

**AN ACCELERATED COMPUTATIONAL METHOD  
FOR TIME-DOMAIN ANALYSIS OF 3D WAVE-BODY INTERACTIONS**

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**INTRODUCTION**

Since the early work of Finkelstein (1957), the linearized time domain analysis of floating bodies has been subject to numerous studies. However, the applications have been restricted for a long time to two-dimensional geometries, (e.g. Adachi & Ohmatsu (1980)), or to bodies with axial symmetry (Newman (1985a)). Despite the recent development of efficient numerical methods for the computation of the 3D time-dependent Green function, the fully 3D time-domain models seem to have been applied only to very simple bodies such as spheres or cylinders. (Jami & Pot (1985): mixed FEM-BIE method; Beck & Liapis (1987): BIE method). This is due to the large CPU time required, during the time-stepping procedure, for the evaluation of the convolution integral, involving numerous computations of the Green function and of its derivatives.

An accelerated method for the linear, time-domain analysis of three-dimensional wave-body interactions in water of infinite depth is described. A substantial reduction of the computing time is obtained using a tabulation technique for the evaluation of the Green function and its derivatives, in place of the direct application of numerical schemes such as those described in Newman (1985b) or Beck & Liapis (1987). This improvement allows the hydrodynamic behaviour of complex structures, such as TLPs, to be analysed in the time domain. As an application of the method, the impulse response functions in heave and surge modes of a submerged torus and of the ISSC TLP have been computed. The continuous curves of added mass and damping coefficients were then evaluated by Fourier transforming the impulse functions, and compared with the results of a conventional frequency-domain panel code. Respectively 400 and 1200 panels are used in the discretization of the complete bodies.

**OUTLINE OF THE METHOD**

We consider a three-dimensional body on the free surface of a perfect fluid. Starting from rest for  $t < 0$ , a prescribed velocity is imposed to the body for  $t > 0$ . Assuming irrotational flow and small motions, a linearized problem for the velocity potential  $\phi$  is defined. Using Green's identity and the impulsive source Green function, an integral equation for the time-dependent potential on the body surface is then derived (Wehausen (1971)). This equation is solved numerically using a panel method in which the body surface is approximated by plane quadrilateral panels over which the source strength is assumed to be constant. The time variable is discretized with constant steps, and the convolution integral is evaluated using a simple trapezoidal rule. The updating of the convolution integral requires the computation of a complete set of influence coefficients at each time step, involving integrals of the time-dependent Green function and its derivatives over the panels. The complete Green function for a submerged source with a Dirac delta function strength is given by:

$$G(M, P, t) = \left( \frac{-1}{4\pi r_1} + \frac{1}{4\pi r_1} \right) \delta(t) - \frac{H(t)}{2\pi} \int_0^\infty (gk)^{1/2} \sin[(gk)^{1/2} t] J_0(kR) e^{k(z+z')} dk$$

$$= G_0(M, P) \cdot \delta(t) + H(t) g^{1/2} r_1^{-3/2} \tilde{G}(\cos\theta, \tau)$$

where  $M(x, y, z)$  and  $P(x', y', z')$  are respectively the source and the field points,  $r_1 = |PM_1|$  with  $M_1$  symmetric point of  $M$  with respect to the free surface,  $R = [(x-x')^2 + (y-y')^2]^{1/2}$ ,  $\cos\theta = -(z+z')/r_1$ ,  $\tau = t(g/r_1)^{1/2}$ ,  $\delta$  the Dirac delta function and  $H$  the Heaviside step function.

Newman (1985b) and Beck & Liapis (1987) gave various formulas which may be used to compute  $\tilde{G}$  and its derivatives, depending on the values of the parameters  $\cos\theta$  and  $\tau$ . Despite the advantage of these formulas over ordinary quadrature methods in terms of computational efficiency, their direct use in fully 3D time domain models leads to highly time-consuming computer codes. The application of such numerical models to complex structures, with accurate space and time discretizations, seems therefore to be only possible on supercomputers. The major computational burden being the evaluation of the Green function, the first problem to be addressed is the minimization of the CPU time required for this task.

