SECOND ORDER VELOCITY POTENTIAL FOR ARBITRARY BODIES IN WAVES

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1 INTRODUCTION

It is well known that second order effects may in many cases be important for the nonlinear hydrodynamic problem arising in ocean engineering. Despite considerable efforts having been made in the past in calculating second order unsteady forces, similar studies are rare for the actual second order velocity potential itself, which is important for the understanding of wave kinematics. A mathematical model has been developed for the calculation of the second order velocity potential for arbitrary bodies. Attention is paid here to the development of fast and accurate numerical solutions.

2 FORMULATION OF THE PROBLEM

Within the framework of potential flow and based on a second order perturbation theory, similar formulations for both the first order and second order problems can be derived in the frequency domain. However, unlike the first order problem the second order problem is characterized by an inhomogeneous free surface boundary condition, involving quadratic products of the first order potentials and their derivatives. It is this inhomogeneous free surface boundary condition that imposes a considerable computational burden on the numerical implementation of the second order theory.

3 FIRST ORDER SOLUTION

It is believed that a first step towards the solution of the second order problem is the accurate evaluation of the first order potential. Through the classical Green function, G, and the new integral identities proposed by Noblesse [1], the first order problem can be recast into an integral equation over the boundary of the body. This integral equation is then solved by the Boundary Element Method, with quadratic elements and use of the program 'FINGREEN' for evaluation of the Green function. In comparison with the classical Green's second identity, Noblesse's formulation permits a weakening, but not complete removal, of the singularities inherent in the numerical integration. Those singularities may be further weakened by employing a triangle polar coordinate transformation [2] about the singular elements. The idea is based on a sequence of coordinate mappings, which not only reduces the degree of singularity of the boundary integral but also enables standard numerical integration to be performed on a square such that the integration points represent the mapping which lumps the points toward the singularity in the original domain. This feature is evidently advantageous for the numerical computations. Numerical results for a vertical circular cylinder with $ka=1.4$, $a/d=1$ are given in table 1, giving the distribution of the first order potential on the surface of the cylinder. It is observed that the comparison with the MacCamy and Fuch's analytic solution is in good agreement even when only two boundary elements are used in one quadrant (two divisions in the circumferential direction per quadrant)

z/d	θ^{o}	ANALYTICAL SOLUTION Re	ANALYTICAL SOLUTION Im	NUMERICAL SOLUTION Re	NUMERICAL SOLUTION Im
0.0	0	-0.1788E+0	0.2572E+0	-0.1798E+0	0.2587E+0
0.0	45	-0.7715E-1	0.1144E+0	-0.7677E-1	0.1150E+0
0.0	90	-0.7104E-1	-0.9172E-1	-0.6994E-1	-0.9184E-1
0.0	135	-0.1878E+0	-0.1906E-1	-0.1880E+0	-0.1852E-1
0.0	180	-0.2090E+0	0.7424E-1	-0.2098E+0	0.7541E-1
-1.0	0	-0.8312E-1	0.1196E+0	-0.8350E-1	0.1196E+0
-1.0	45	-0.3587E-1	0.5317E-1	-0.3582E-1	0.5323E-1
-1.0	90	-0.3303E-1	-0.4264E-1	-0.3253E-1	-0.4245E-1
-1.0	135	-0.8732E-1	-0.8859E-2	-0.8713E-1	-0.8730E-2
-1.0	180	-0.9716E-1	0.3452E-1	-0.9733E-1	0.3464E-1

Table 1 : first order results for a fixed vertical circular cylinder based on two boundary elements per quadrant

4 SECOND ORDER SOLUTION

For the second order problem, the solution procedures are quite similar to the method used for the first order problem except that special techniques are required to calculate efficiently the additional free surface integral which decays slowly to infinity in a highly oscillatory manner. In the present study, the free surface is divided into two regions in which the integrals are treated differently. The first region, S_f, is bounded by the waterline Γ_w and a circular external boundary Γ_i. Within this region, the free surface is discretized into planar panels and integrated by numerical quadrature. Outside this region, a simplification is possible by exploiting vertical axisymmetry, which allows one to develop the first order potential and Green function as Fourier series in the polar angle θ. After integration in the circumferential direction and use of orthogonality for each Fourier mode, the 2D free surface integral can be reduced to a series of 1D radial line integrals in which integration can be performed numerically. In order to speed up convergence, numerical quadrature is only employed up to a finite range but complemented by an analytical integration to infinity. This infinite integral can be evaluated by employing Hankel's asympotic expansions of the different kinds of Bessel functions in each term of the integrand. Eventually, the integrand can be represented by summations of polynomials with various orders. Integration of each term of the polynomials is found to satisfy a simple recurrence relationship from which its value can be easily calculated.

