Nonlinear Free-Surface Flow at a Two-Dimensional Bow

by

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This paper investigates the nonlinear behavior of the free surface at the bow of a two-dimensional, translating body. A modified Eulerian-Lagrangian technique is used to calculate the evolution of the free surface. Because the shape of the free surface is defined by Lagrangian particles, solutions in which the bow wave overturns and breaks are admissible.

Dagan & Tulin (1972) studied two-dimensional, steady flow past a semi-infinite body. Specifically, they solved the problem of flow past a vertical step by a perturbation expansion based on small Froude number up to second order. Vanden Broeck & Tuck (1977) generalized Dagan & Tulin's formulation to include plane faces of arbitrary angle and derived a method to continue the asymptotic series indefinitely. They found that the asymptotic series has a zero radius of convergence. They were able to sum the series using Shank's transformations, but the resulting free-surface profiles were only piecewise continuous. They concluded that no continuous free-surface profile exists for a bow with a plane face, and they speculated that the form of the solution is that of an overturning jet.

The possible absence of a nonlinear, steady-state solution in bow flow suggests that a formulation in the time domain is desirable. Our method is based on Vinje & Brevig (1981) with modifications made by Lin et al. (1984) for handling solid boundaries. The novelty in our approach is the treatment of the initial conditions and the far-field closure condition.

Lagrangian marker particles are distributed on both the free-surface and the far-field boundary. Evolution equations, describing how the particles move and how their potential changes, are derived for the points on the far-field boundary. The evolution equation for the potential is exact for particles on the free surface; it is approximate for particles on the far-field boundary. The pressure of the far-field particles is kept constant with time. It is shown that this assumption gives an error on the order of the inverse of the distance from the body. Calculations using this method were found to give a relative error in energy of less than 5%. This error does not accumulate.

An inviscid *double-body* solution is taken as the initial condition. It is argued that this can be a realistic condition to use in calculating the unsteady flow of a body that has been moving for a long time. The alternative of setting the potential in the fluid equal to zero at $t = 0$ and starting the body up from rest is more reasonable for studying impulsive motion. Such conventional treatment gives rise to an unbounded wave elevation at the intersection point between the body and the free surface (see, for example, Lin et al., 1984). An important consequence of using a double-body solution is that the wave elevation at the bow of the body is finite for all time. We show that our initial condition creates a jump in the pressure on the free surface at time $t = 0$, but that it does not create a jump in the potential.

Solutions for the bow-flow problem are obtained over a range of Froude numbers for bodies of three different shapes: a vertical step, a faired profile, and a bulbous bow. Sample calculations for a vertical step are shown in Figure 1.

