nature of the problem suggests that the two end conditions should be imposed upstream, while no condition is imposed downstream. The conditions that are used are $\phi_x = \phi_{xx} = 0$ at the most upstream points of the domain. This way upstream waves and downstream wave-reflections are prevented.

Figure (1) is a plot of the velocity potential of single 3-D Havelock along a cut parallel to the direction of motion. The results of the quadratic spline scheme, a finite difference scheme, as well as the exact values, are included. The advantages of the present scheme become apparent especially with respect to numerical damping and dispersion.

Similar results have been computed for the case of a 'thin ship', which can be considered as a superposition of Havelock sources.

The submerged spheroid.

The Neumann-Kelvin linearization can be more easily justified for the case of a submerged body. Moreover Farell (1973) has given an analytical solution for a spheroidal body, which provides a reference solution for convergence analysis of the scheme.

In the present case the shape function is quadratic spline in both directions. The end-conditions in the logitudinal direction are identical with the ones discribed in the case of a single source. In the transverse direction the condition $\phi_{yy} = 0$ (natural spline end-condition) is employed at the two ends. It has been found that the latter end-conditions have no essential effect on the solution. The body surface is discretized by plane quadrilateral panels while the same quadratic spline shape function is used for the approximation of the velocity potential on the body. The natural spline end-condition is employed at the body's edges.

Extensive convergence analysis proves that 8 panels per characteristic wavelength $(2\pi U^2/g)$ in the longitudinal direction are sufficient to provide three significant digits convergence in the wave resistance coefficient. Moreover the transverse grid-spacing can be as large as the body's beam.

In figure (2) the wave resistance coefficient (c_w) is plotted against the Froude Number (Fr) for a spheroid with beam-to-length ratio 1:6 at two mean submergences : $0.125L$ and $0.250L$, where L is the body's length. Farell's results are also included for comparison purposes.

The surface piercing body

The scheme that has been described above is, finally, applied to the case of a surface piercing body.

There have been many discussions on the required free surface boundary condition (FSBC) The present scheme can, in principle, handle all of the different linearized versions of the FSBC.

The Neumann-Kelvin condition is first employed. Systematic analysis proves lack of convergence close to the waterline of the body, while the wake-off calculations are convergent. Moreover significant discontinuities of ϕ_x are encountered on the waterline, when these are computed by interpolation on the free surface on one hand, and on the the body on the other hand. It should be pointed out that such dicontinuities are not present in the value of the potential ϕ. This result is in agreement with Suzuki's suggestions (1979-81). It has also been found that, unlike the case of the submerged body, the end conditions of the transverse direction on the free surface has great effect on the solution close to the body and its track downstream.

As the next step the Dawson's FSBC is used. Work is currently in progress on this subject. A point of significant interest is the proper treatment of the very fast variation of the derivatives of the double-body velocity potential close to the bow and stern.

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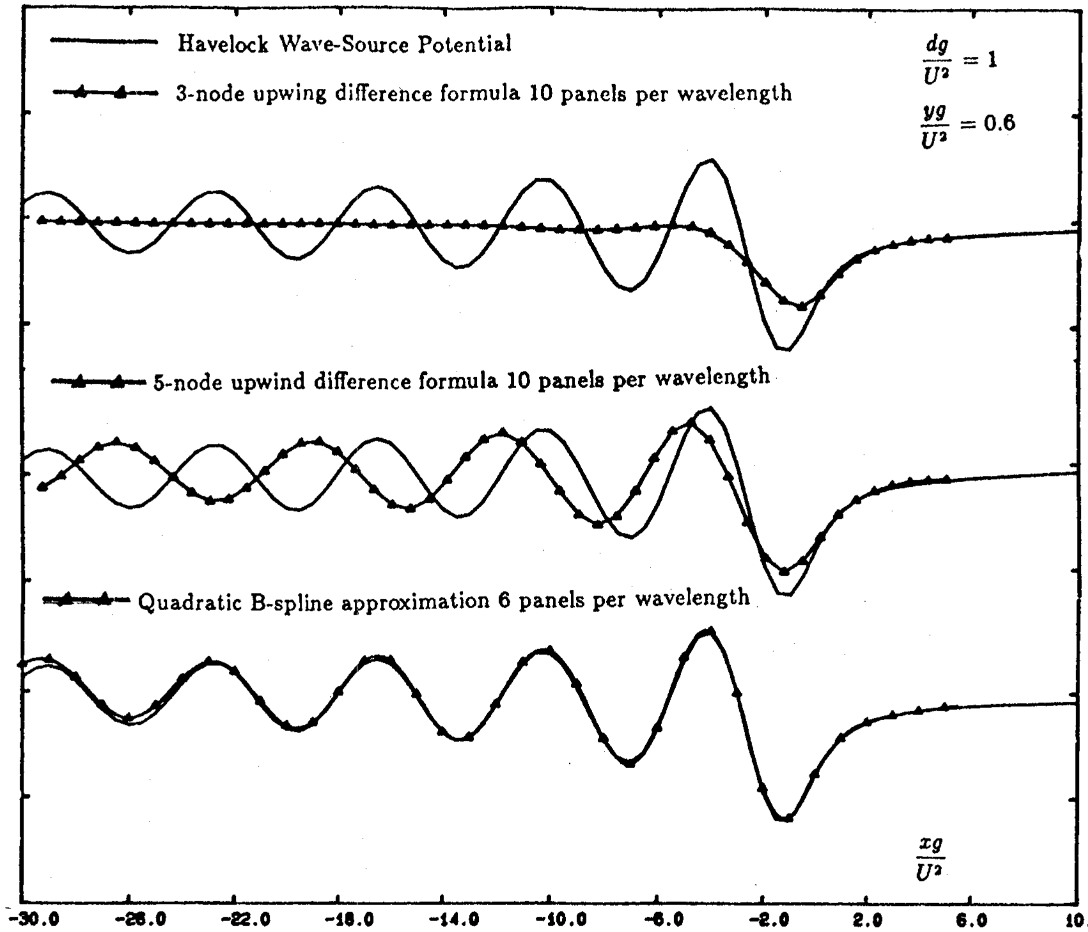

