NONLINEAR MOTIONS IN THE TIME DOMAIN

by

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At the University of Michigan research has been conducted to investigate the use of time-domain analysis to solve linear seakeeping problems for ships with forward speed. A series of papers and reports have been published (see for example King (1987), King and Beck (1987)). In the technique, an unsteady Neumann-Kelvin type problem is posed in which the body boundary condition is solved on the three-dimensional mean body surface and the free surface is linearized about the calm water plane. An ideal fluid is assumed so that the Laplace equation governs the flow in the fluid domain. Using Green's theorem and the Green function for a time dependent source, an integral equation is developed to solve for either the radiation or diffraction perturbation potentials. To solve the integral equation, a boundary integral (or panel) method is used. The body surface is discretized into a number of quadrilateral elements over which the perturbation potential is assumed constant. The integral equation reduces to a series of simultaneous equations which must be solved at each time step. In the solution, the coefficient matrix must be inverted only once, but there are convolution integrals which must be evaluated at each time step. Because the system is stationary, the Green function can be stored and re-used, and need only be evaluated at one new time level per time step.

At zero forward speed, the time-domain computations are not as fast as the conventional frequency-domain calculations because of the convolution integrals and Green function evaluations. However, at forward speed the time-domain method appears to be significantly faster because the Green function is much simpler to compute. Comparisons between the results of time-domain analysis and strip theory indicate that in general the exciting forces, added masses, and damping coefficients are very similar. Depending on the specific variables being considered, one method or the other may give better results when compared with experiments. These results are not too surprising since almost all of the comparisons have been for vertical plane motions in head seas, cases for which strip theory works very well. There are indications from the few comparisions so far available, that predictions for the horizontal plane motions will be improved by the use of the three-dimensional time-domain method. To date, results have been obtained for simple geometric forms, a Series 60 (Cb=.70) model, and a Wigley hull form being tested by Gerritsma.

Work is presently starting on the use of time-domain techniques to investigate large amplitude, nonlinear motion problems. In particular, the hydrodynamic forces due to large amplitude translations of a rigid body below a free surface will be discused in this abstract. This is the first step in a continuing investigation which will eventually examine the large amplitude motions of a body on the free surface.

The use of the Neumann-Kelvin type approximation in which a linearized free surface boundary condition is used in conjunction with the exact body boundary condition is in general inconsistent. However, it is the use of the exact body boundary condition which gives the nonlinear character to the problem. For bodies which are sufficiently submerged the use of the linearized free surface condition appears reasonable. It is hoped that by starting with a deeply submerged body and progressing to the case of a floating body much valuable information can be gained about the nonlinear effects of large body motions.

To formulate the problem, an inertial coordinate system is chosen with the z-axis positve upwards and the origin fixed at the calm water level. The region of fluid is bounded by the free surface, S_f, the body surface, S_h, and the bounding surface at infinity, S_∞. Because of the ideal fluid and irrotational flow assumptions, the velocity potential must satisfy the Laplace equation. On the plane z=0 a linearized free surface boundary condition is applied such that:

$$[(\partial/\partial t)^2 + g\, \partial/\partial z]\, \phi = 0 \qquad \text{on } z=0 \tag{1}$$

The body boundary condition is the no penetration condition applied on the *exact* moving body surface, S_h. Thus,

$$\partial\phi/\partial n = U(t) \bullet \mathbf{n} \qquad \text{on } S_h \qquad (2)$$

where,

$U(t)$ = the vector of the body translational velocity
n = the unit normal to the body surface out of the fluid

This body boundary condition is nonlinear and is a significant departure from the more usual linear condition in which small oscillatiions are assumed, the velocity is expanded in a Taylor series, and the body boundary condition is developed on the mean hull position.

The condition at infinity is that

$$\nabla\phi \rightarrow 0 \qquad \text{as } r \rightarrow \infty$$

and an initial value problem is posed such that

$$\phi, \partial\phi/\partial t \rightarrow 0 \qquad \text{as } t \rightarrow -\infty$$

As described by King (1987), an integral equation to determine the velocity potential is developed by applying Green's theorem to the fluid domain and integrating both sides with respect to time. The appropriate time-domain Green function is given by

$$G(P, Q, t, \tau) = (1/r - 1/r')\,\delta(t-\tau) + H(t-\tau)\,\tilde{G}(P, Q, t, \tau)$$

$$\tilde{G}(P, Q, t, \tau) = \int_0^\infty dk \sqrt{kg}\, \sin(\sqrt{kg}(t-\tau))\; e^{k(z+\zeta)} J_0(kR)$$

$$P = (x,y,z)$$

$$Q = (\xi, \eta, \zeta)$$

$$r = [(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)]^{1/2}$$

$$r' = [(x-\xi)^2 + (y-\eta)^2 + (z+\zeta)]^{1/2}$$

$$R = [(x-\xi)^2 + (y-\eta)^2]^{1/2}$$

The final result for the integral equation is

$$\phi(P,t) + 1/2\pi \int_{S_h(t)} (\phi(Q,t)\, \partial/\partial n_Q (1/r - 1/r')\, dS = -1/2\pi \int_{S_h(t)} (1/r - 1/r')\, \partial/\partial n_Q\, \phi(Q,t))\, dS$$

$$-1/2\pi \int_{-\infty}^{t} \int_{S_h(\tau)} (\phi(Q,\tau)\, \partial/\partial n_Q\, \tilde{G}(P,Q,t,\tau) - \tilde{G}(P,Q,t,\tau)\, \partial/\partial n_Q\, \phi(Q,\tau))\, dS\, d\tau \qquad (3)$$