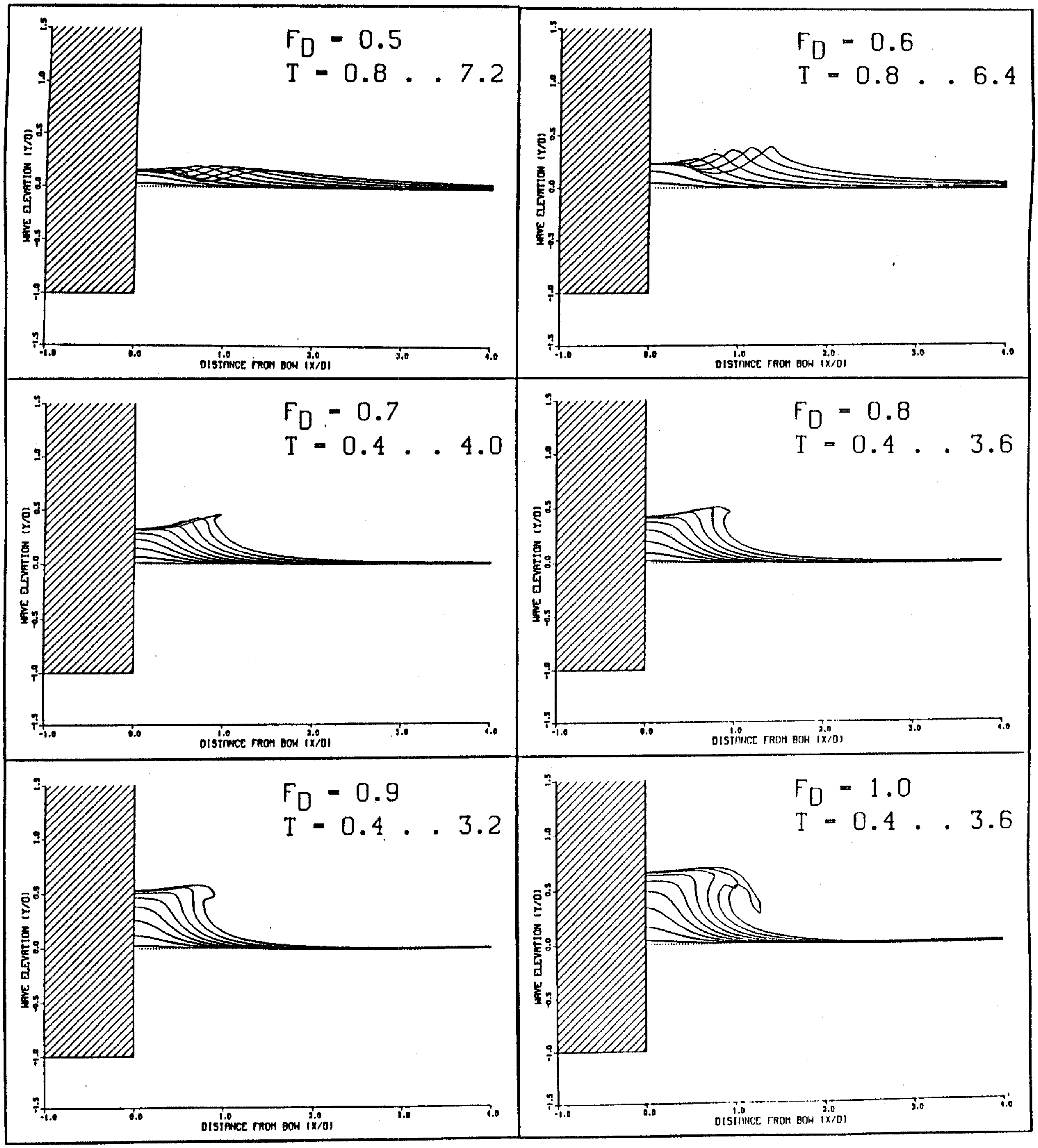

Figure 1: Bow-wave evolution for vertical step for Froude numbers F_d = 0.5 *to* 1.0 *.(Viewed in a coordinate system moving with the body.)*

A transition Froude number exists at which the bow wave begins to overturn and break. The value of the transition Froude number depends on the bow shape. The fuller the profile geometry, the larger is the value. Thus, a bulbous bow delays wave breaking. A scaling, based on the variation in the transition numbers, is proposed which successfully explains the differences in the spatial dimensions of the bow waves and the times over which the waves evolve for the three bow shapes.

For all cases studied, an isolated stagnation point is observed to be present below the free surface during the initial stage of the wave formation. For flows occurring above the transition Froude number, the stagnation point remains trapped below the free surface as the wave overturns. Below the transition Froude number, the stagnation point rises to the surface as the crest of the transient bow wave moves upstream and away from the body. These calculations suggest that, at sufficiently low Froude numbers, calculations with accurate spatial resolution could approach steady-state conditions.

References

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Wu: This is a very interesting paper, showing what some of us have contended, that this problem may generally have no steady-state limit as a solution. Would you please elaborate on the body shape factors used in your attempt to obtain a similarity form for the different solutions in the various cases tried. Another question is for Prof. Tuck: if he would provide his bow shape for which he has obtained a steady-state solution for comparison with your similarity family forms.