Figure 1: Velocity potential on the free surface (z = 0) along yg/U^2 = 0.6 due to a unit source located at a distance d below the free surface and translating at a speed U. A comparison is made between a 3-node and 5-node upwind difference formulae, a quadratic B-spline approximation and the exact solution obtained from the evaluation of the Havelock wave source potential.

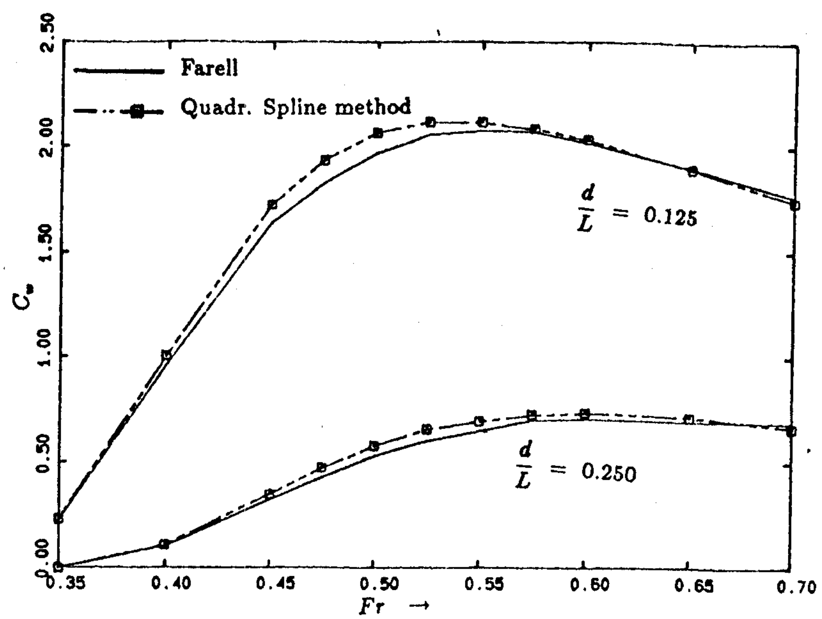

Figure 2 : The wave resistance coefficient for a spheroid with beam-to-length ratio 1:6 and for two submergences 0.125 L and 0.250 L.

Reed: Your results show increasing oscillations on the centerline with increasingly fine discretization. These results are consistent with the singularities/irregularities which occur for the continuous distributions of Havelock sources. The fact that your discrete free-surface panel method reproduces similar singularities should be considered a significant technical achievement. It shows the rigor and robustness of your results relative to the continuous distribution methods. A question which should be asked is why the continuous and discretized problems have singularities on the free surface, is it due to the linearized verses nonlinear free-surface condition, or due to neglecting surface tension and or viscosity?

Nakos: The lack of numerical damping is a crucial characteristic of the quadratic spline scheme. Thus the numerics *will* model the highly oscillatory behavior of the analytical solution, preventing convergence in the case of a surface piercing body. A similar behavior was found regardless of whether the Neumann-Kelvin or Dawson free-surface conditions were used. We think that even the use of the nonlinear free-surface condition will not circumvent the problem. A judicious "distribution" of damping and/or surface tension in the numerics appears to be more promising and it is currently under investigation.