Taking into account the fact that the non-trivial terms to be evaluated may be reduced to the bivariate function  $\tilde{G}$  and its derivatives, we developed a tabulation procedure for these functions. The 2D domain described by  $\cos\theta$  ( $0 < \cos\theta < 1$ .) and  $\tau$  ( $0 < \tau < \infty$ ) is truncated at a large value  $\tau_{max}$ , and the remaining bounded domain is mapped by a discrete set of equispaced  $(\cos\theta, \tau)$  points for which  $\tilde{G}$  and its derivatives are computed by numerical schemes very similar to those described by Newman (1985b). This tabulation procedure has to be performed only once, and the results are stored in permanent data files. When a time-domain simulation is performed for a given body geometry and motion, the resulting computations of the Green function and its derivatives are replaced by first order bivariate interpolations of the stored data. The tabulated part of the variables plane is sufficiently extended to allow the use of a large time asymptotic expansion if  $\tau > \tau_{max}$ . However, the use of this particular procedure only occurs for a very small part of the computation, when the mutual influence coefficients of panels intersecting the free surface are evaluated at large time, and these few extra computations do not affect the overall efficiency of the method.

The large saving in CPU time obtained using this tabulation technique allows time-domain analysis to be performed on complex 3D structures within a reasonable computing time.

## RESULTS AND COMMENTS

Results presented in this paper concern the application of the code to the determination of the heave and surge hydrodynamic coefficients of two different bodies, using the impulse response method. All computations were run on a VAX 8700 scalar computer.

The first one is a submerged torus, which may be regarded as the submerged part of a semi-submersible. This body has already been studied in the frequency domain by Hulme (1985). We give in figures 1 and 2 the heave and surge impulse response functions of the body, discretized by a total of 400 panels (Fig. 9). For the heave mode, the cyclic symmetry of the discretization has been taken into account to avoid redundant evaluations of influence coefficients. The reduced time step is  $dt=0.1$ , and  $Nt=200$  time steps have been used for the computation. The CPU time for this case is of about 2 minutes. For the surge mode, only two symmetries are exploited, and the code has been run for  $dt=0.2$  and  $Nt=150$ . The required CPU time is of about 20 minutes. Figures 3 and 4 exhibit the continuous curves of added mass and damping coefficients obtained by Fourier transforming the impulse response functions. The discrete points have been obtained by a direct computation of the coefficients using a conventional frequency-domain panel code, with the same body discretization. The agreement is very satisfactory for both modes. A version of the code with direct computation of the Green function and its derivatives has been run on these cases. No significant discrepancy has been observed on the results, compared to that of the accelerated code.

At last, the code has been applied to the well-documented ISSC TLP (Eatock-Taylor & Jefferys 1986). A total of 1200 panels has been used to discretize this rather complex body (see Fig. 10). We give in figure 5 and 6 the heave impulse response function and the corresponding frequency-domain hydrodynamic coefficients obtained by Fourier transform. For this case,  $dt=.12$  and  $Nt=200$ . The CPU time was 3h30. Figures 7 and 8 give the same results for the surge mode, obtained with  $dt=.20$  and  $Nt=200$ , and

the same CPU time. Note that the impulse response functions for all modes may be obtained in a single run of the program, with a small extra CPU requirement compared to a single mode computation, provided enough out-of-core storage is available.