Another difficulty encountered in the second order solution is the treatment of the double derivative of the first order potential, which occurs in one of the free surface integral terms. High accuracy of this double derivative is rather difficult to achieve, especially in the region around the body, because the first order potential itself is obtained numerically. Using Laplace's equation and integration by parts in two dimensions(Green's theorem), it is, however possible to express the integral containing the double derivative as one containing only first order derivatives plus two line integrals taken along the boundaries of S_f. Considering one of the typical terms as an example, we use the transformation

$$\iint_{S_f} \phi \frac{\partial^2 \phi_s}{\partial x^2} G \, dxdy = - \iint_{S_f} \frac{\partial \phi_s}{\partial x} \left[\frac{\partial \phi}{\partial x} G + \frac{\partial G}{\partial x} \phi \right] dxdy + \oint_{\Gamma_w + \Gamma_j} \phi G \frac{\partial \phi_s}{\partial x} n_x \, d\Gamma$$

$$= - \iint_{S_f} \frac{\partial \phi_s}{\partial x} \frac{\partial \phi}{\partial x} G \, dxdy$$

$$- \iint_{S_f} \left[\phi \frac{\partial \phi_s}{\partial x} - \phi(\xi) \frac{\partial \phi_s(\xi)}{\partial x} \right] \frac{\partial G}{\partial x} \, dxdy$$

$$+ \oint_{\Gamma_w + \Gamma_j} \left[\phi \frac{\partial \phi_s}{\partial x} - \phi(\xi) \frac{\partial \phi_s(\xi)}{\partial x} \right] G \, n_x \, d\Gamma$$

The right hand side of this expression is evidently more efficient to compute than the left hand side, and the singularity when $\vec{x} \rightarrow \vec{\xi}$ has been weakened.

5 NUMERICAL RESULTS

Based on the above formulation, numerical results for the distribution of the second order velocity potential on the surface of a fixed vertical circular cylinder have been obtained. They are in good agreement with results from the explicit method [3] which has been developed earlier by using a different approach. Typical comparisons are shown in Fig. 1 based on the element mesh shown in Fig. 2. Encouraged by this, more runs will be made for various geometries in the near future so as to assess the validity of the present numerical model, and to provide some light on the physical interpretation of the second order theory.

6 REFERENCES

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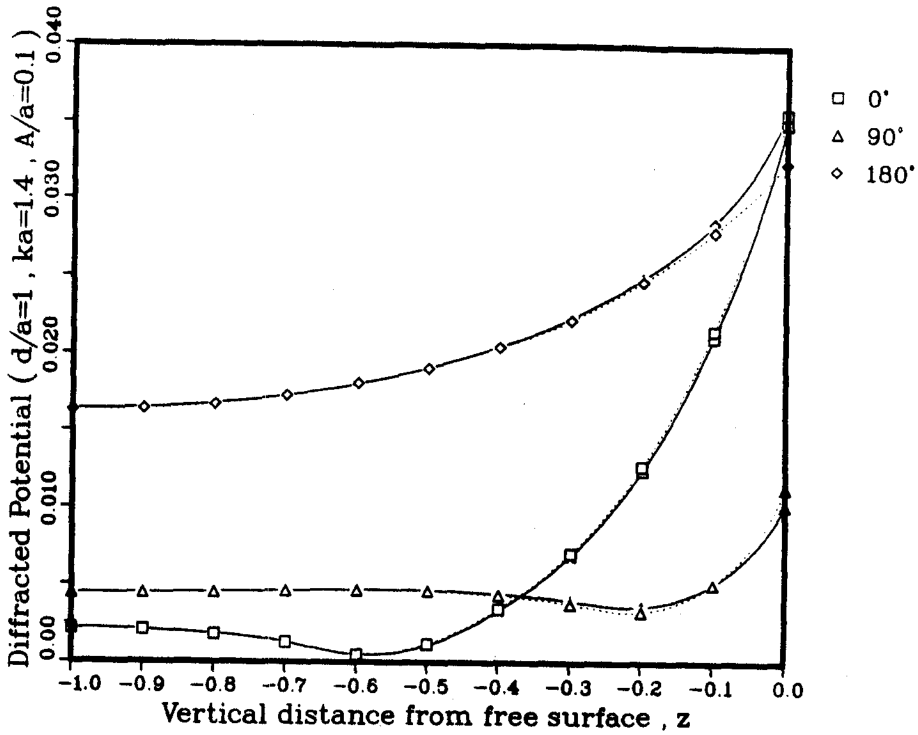

