Critical Condition for the Appearance of Sub-Breaking Waves and Numerical Simulation

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1. Steady breakers

Experiments suggest that steady breakers behind a submerged foil are not overturning but intensively fluctuating. They are rather turbulent free-surface flows without any backward flows [1] [2]. They are called "sub-breaking waves" there to be distinguished from plunging-type breakers.

2. Critical condition for the appearance of sub-breaking waves

Assuming the sub-breaking waves as turbulent free-surface flows, an instability analysis is invoked to derive a critical condition for their appearance [2].

The condition for an introduced disturbance to develop temporally is given by,

$$\frac{U}{M}\frac{\partial M}{h\partial s} - \frac{\partial U}{h\partial s} - \frac{1}{n_z}U\frac{\partial n_z}{h\partial s} > 0 \qquad (1)$$

where $M = (\kappa U^2 - n_z g) n_z$ (2)

s is the streamline coordinate along the free-surface and h is its metric coefficient, while n is the normal ; n_z the direction cosine of n to z. T is time, U and W are the velocity components in the s- and n- directions respectively ; κ is the curvature of the free-surface and g the gravity acceleration. Limiting ourselves to a narrow region around the wave crest, we can assume $n_z \fallingdotseq 1$ and $\frac{\partial}{h\partial s} \fallingdotseq \frac{\partial}{\partial x}$; then (1) and (2) can be reduced approximately into

$$\frac{U^2}{M}\frac{\partial}{\partial x}\frac{M}{U} > 0 \qquad (3)$$

$$M = \kappa U^2 - g \qquad (4)$$

Velocity and wave-height measurements are carried out for two cases ; one is without breaking and the velocity is far below for its appearance (NON) and the other is the case whose oncoming velocity is the highest for the sub-breaking waves not to appear, in other words, at a just lower velocity than that where a breaking appears(JB). Fig.1 shows the distribution of M/U versus x which is calculated from the measured data, where ζ and M/U are non-dimensionalized based on chord length and uniform velocity. The negative gradient observed in the case of JB which means the flow can be unstable and sub-breaking can appear (note here M is negative). On the other hand, M/U remains almost constant with respect to x in the case of NON, which means the sub-breaking can never take place.