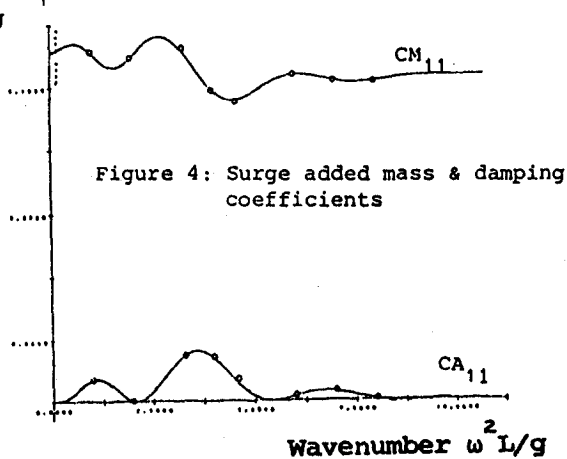
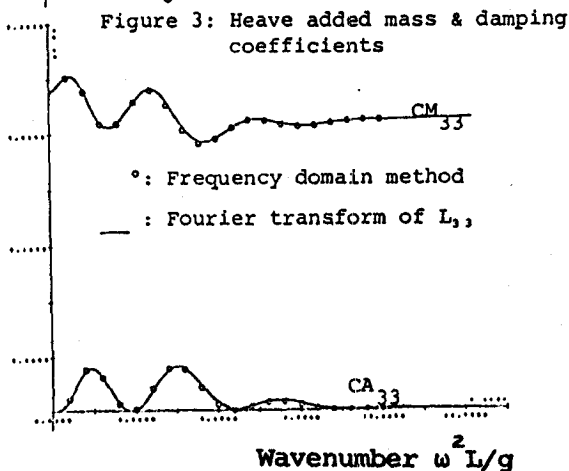
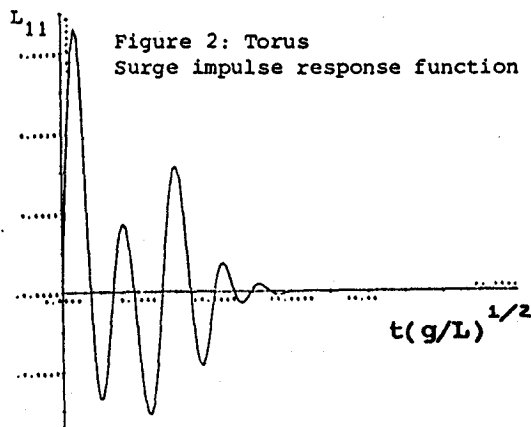
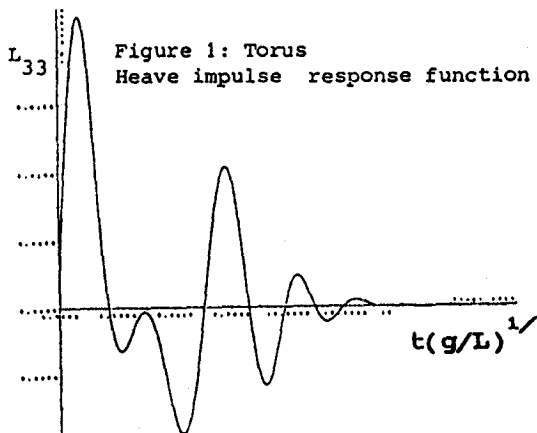
A perspective view of the  $\tilde{G}$ , plotted as a function of  $\cos\theta$  and  $\tau^2$ , is given in figure 11, for  $0 \leq \tau^2 \leq 150$ .

A striking feature of the method is that a large part of the CPU time is now dedicated to the computation of the convolution integral. Since this part of the computation is directly subject to vectorization, new significant savings are anticipated when running the program on a vector computer. Improvements of the tabulation technique are also planned, combining finer tabulations and simpler interpolation schemes.

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## REFERENCES

- 1- ADACHI A., OHMATSU S. (1980): 13<sup>th</sup> Symp. on Naval Hydrodynamics. Washington.
- 2- BECK R.F., LIAPIS S. (1987): J. S. R., Vol.31 n°3. Sept. 1987. pp. 164-176.
- 3- EATOCK-TAYLOR R., JEFFERYS E.R. (1986): Ocean Engg. Vol. 13., pp.449-490.
- 4- FINKELSTEIN A. B. (1957): Comm. on Pure and Applied Math. Vol. 10, pp. 511-522.
- 5- HULME A. (1985): J. Fluid Mech., Vol. 155, pp. 511-530.
- 6- JAMI A., POT G. (1985): 4<sup>th</sup> Int. Conf. on Num. Ship Hydrodynamics. Washington.
- 7- NEWMAN J.N. (1985): J. Fluid Mech., Vol. 157. pp 17-33.
- 8- NEWMAN J.N. (1985): 4<sup>th</sup> Int. Conf. on Numerical Ship Hydrodynamics. Washington.
- 9- WEHAUSEN J.V. (1971): The motion of floating bodies. Annual Review of Fluid Mechanics. Vol. 3, pp. 237-268.