This is the equation which must be solved at each time step. It is a Fredholm integral equation of the 2nd kind in space and a Volterra equation in time. For the radiation problem, G, $\partial G/\partial n$, and $\partial\phi/\partial n$ are all known while $\phi(t)$ is the unknown.

At each time step, the equation is solved using a panel method with the unknown potential assumed to be constant over each quadrilateral on the body surface. The solution must be time stepped with the time integrals evaluted over the entire past history of the motion. Note that these integrals are no longer convolutions in the usual sense since they depend on t and τ explicitly and not only on the difference. Because the body position moves the system is no longer stationary and the Green function must be recomputed at each time step for all previous time. However, because the long-past history does not contribute significantly, the integration can be truncated at some point.

The hydrodynamic forces acting on the body are found by integrating the pressure as

$$\mathbf{F} = \int_{S_h} P\, \mathbf{n}\, dS \tag{4}$$

where P is the dynamic pressure determined from Bernoulli's equation in a moving coordinate system

$$P = -\rho\, (\partial\phi/\partial t\ - \mathbf{U} \bullet \nabla\phi + 1/2\, |\nabla\phi|^2\,) \tag{5}$$

As a preliminary investigation into the free surface problem, the zero and infinite frequency limits were studied. In these two cases the Green function is (1/r + 1/r') and (1/r - 1/r'), and the free surface condition reduces to $\partial\phi/\partial n = 0$ (a rigid wall), and $\phi = 0$ (a flat free surface), respectively. In Figure 1 the dotted lines show the heave added mass, calculated using the $\partial\phi/\partial t$ term in Bernoulli's equation for various depths of submergence using 514 panels on one quarter of the sphere. The x-axis is the logarithm of the distance from the top of the sphere to the surface. Also shown is an analytic solution (solid lines) due to Miloh (1977). The upper curves are for the rigid wall boundary condition, and the limiting values for small and large depth for both limits are denoted by (N)umerical and (A)naltyic.

Figure 2 shows the force on a sphere heaving beneath a free surface, calculated by three different methods. The mean depth of submergence at the center is 2R, and the amplitude of the sinusoidal motion is 0.5R, where R is the radius of the sphere. The solid line is the time-domain (TD) solution including the nonlinear pressure term. 74 panels were used on one quarter of the sphere, and the nondimensional time step was 0.3. Ferrant (1987) calculated the nonlinear force using only $\partial\phi/\partial t$ for the pressure and a frequency-domain perturbation approach. Kang and Troesch (1988) used axisymmetric curved panels and a Rankine source distribution over the body and moving free surface to solve the exact nonlinear problem in the time-domain.

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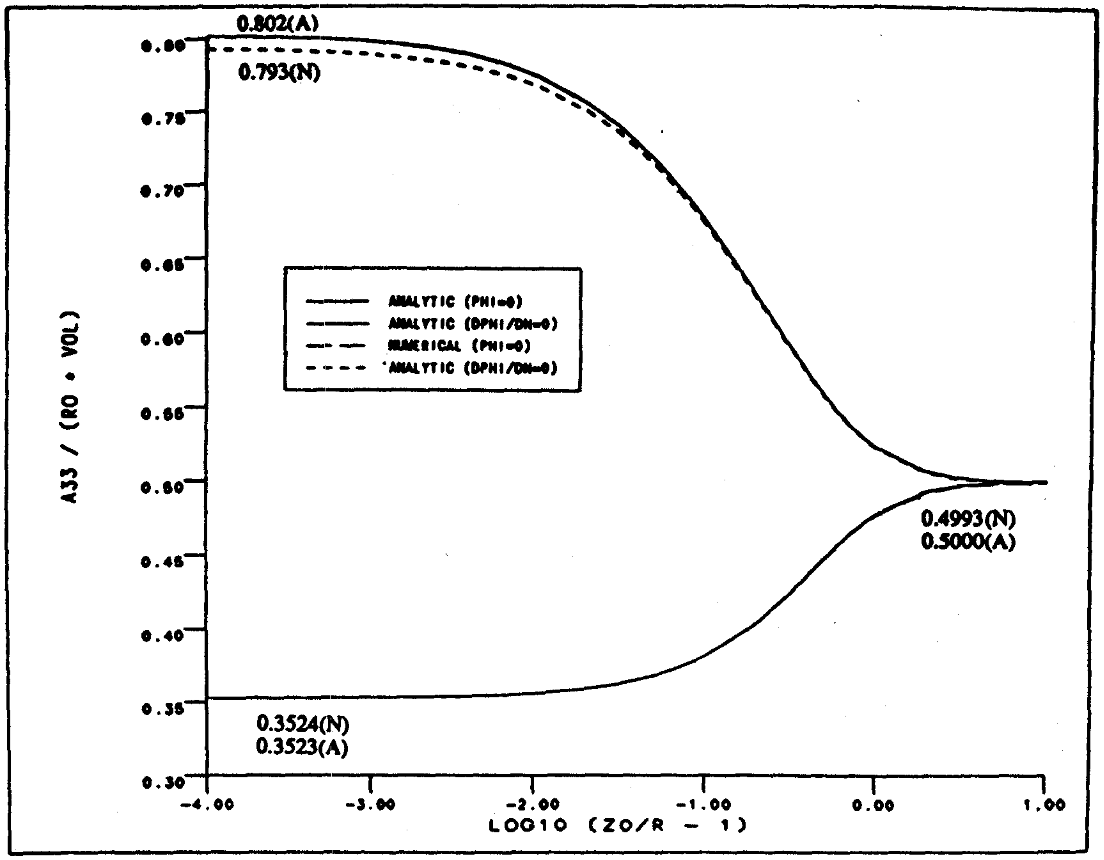