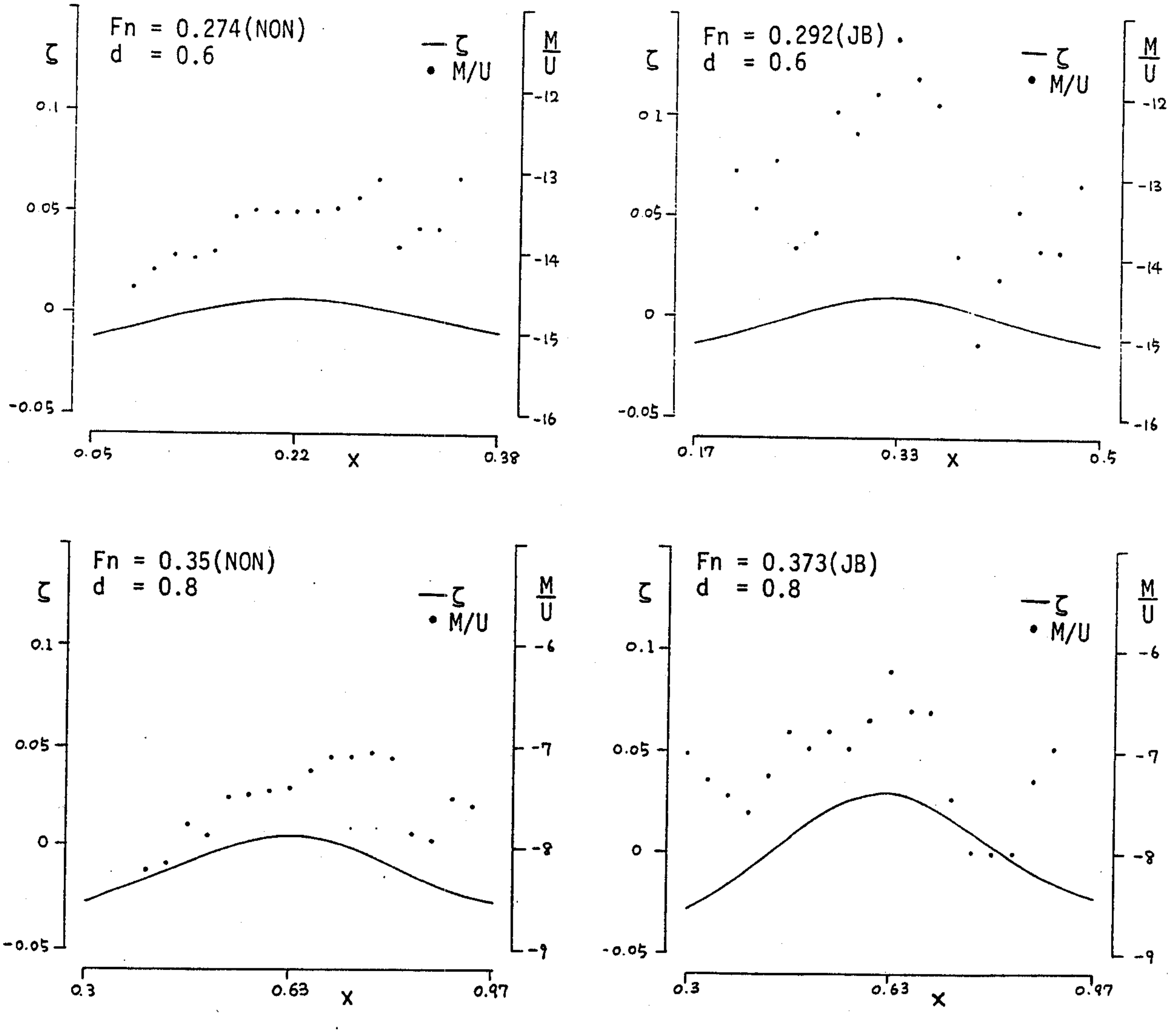

Fig.1 Distribution of M/U (experimental result, $\alpha=5°$)

3. Numerical simulation

The free-surface flow is simulated by solving the N-S equation. The numerical scheme is a direct finite difference method based on MAC method whose precise description is found in [3] .

Fig.2 is the computed flow field around a submerged NACA-0012 foil which is set at an angle of 5 degree. The Froude number is 0.567 and the immersion depth d is 1.29. It is quite well simulated.