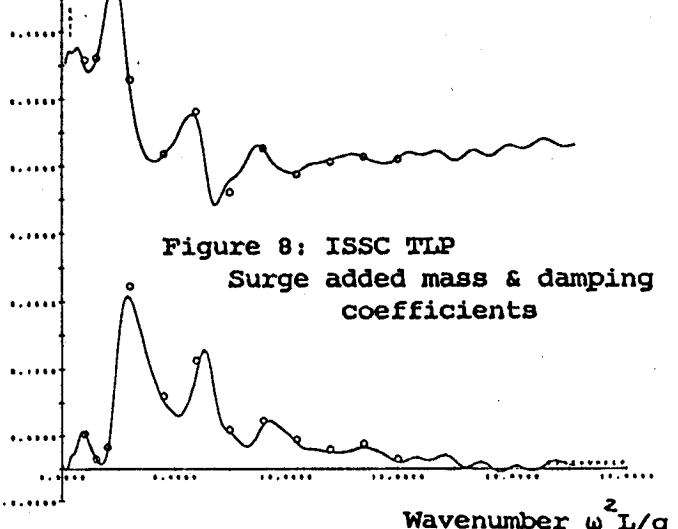
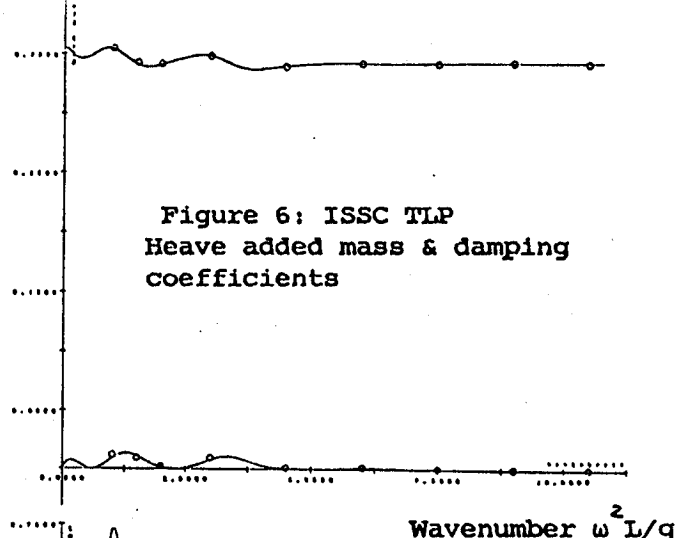
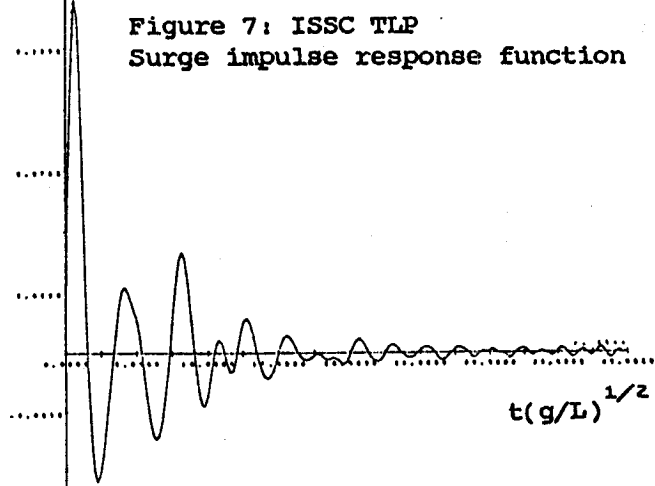
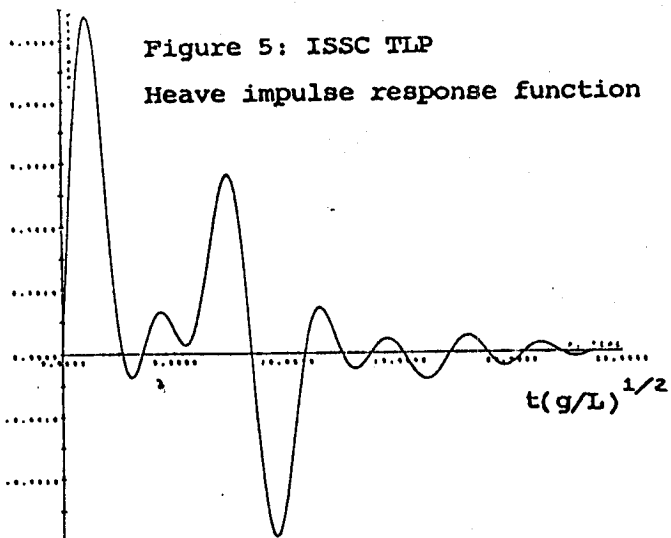