Figure 1. Heave added mass for a sphere at zero frequency (upper) and infinite frequency (lower) for various depths Z0 at the center.

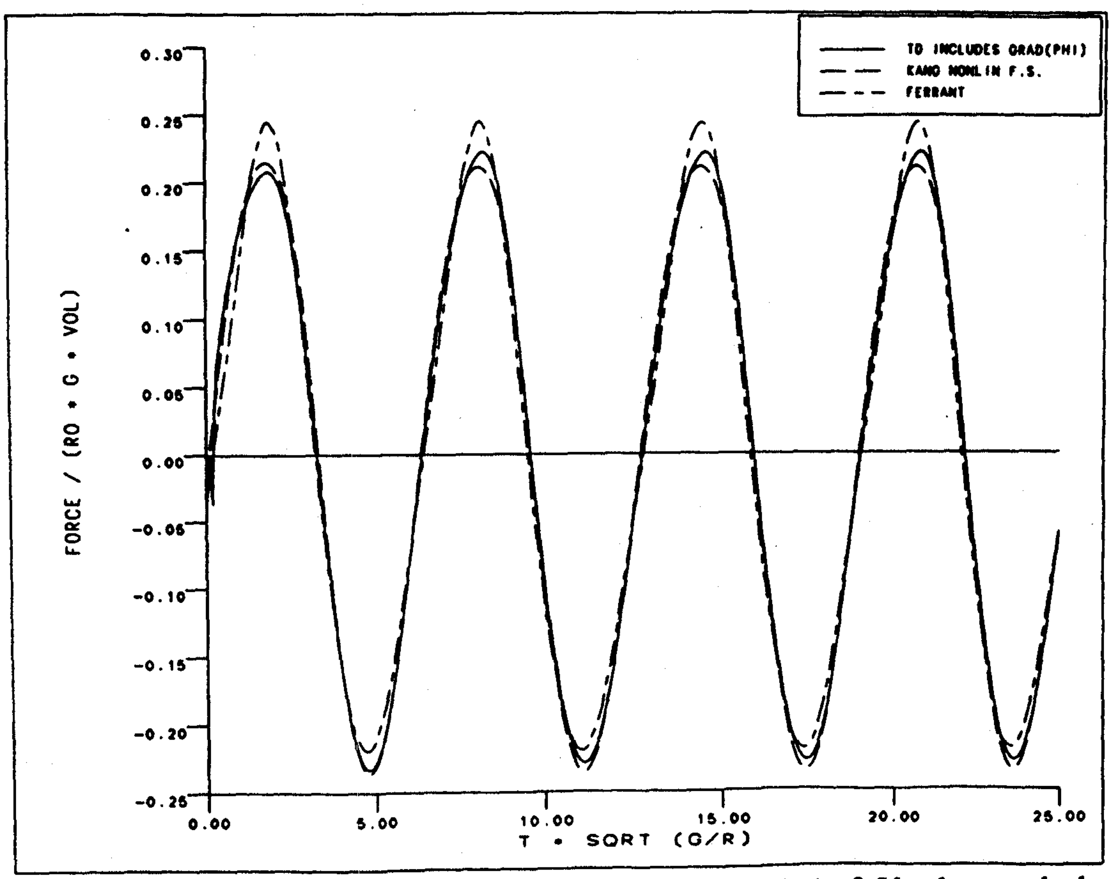

Figure 2. Force on a sphere heaving near a free surface Z0/R = 2 A= 0.5 by three methods.