Grosenbaugh & Yeung: The scaling that we refer to is qualitative and not exact. The transition from a breaking to a nonbreaking flow regime occurs at different Froude numbers depending on the bow shape. In the original calculations, the Froude number was based on the draft of the body at the downstream end of the fluid domain. In these calculationns, the downstream draft was the same for all the bodies and the transition Froude number was different. If we assume instead that the transition Froude number should be the same for all bodies (i.e. the transition from breaking to nonbreaking flow regime is a self-similar feature), then we can define a local length scale that will be different depending on bow shape. We will elaborate on this qualitative similarity at the 17th Symposium on Naval Hydrodynamics this summer.

Cooker: How much computer time does a calculation take?

Grosenbaugh & Yeung: Each computer run uses approximately 400 time steps. One time step takes about 7 CPU seconds on an IBM 3090 (using 300 nodal points distributed around the boundary).

Tuck: I am glad to see some evidence that breaking always happens, since work done by my students and I over the last ten or so years has indicated that there is no steady solution except for some very special bulbous bows which do not include the bulb used by the authors.

Why is it necessary to stop the computation? Could the plunging breaker not be allowed to pass into the fluid below perhaps without even disturbing it, e.g. as if on a different Riemann sheet?

My student Madurasinghe is about to have a paper in The Journal of Ship Research giving examples of bulbous shapes for which steady flows do exist with a free surface stagnation point. I have confidence in the accuracy of these results. However, I have no proof of nonexistence of steady flows for other shapes.

In principle, one could start a flow like the authors did by suddenly melting a thin layer of ice under which there was an already-existing flow. If that layer was not plane (as the authors have assumed), the body shape was one of those computed by Madurasinghe, the Froude number was correctly chosen, and the steady flow was stable to small perturbations; then the authors' program should verify that nothing then happens!

Grosenbaugh & Yeung: Our contention is that for a given shape and a sufficiently low Froude number there is a possibility of a steady-state solution if we could have a better numerical resolution of the increasingly shorter wave lengths.

The calculations are unable to continue after the tip of the overturning wave finally touches the free surface and the fluid domain is no longer simply connected. The treatment of the reentrant plunger in the fluid requires additional work and in principle is possible. We do not think the solution would be very realistic because a reentrant jet would surely interact with the fluid and produce vorticity.

We would be happy to try (as also suggested by Prof. Wu) the waveless bow that will appear in Prof. Tuck's upcoming paper. However, both of us can be correct at the same time since in nonlinear problems the initial conditions could dictate different final states.

Yue: You have used two useful techniques for your computations. However, they pose difficulties in interpreting the results. On the one hand, your starting condition on the free surface does not allow one to obtain any understanding of the related and very important transient phenomenon associated with a wavemaker starting from rest (e.g. are there two classes of solutions, depending on the value of the acceleration, leading to breaking and nonbreaking waves?). On the other hand, since you do not find steady results (in which case the details of the starting condition may not be too critical), the physical significance of your transient result is difficult to establish. Did you try to systematically look for steady solutions (I refer to the work of Tuck and his colleagues)?

Grosenbaugh & Yeung: On the point related to the assumption of a steady-state initial condition: the new approach is taken to circumvent a classic initial condition (namely $\phi = 0$ on the free surface) that has not yielded much information in the past except that a splash will be generated. We are not interested in the wavemaker problem. We do not see any contradiction in starting with a steady-state initial condition. The initial steady state and the final steady state (if it exists) are for different problems. If one had started with an impulsive motion and a steady-state solution was achieved, it would not be clear that the same steady-state is achievable if the initial-condition is slightly varied.

Palm: In your far-field boundary condition you cancel Dp/Dt because this quantity decays as $1/R$ (R being the distance), but what about the quantities you retain, are they not also very small?

Grosenbaugh & Yeung: The far-field condition is not exact. The term which is quadratic in velocity is neglected.

Joo: You used $\phi_y = 0$ at $t = 0^+$ as the free-surface boundary condition when the lid has been removed, by arguing that ϕ_y should be continuous. However you are allowing ϕ_x to be discontinuous at $t = 0$ on the solid body which means the intersection has a jump in the x-velocity component. Can you justify this?

Grosenbaugh & Yeung: There is no jump in the velocity in the x-direction along the body and that includes the free-surface/body intersection. At $t = 0^-$ and $t = 0^+$, the velocity field is continuous.