Figure 10:

TLP discretization

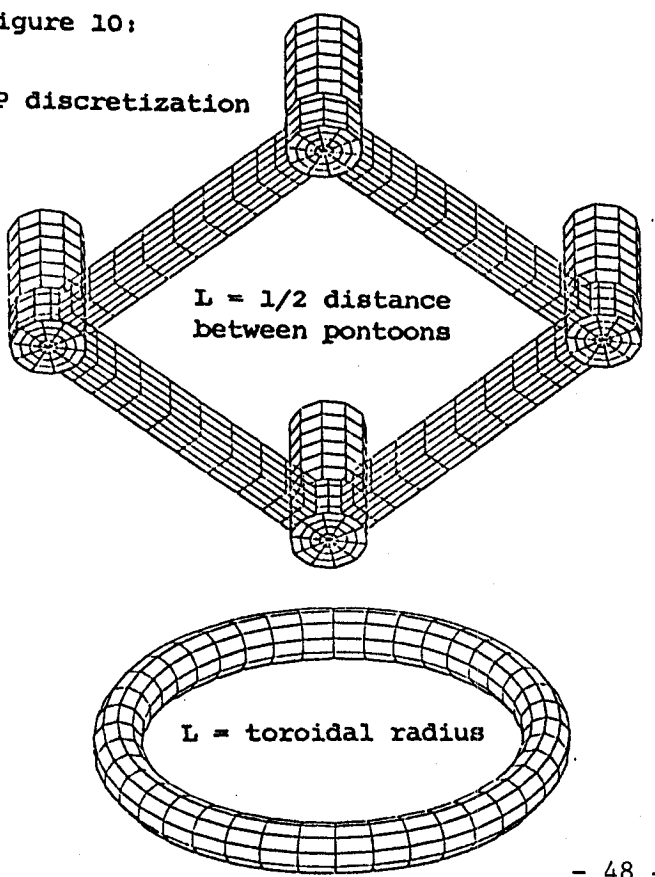


Figure 9: Torus discretization  
( see Hulme [5] for the dimensions )

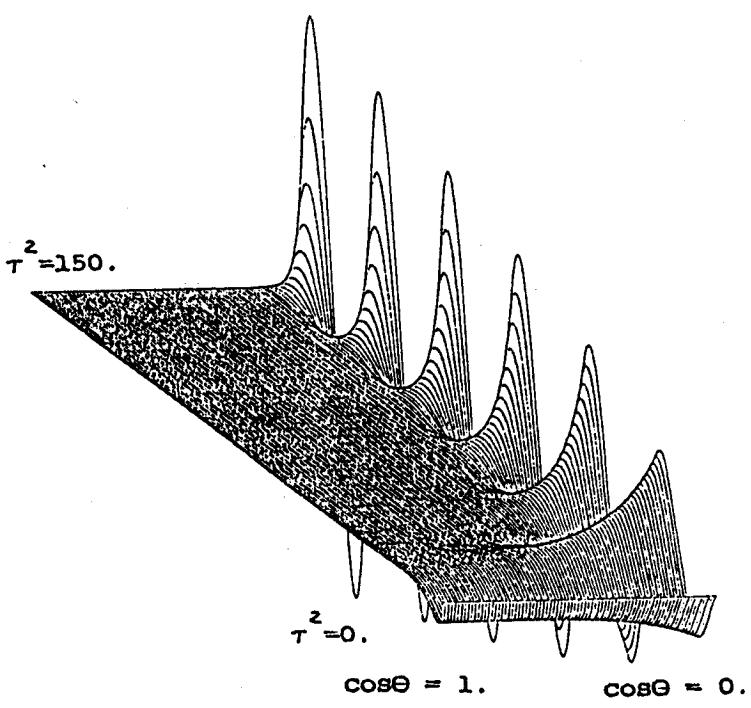


Figure 11: Plot of  $\tilde{G}(\cos\theta, \tau^2)$