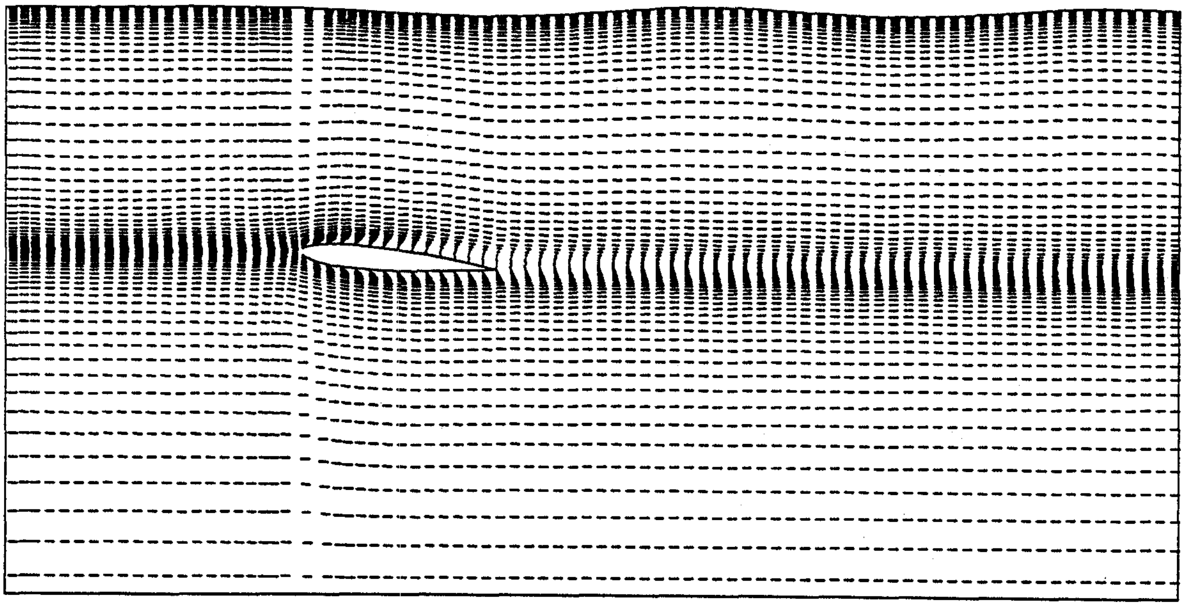

Fig.2 Velocity field (d=1.29, α=5°, Fn=0.567, Rn=2000, T=22)

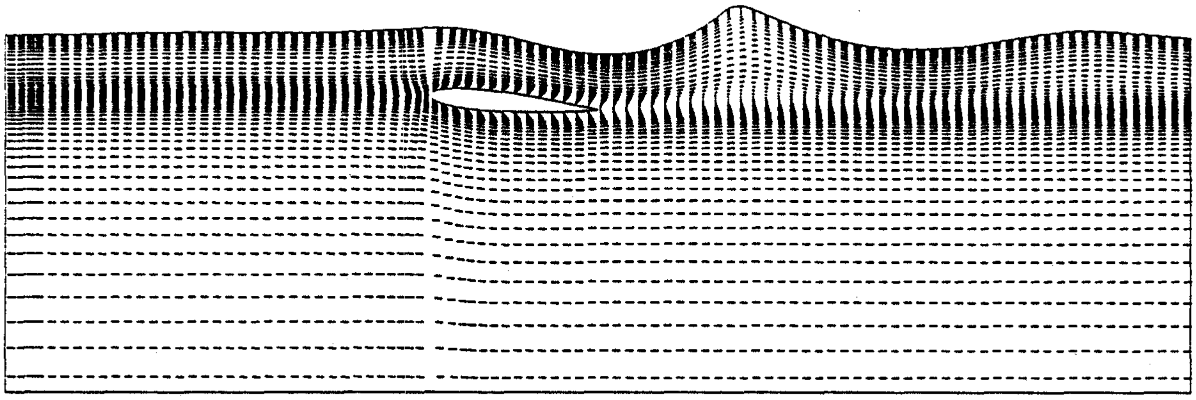

Fig.3 Velocity field (d=0.4, α=5°, Fn=0.567, Rn=2000, T=7)

Fig.3 is the results for the case at d=0.4. Although the computation continues without any troubles up to T=7 where T is the non-dimensionalized time. However, it is quite obvious that such steep waves cannot exist actually ; the angle of slope is well beyond 30° .

Fig.4 shows the distributions of M/U for the two computed cases, where ζ and M/U are non-dimensionalized based on wave length and uniform velocity. Although the flow of d=0.4 is still under developing and the application of (3) may contradict its derivation where the basic flow is assumed steady, the time dependence of the basic flow may be much less than that of the introduced disturbance. As clearly seen, M/U remains almost constant for d=1.29, while it shows a steep negative gradient for the latter case, which means such waves cannot exist stably, in other words, sub-breaking waves may take place.