Figure 1 Distribution of $|\phi_s^{(2)}|$ along three generators of a vertical cylinder : ——— numerical ; .... direct

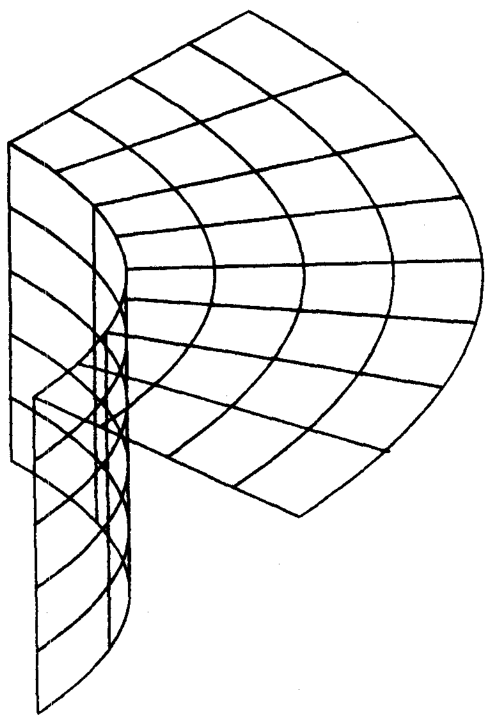

Figure 2 Mesh for cylinder, showing body surface and free surface elements

Kleinman: Since the integral equation for the second-order potential involves a free-surface integral, would it not simplify the numerical computation to employ the simple source (or combination of simple sources) rather than the Green function which eliminates the free-surface integral in the first-order problem?

Chau: If only Rankine sources are used for the second order problem, they must be distributed over the entire boundary of the fluid domain, including the radiation boundary which may not be simply truncated at some finite distance from the body surface. Hence, as suggested by Korsmeyer, this would increase the number of unknowns in the discretized integral equation. Moreover, the main difficulty in evaluating the free surface Green function has been overcome by employing the subroutine "FINGREEN."

Kim:

1. From our experience, the convergence of the asymptotic expansion of the Hankel function for large n is not uniform, and one may need many terms in the series to have reasonable accuracy. How many Fourier modes and terms in the asymptotic expansion did you use for your numerical result?

2. The determination of the partition between the asymptotic and analytical evaluations of the free-surface integral is complicated by the fact that the asymptotic expansion depends on each Fourier mode. How did you determine the partition radius?

Chau: Outside the 2-D numerical integration region (with radius equal to few times the water depth), the free-surface integral can be reduced to a series of 1-D infinite line integrals with respect to each Fourier mode. These integrals may be split in such a way that numerical evaluations are only required over finite intervals, while complemented by explicit integrations up to infinity employing asymptotic expansions of the Hankel functions. Because these expansions are not uniform with respect to order, the bound for the finite range, Rm, is an increasing function of the Fourier mode. From numerical experience, a good choice of Rm may be $(m+2)$ times half the incident wave length. Since Hankel expansions are an asymptotic series, high accuracy of the approximation can be obtained by truncating at their optimal values. The maximum number of Fourier modes used in our analysis may be as high as 15.

Papanikolaou: In the evaluation of the second-order, free-surface inhomogeneity, you divided the free surface into two regions, one near the body and the second one outside a circle of a certain radius where analytical integration is performed on the basis of Fourier expressions of the related functions. Are these expressions valid for the non-axisymmetric case?

Chau: This method of evaluating the free-surface integral is equally valid for arbitrary bodies, since in the inner region, the integration is performed by 2-D quadrature. Indeed, calculations are now in progress for general body shapes.