Once the sub-breaking takes place, the averaged momentum equation is given by ;

$$\frac{\partial W}{\partial t} + U\frac{\partial W}{h\partial s} + W\frac{\partial W}{\partial n} = -\frac{1}{\rho}\frac{\partial P}{\partial n} - n_z g - \frac{\partial \overline{W'^2}}{\partial n} \qquad (5)$$

where P is the pressure and $\overline{W'^2}$ is the turbulence intensity in the n-direction. Experiments suggest that $\frac{\partial}{\partial n}\overline{W'^2}$ is positive and about 10 % of the gravity [4] , which means the gravity acceleration becomes greater apparently by $-\frac{\partial}{\partial n}\overline{W'^2}$ due to the transport of the turbulent energy. Then the inclusion of $-\frac{\partial}{\partial n}\overline{W'^2}$ in calculation may yield less steep waves profiles as observed experimentally.

It may be important to take into account the sub-breaking waves in the numerical simulation, otherwise, the simulation may mislead us.

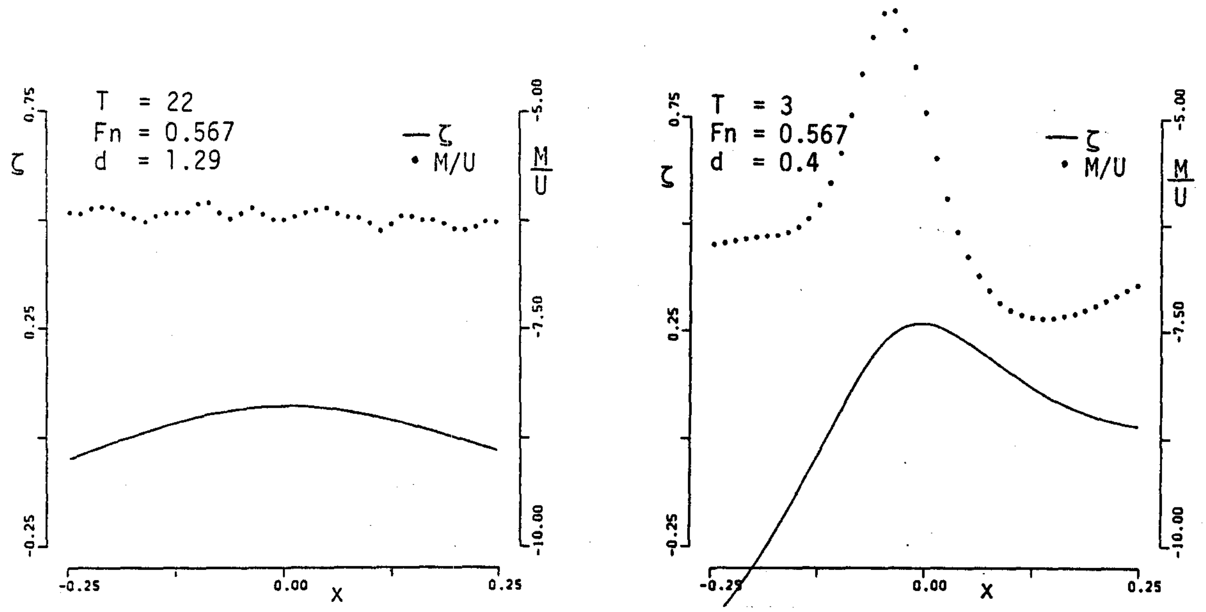

Fig.4 Distribution of M/U (calculation result, α=5°)

References

[1] Mori, K., "Sub-Breaking Waves and Critical Condition for their Appearance", J. Soc. Nav. Archt., Vol 159, pp. 1-8 (1986).

[2] Mori, K., "Critical Condition for Their Appearance of Steady Breakers on 2-Dimensional Wave Generated by Submerged Foil", Proc. of IUTAM (1987) (under printing).

[3] Shin, M. and Mori, K., "Numerical Simulation of a Flow around a Submerged Wing", Proc. of the 1st Symp. on Computational Fluid Dynamics, pp. 21-24 (1987) (in Japanese).

[4] Mori, K. and Doi, Y., "Flow Characteristics of 2-Dimensional Sub-Breaking Waves, Turbulence Measurements and Flow Modeling, Hemisphere Publishing Corporation, pp. 69-78 (1987).